Q1

6.5. Consider a sequence

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$$

(*a*) Sketch $x[n]$.

(*b*) Find the Fourier coefficients c_k of $x[n]$.

Q2

6.6. Determine the discrete Fourier series representation for each of the following sequences:

(*a*) $x[n] = \cos \frac{\pi}{4} n$

(*b*) $x[n] = \cos \frac{\pi}{3} n + \sin \frac{\pi}{4} n$

(*c*) $x[n] = \cos^2 \left(\frac{\pi}{8} n \right)$

Q3

6.11. Find the Fourier transform of

$x[n] = -a^n u[-n - 1]$ a real

Q4

6.12. Find the Fourier transform of the rectangular pulse sequence (Fig. 6-10)

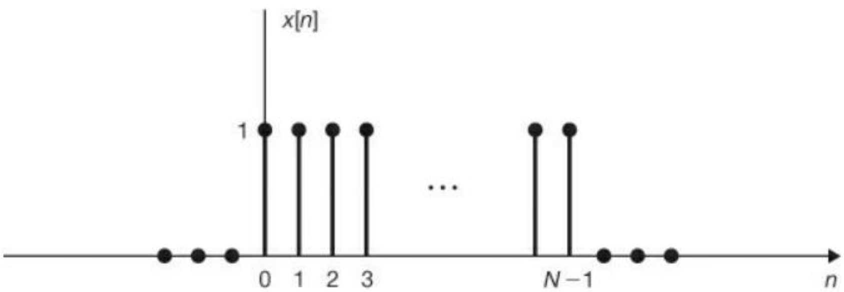

Fig. 6-10

Solution Q1

(a) The sequence $x[n]$ is sketched in Fig. 6-9(a). It is seen that $x[n]$ is the periodic extension of the sequence {1, 0, 0, 0} with period $N_0 = 4$.

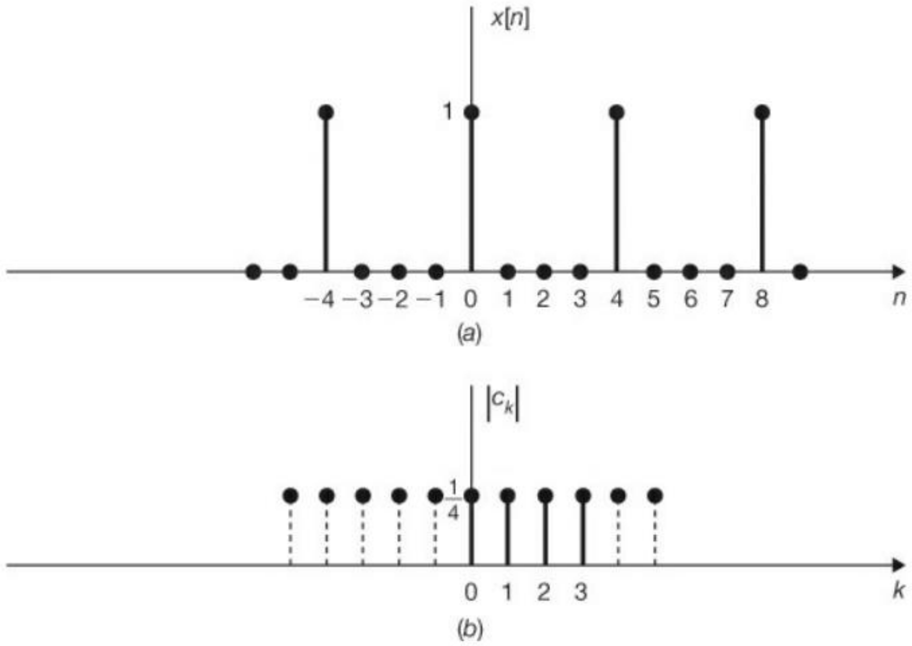

Fig. 6-9

(b) From Eqs. (6.7) and (6.8) and Fig. 6-9(a) we have

$$x[n] = \sum_{k=0}^{3} c_k\, e^{jk(2\pi/4)n} = \sum_{k=0}^{3} c_k\, e^{jk(\pi/2)n}$$

and
$$c_k = \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-jk(2\pi/4)n} = \frac{1}{4} x[0] = \frac{1}{4} \qquad \text{all } k$$

since $x[1] = x[2] = x[3] = 0$. The Fourier coefficients of $x[n]$ are sketched in Fig. 6-9(b).

Q2

(a) The fundamental period of $x[n]$ is $N_0 = 8$, and $\Omega_0 = 2\pi/N_0 = \pi/4$. Rather than using Eq. (6.8) to evaluate the Fourier coefficients c_k, we use Euler's formula and get

$$\cos\frac{\pi}{4}n = \frac{1}{2}(e^{j(\pi/4)n} + e^{-j(\pi/4)n}) = \frac{1}{2}e^{j\Omega_0 n} + \frac{1}{2}e^{-j\Omega_0 n}$$

Thus, the Fourier coefficients for $x[n]$ are $c_1 = \frac{1}{2}$, $c_{-1} = c_{-1+8} = c_7 = \frac{1}{2}$, and all other c_k 0. Hence, the discrete Fourier series of $x[n]$ is

$$x[n] = \cos\frac{\pi}{4}n = \frac{1}{2}e^{j\Omega_0 n} + \frac{1}{2}e^{j7\Omega_0 n} \qquad \Omega_0 = \frac{\pi}{4}$$

(b) From Prob. 1.16(i) the fundamental period of $x[n]$ is $N_0 = 24$, and Ω_0 $2\pi/N_0 = \pi/12$. Again by Euler's formula we have

$$x[n] = \frac{1}{2}(e^{j(\pi/3)n} + e^{-j(\pi/3)n}) + \frac{1}{2j}(e^{j(\pi/4)n} - e^{-j(\pi/4)n})$$
$$= \frac{1}{2}e^{-j4\Omega_0 n} + j\frac{1}{2}e^{-j3\Omega_0 n} - j\frac{1}{2}e^{j3\Omega_0 n} + \frac{1}{2}e^{j4\Omega_0 n}$$

Thus, $c_3 = -j(\frac{1}{2}), c_4 = \frac{1}{2}, c_{-4} = c_{-4+24} = c_{20} = \frac{1}{2}, c_{-3} = c_{-3+24} = c_{21} = j(\frac{1}{2})$, and all other $c_k = 0$. Hence, the discrete Fourier series of $x[n]$ is

$$x[n] = -j\frac{1}{2}e^{j3\Omega_0 n} + \frac{1}{2}e^{j4\Omega_0 n} + \frac{1}{2}e^{j20\Omega_0 n} + j\frac{1}{2}e^{j21\Omega_0 n} \qquad \Omega_0 = \frac{\pi}{12}$$

(c) From Prob. 1.16(j) the fundamental period of $x[n]$ is $N_0 = 8$, and $\Omega_0 = 2\pi/N_0 = \pi/4$. Again by Euler's formula we have

$$x[n] = \left(\frac{1}{2}e^{j(\pi/8)n} + \frac{1}{2}e^{-j(\pi/8)n}\right)^2 = \frac{1}{4}e^{j(\pi/4)n} + \frac{1}{2} + \frac{1}{4}e^{-j(\pi/4)n}$$
$$= \frac{1}{4}e^{j\Omega_0 n} + \frac{1}{2} + \frac{1}{4}e^{-j\Omega_0 n}$$

Thus, $c_0 = \frac{1}{2}, c_1 = \frac{1}{4}, c_{-1} = c_{-1+8} = c_7 = \frac{1}{4}$, and all other $c_k = 0$. Hence, the discrete Fourier series of $x[n]$ is

$$x[n] = \frac{1}{2} + \frac{1}{4}e^{j\Omega_0 n} + \frac{1}{4}e^{j7\Omega_0 n} \qquad \Omega_0 = \frac{\pi}{4}$$

Q3

From Eq. (4.12) the z-transform of $x[n]$ is given by

$$X(z) = \frac{1}{1 - az^{-1}} \qquad |z| < |a|$$

Thus, $X(e^{j\Omega})$ exists for $|a| > 1$ because the ROC of $X(z)$ then contains the unit circle. Thus,

$$X(\Omega) = X(e^{j\Omega}) = \frac{1}{1 - ae^{-j\Omega}} \qquad |a| > 1 \tag{6.130}$$

Q4

$$x[n] = u[n] - u[n - N]$$

Using Eq. (1.90), the z-transform of $x[n]$ is given by

$$X(z) = \sum_{n=0}^{N-1} z^n = \frac{1 - z^N}{1 - z} \qquad |z| > 0 \tag{6.131}$$

Thus, $X(e^{j\Omega})$ exists because the ROC of $X(z)$ includes the unit circle. Hence,

$$X(\Omega) = X(e^{j\Omega}) = \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} = \frac{e^{-j\Omega N/2}(e^{j\Omega N/2} - e^{j\Omega N/2})}{e^{-j\Omega/2}(e^{j\Omega/2} - e^{-j\Omega/2})}$$

$$= e^{-j\Omega(N-1)/2} \frac{\sin(\Omega N/2)}{\sin(\Omega/2)} \tag{6.132}$$

Q5