Test 2

Q1

9-3 Find the Laplace Transform of:

$$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t).$$

Q2

3-20-b By using Partial Fraction, find the Inverse Laplace Transform of:

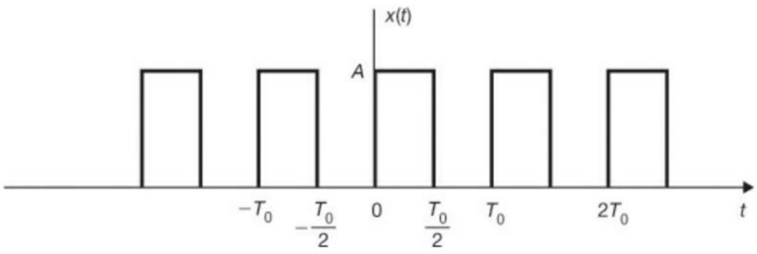
$$X(s) = \frac{s^2 + 6s + 7}{s^2 + 3s + 2}, \ \mathrm{Re}(s) > -1$$

(hint: change to this form first……)

$$\frac{s^2 + 6s + 7}{s^2 + 3s + 2} = 1 + \frac{\dots\dots}{s^2 + 3s + 2}$$

Q3

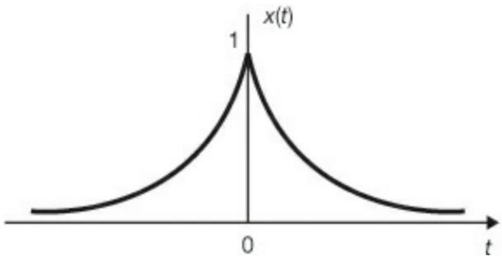
5-5-a Find the Complex Fourier Series of the waveform shown below:



Q4

5-21 Find the Fourier Transform of the signal shown below:

$$x(t) = e^{-a|t|} \qquad a > 0$$



SOLUTION

Q1

$$X(s) = \int_{-\infty}^{\infty} \left[3e^{-2t}u(t) - 2e^{-t}u(t)\right] e^{-st}\, dt$$
$$= 3\int_{-\infty}^{\infty} e^{-2t}e^{-st}u(t)\, dt - 2\int_{-\infty}^{\infty} e^{-t}e^{-st}u(t)\, dt.$$

From the lookup table…

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}.$$

To determine the ROC we note that $x(t)$ is a sum of two real exponentials, and from eq. (9.21) we see that $X(s)$ is the sum of the Laplace transforms of each of the individual terms. The first term is the Laplace transform of $3e^{-2t}u(t)$ and the second term the Laplace transform of $-2e^{-t}u(t)$. From Example 9.1, we know that

$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, \qquad \mathcal{R}e\{s\} > -1,$$
$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \qquad \mathcal{R}e\{s\} > -2.$$

The set of values of $\mathcal{R}e\{s\}$ for which the Laplace transforms of both terms converge is $\mathcal{R}e\{s\} > -1$, and thus, combining the two terms on the right-hand side of eq. (9.22), we obtain

$$3e^{-2t}u(t) - 2e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s-1}{s^2+3s+2}, \quad \mathcal{R}e\{s\} > -1. \qquad (9.23)$$

Q2

(*b*) Performing long division, we have

$$X(s) = \frac{s^2+6s+7}{s^2+3s+2} = 1 + \frac{3s+5}{s^2+3s+2} = 1 + \frac{3s+5}{(s+1)(s+2)}$$

$$X_1(s) = \frac{3s+5}{(s+1)(s+2)} = \frac{c_1}{s+1} + \frac{c_2}{s+2}$$

where

$$c_1 = (s+1)X_1(s)\big|_{s=-1} = \frac{3s+5}{s+2}\bigg|_{s=-1} = 2$$
$$c_2 = (s+2)X_1(s)\big|_{s=-2} = \frac{3s+5}{s+1}\bigg|_{s=-2} = 1$$

Hence,

$$X(s) = 1 + \frac{2}{s+1} + \frac{1}{s+2}$$

The ROC of $X(s)$ is $\mathrm{Re}(s) > -1$. Thus, $x(t)$ is a right-sided signal and from Table 3-1 we obtain

$$x(t) = \delta(t) + (2e^{-t} + e^{-2t})u(t)$$

Q3

(*a*) Let

$$x(t)=\sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t} \qquad \omega_0=\frac{2\pi}{T_0}$$

Using Eq. (5.102a), we have

$$\begin{aligned} c_k &= \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_0^{T_0/2} A\, e^{-jk\omega_0 t}\, dt \\ &= \frac{A}{-jk\omega_0 T_0} e^{-jk\omega_0 t}\Big|_0^{T_0/2} = \frac{A}{-jk\omega_0 T_0}(e^{-jk\omega_0 T_0/2}-1) \\ &= \frac{A}{jk2\pi}(1-e^{-jk\pi}) = \frac{A}{jk2\pi}\left[1-(-1)^k\right] \end{aligned}$$

since $\omega_0 T_0 = 2\pi$ and $e^{-jk\pi} = (-1)^k$. Thus,

$$\begin{aligned} c_k &= 0 && k=2m\neq 0 \\ c_k &= \frac{A}{jk\pi} && k=2m+1 \\ c_0 &= \frac{1}{T_0}\int_0^{T_0} x(t)\, dt = \frac{1}{T_0}\int_0^{T_0/2} A\, dt = \frac{A}{2} \end{aligned}$$

Hence,

$$c_0=\frac{A}{2} \qquad c_{2m}=0 \qquad c_{2m+1}=\frac{A}{j(2m+1)\pi}$$

and we obtain

$$x(t)=\frac{A}{2}+\frac{A}{j\pi}\sum_{m=-\infty}^{\infty}\frac{1}{2m+1}e^{j(2m+1)\omega_0 t}$$

Q4

Signal $x(t)$ can be rewritten as

$$x(t)=e^{-a|t|}=\begin{cases} e^{-at} & t>0 \\ e^{at} & t<0 \end{cases}$$

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{0} e^{at}e^{-j\omega t}\, dt + \int_0^{\infty} e^{-at}e^{-j\omega t}\, dt \\ &= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_0^{\infty} e^{-(a+j\omega)t}\, dt \\ &= \frac{1}{a-j\omega}+\frac{1}{a+j\omega}=\frac{2a}{a^2+\omega^2} \end{aligned}$$

Hence, we get

$$e^{-a|t|} \leftrightarrow \frac{2a}{a^2+\omega^2} \qquad (5.138)$$

The Fourier transform $X(\omega)$ of $x(t)$ is shown in Fig. 5-18(*b*).(5.138)

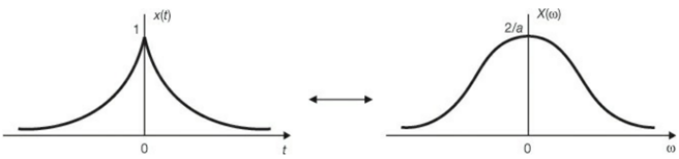


Fig. 5-18 $e^{-|a|t}$ and its Fourier transform.