1-03-h Tutorial

Question 1

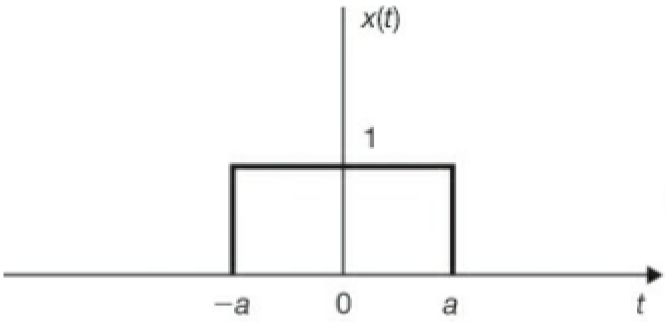
5.16. Verify the time-shifting property of Fourier Transform

$$x(t-t_0) \leftrightarrow e^{j\omega t_0} X(\omega)$$

Question 2

5.19. Find the Fourier transform of the rectangular pulse signal x(t) defined by

$$x(t) = p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases} \tag{5.135}$$



Question 3

5.20. From Q2, find the Fourier transform of the signal

$$x(t) = \frac{\sin at}{\pi t}$$

Question 4

5.23. Find the Fourier transforms of the following signals:

(*a*) $x(t) = 1$

(*b*) $x(t) = e^{-j\omega_0 t}$

(*c*) $x(t) = e^{-j\omega_0 t}$

(*d*) $x(t) = \cos \omega_0 t$

(*e*) $x(t) = \sin \omega_0 t$

Solution

Q 1

By definition (5.31)

$$\mathscr{F}\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0)e^{-j\omega t}\, dt$$

By the change of variable $\tau = t - t_0$, we obtain

$$\mathscr{F}\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(\tau)e^{-j\omega(\tau+t_0)}\, d\tau$$
$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}\, d\tau = e^{-j\omega t_0}\, X(\omega)$$

Hence,

$$x(t-t_0) \leftrightarrow e^{-j\omega t_0}\, X(\omega)$$

Q2

$$x(t) = p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases} \tag{5.135}$$

By definition (5.31)

$$X(\omega) = \int_{-\infty}^{\infty} p_a(t)e^{-j\omega t}\, dt = \int_{-a}^{a} e^{-j\omega t}\, dt$$
$$= \frac{1}{j\omega}(e^{j\omega a} - e^{-j\omega a}) = 2\frac{\sin \omega a}{\omega} = 2a\frac{\sin \omega a}{\omega a}$$

Hence, we obtain

$$p_a(t) \leftrightarrow 2\frac{\sin \omega a}{\omega} = 2a\frac{\sin \omega a}{\omega a} \tag{5.136}$$

The Fourier transform $X(\omega)$ of $x(t)$ is sketched in Fig. 5-16(b).

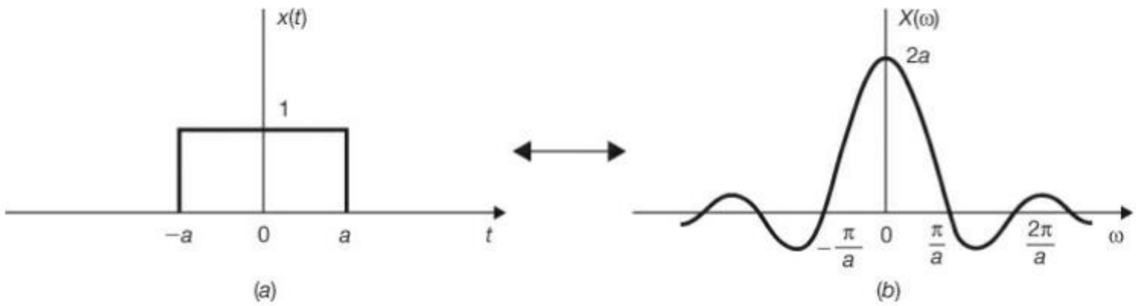


Fig. 5-16 Rectangular pulse and its Fourier transform.

Q3

$$x(t) = \frac{\sin at}{\pi t}$$

From Eq. (5.136) we have

$$p_a(t) \leftrightarrow 2\frac{\sin \omega a}{\omega}$$

Now by the duality property (5.54), we have

$$2\frac{\sin at}{t} \leftrightarrow 2\pi p_a(-\omega)$$

Dividing both sides by 2π (and by the linearity property), we obtain

$$\frac{\sin at}{\pi t} \leftrightarrow p_a(-\omega) = p_a(\omega) \tag{5.137}$$

where $p_a(\omega)$ is defined by [see Eq. (5.135) and Fig. 5-17(*b*)]

$$p_a(\omega) = \begin{cases} 1 & |\omega| < a \\ 0 & |\omega| > a \end{cases}$$

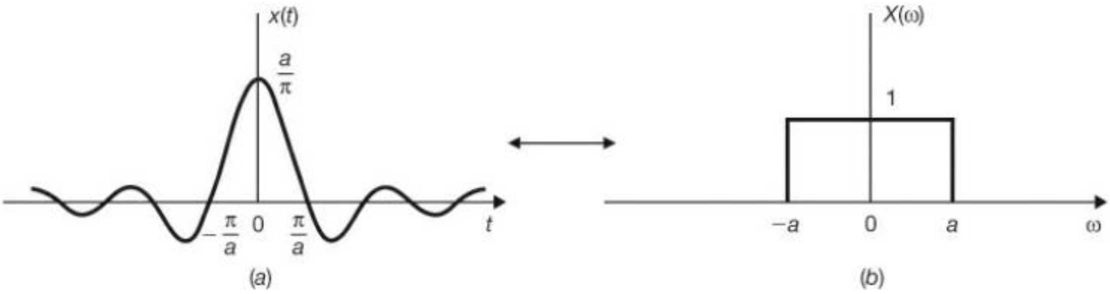


Fig. 5-17 sin at/πt and its Fourier transform.

Q4

(*a*) By Eq. (5.43) we have

$$\delta(t) \leftrightarrow 1 \tag{5.140}$$

Thus, by the duality property (5.54) we get

$$1 \leftrightarrow 2\pi\delta(-\omega) = 2\pi\delta(\omega) \tag{5.141}$$

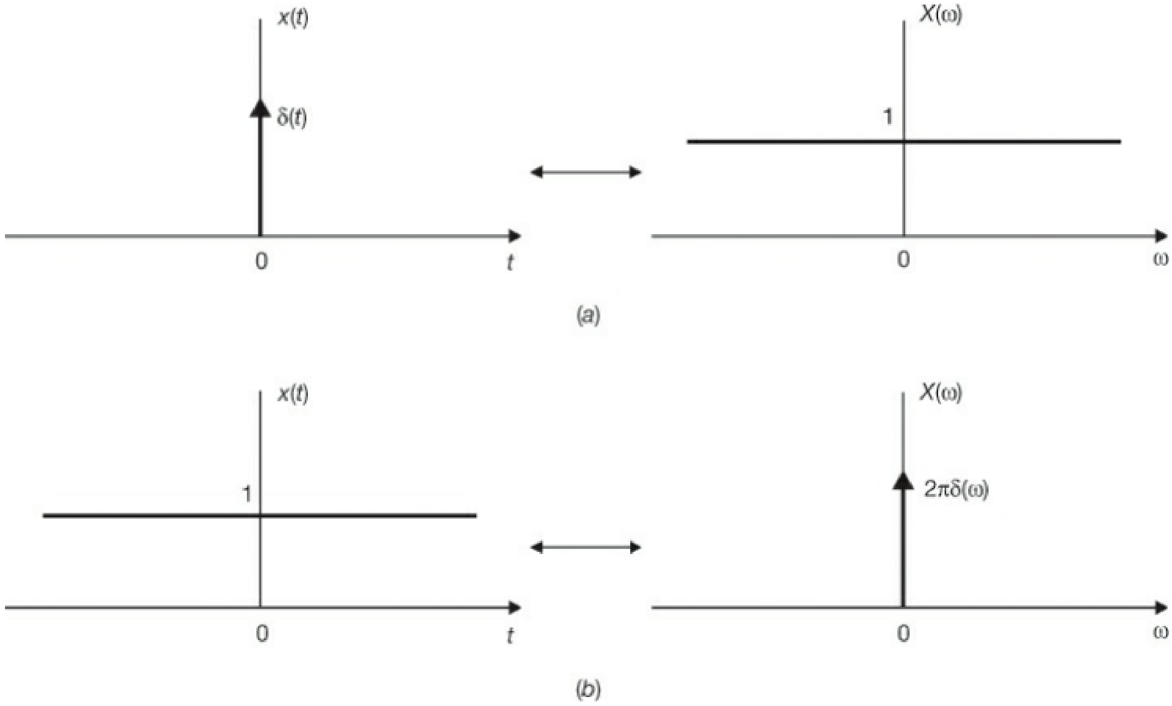


Fig. 5-20 (*a*) Unit impulse and its Fourier transform; (*b*) constant (dc) signal and its Fourier transform.

(*b*) Applying the frequency-shifting property (5.51) to Eq. (5.141), we get

$$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0) \tag{5.142}$$

(*c*) From Eq. (5.142), it follows that

$$e^{-j\omega_0 t} \leftrightarrow 2\pi\delta(\omega + \omega_0) \tag{5.143}$$

(*d*) From Euler's formula we have

$$\cos \omega_0 t = \frac{1}{2}(e^{j\omega_0 t} + e^{j\omega_0 t})$$

Thus, using Eqs. (5.142) and (5.143) and the linearity property (5.49), we get

$$\cos \omega_0 t \leftrightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \tag{5.144}$$

Fig. 5-21 illustrates the relationship in Eq. (5.144).

(*e*) Similarly, we have

$$\sin \omega_0 t = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$$

and again using Eqs. (5.142) and (5.143), we get

$$\sin \omega_0 t \leftrightarrow -j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] \tag{5.145}$$

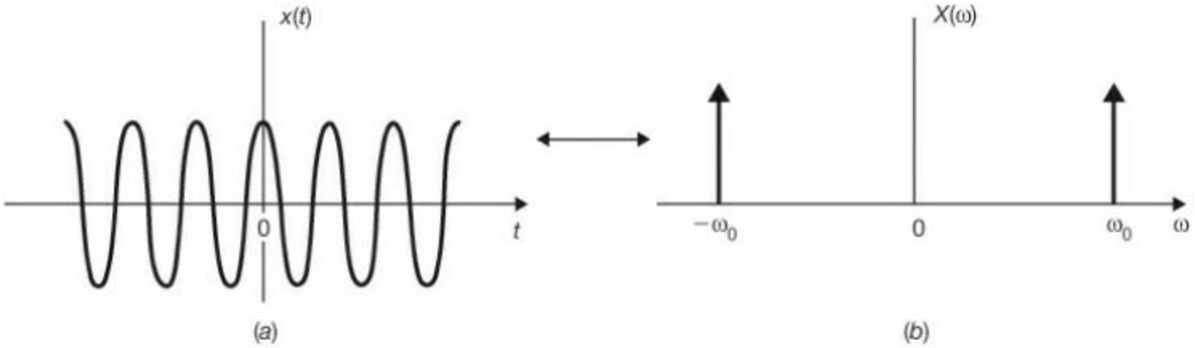


Fig. 5-21 Cosine signal and its Fourier transform.