Tutorial

Q1

A continuous time band pass filter has the following equation:

$$H(s) = \frac{2s}{s^2 + 2s + 100}$$

Synthesis the above filter into digital form *H(z)*, by using bi-linear transformation.

Q2

(b) Implement the following difference equation into a direct form 1(DF-1) IIR filter.

$$y(n) + \frac{2}{3}y(n-1) - \frac{1}{5}y(n-2) = x(n) - \frac{1}{2}x(n-1)$$

(c) Convert your DF-1 IIR filter in (b), to a direct form 2 (DF-2) IIR filter.

Q3

Implement the following IIR filter with the minimum hardware:

$$H(z) = \frac{\sum_{n=0}^{M} b(n)z^{-n}}{1 + \sum_{n=1}^{N} a(n)z^{-n}} \quad M=3;\ N=3$$

SOLUTION

Q1

$$H(s) = \frac{2s}{s^2 + 2s + 100}$$

The appropriate bilinear transformation is

$$s = \frac{20(z-1)}{z+1}$$

Substitution of this relation into the original s-domain transfer function yields

$$H(z) = \frac{\dfrac{40(z-1)}{z+1}}{400\left(\dfrac{z-1}{z+1}\right)^2 + (2)(20)\left(\dfrac{z-1}{z+1}\right) + 100}$$

Upon cleaning up the algebra yields a z-domain transfer function is

$$H(z) = \frac{0.0740(z-1)(z+1)}{z^2 - 1.111z + 0.8519}$$

Q2 (a)

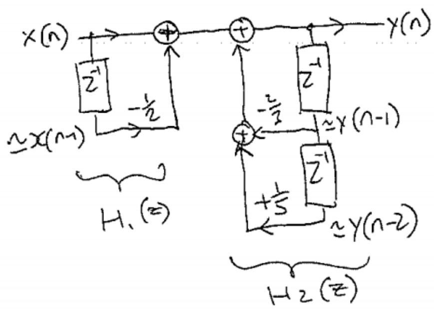
$$H(z) = H_1(z) \cdot H_2(z)$$
$$= \frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)}$$

z transform

$$Y(z) + \frac{2}{3} z^{-1} Y(z) - \frac{1}{5} z^{-2} Y(z) = X(z) - \frac{1}{2} z^{-1} X(z)$$

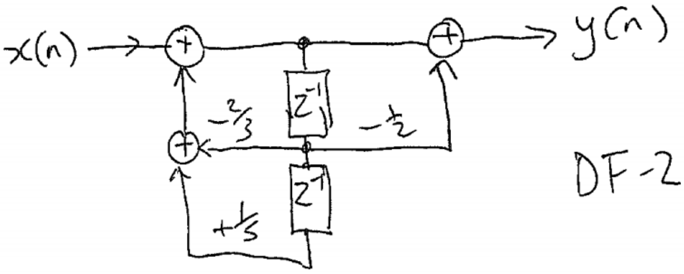
$$Y(z)\left[1 + \frac{2}{3} z^{-1} - \frac{1}{5} z^{-2}\right] = X(z)\left[1 - \frac{1}{2} z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\overset{a_0}{1} - \overset{a_1}{\frac{1}{2}} z^{-1}}{\underset{b_0}{1} + \underset{b_1}{\frac{2}{3}} z^{-1} - \underset{b_2}{\frac{1}{5}} z^{-2}} \quad \begin{matrix} \} \to H_1(z) \\ \} \to H_2(z) \end{matrix}$$



DF-1

(b)



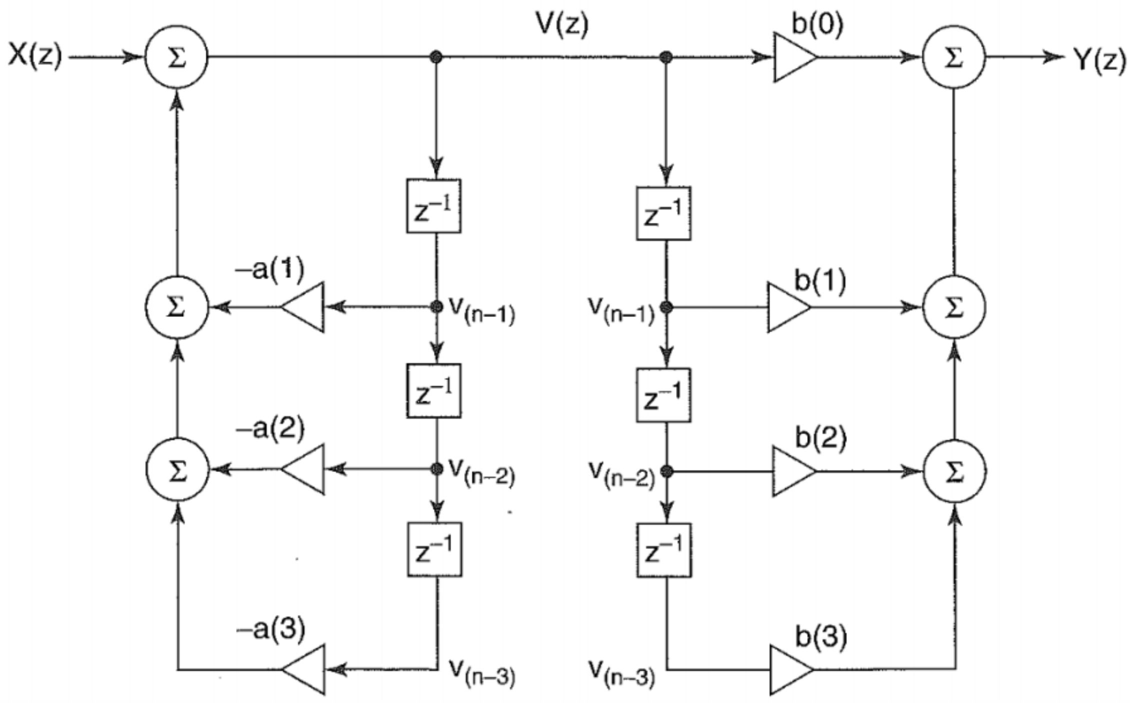
<u>Q3</u>

$$H(z) = \frac{\sum_{n=0}^{M} b(n)z^{-n}}{1 + \sum_{n=1}^{N} a(n)z^{-n}}$$

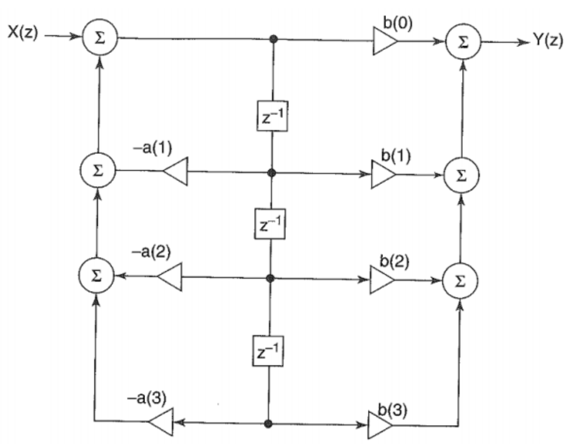
$$= H_1(z)H_2(z) = \left[\frac{1}{1 + \sum_{n=1}^{N} a(n)z^{-n}}\right]\left[\sum_{n=0}^{M} b(n)z^{-n}\right]$$

$$\text{H2(z)} = b_0 + b(1)z^{-1} + b(2)z^{-2} + b(3)z^{-3}$$

$$H_1(z) = \frac{1}{1 + a(1)z^{-1} + a(2)z^{-2} + a(3)z^{-3}}$$



Further simplify….



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