Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 03-h

Fourier Transform

Contents

Reference:
Signals and Systems 2nd Edition – Oppenheim, Willsky
Schaum's Outline Series: Signals and Systems

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1. The Fourier Transform

A. From Fourier Series to Fourier Transform:

Let $x(t)$ be a nonperiodic signal of finite duration; that is,

$$x(t) = 0 \quad |t| > T_1$$

Such a signal is shown in Fig. 5-1(*a*). Let $x_{T_0}(t)$ be a periodic signal formed by repeating $x(t)$ with fundamental period T_0 as shown in Fig. 5-1(*b*). If we let $T_0 \to \infty$, we have

$$\lim_{T_0 \to \infty} x_{T_0}(t) = x(t) \tag{5.22}$$

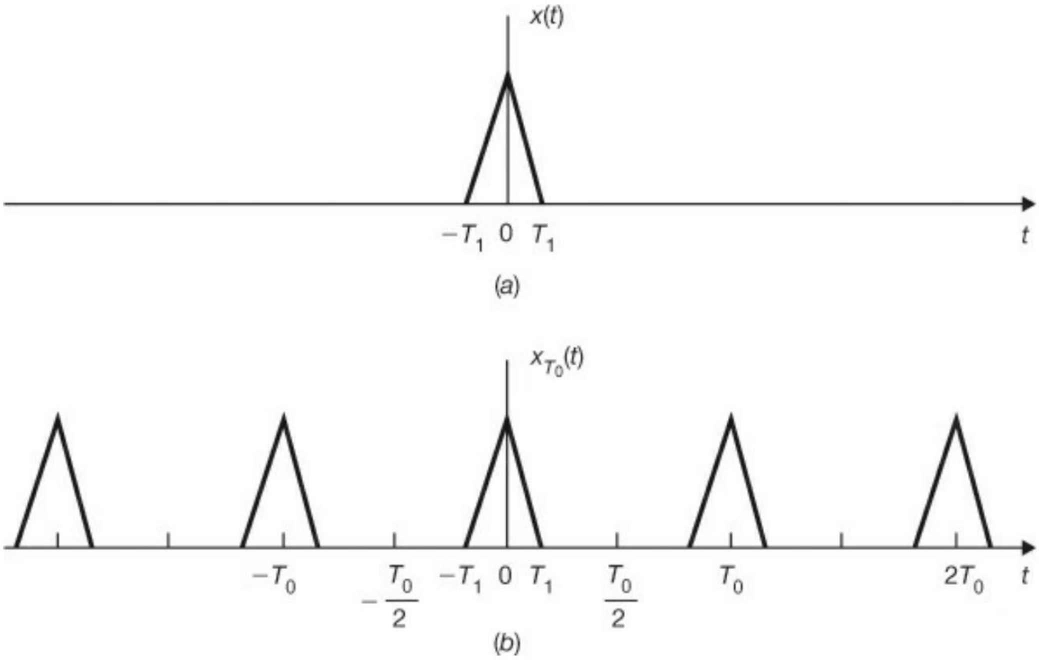

Fig. 5-1 (*a*) Nonperiodic signal $x(t)$; (*b*) periodic signal formed by periodic extension of $x(t)$.

The complex exponential Fourier series of $x_{T_0}(t)$ is given by

$$x_{T_0}(t) = \sum_{k=-\infty}^{\infty} C_k \, e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0} \tag{5.23}$$

where

$$c_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x_{T_0}(t) \, e^{-jk\omega_0 t} \, dt \tag{5.24a}$$

Since $x_{T0}(t) = x(t)$ for $|t| < T_0/2$ and also since $x(t) = 0$ outside this interval, Eq. (5.24a) can be rewritten as

$$c_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\infty}^{\infty} x(t)\, e^{-jk\omega_0 t}\, dt \tag{5.24b}$$

Let us define $X(\omega)$ as

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \tag{5.25}$$

Then from Eq. (5.24b) the complex Fourier coefficients c_k can be expressed as

$$c_k = \frac{1}{T_0} X(k\omega_0) \tag{5.26}$$

Substituting Eq. (5.26) into Eq. (5.23), we have

$$x_{T_0}(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} X(k\omega_0)\, e^{jk\omega_0 t}$$

$$x_{T_0}(t) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(k\omega_0)\, e^{jk\omega_0 t}\omega_0 \tag{5.27}$$

or

As $T_0 \to \infty$, $\omega_0 = 2\pi/T_0$ becomes infinitesimal ($\omega_0 \to 0$). Thus, let $\omega_0 = \Delta\omega$. Then Eq. (5.27) becomes

$$x_{T_0}(t)\Big|_{T_0\to\infty} \to \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(k\Delta\omega)\, e^{jk\Delta\omega t}\Delta\omega \tag{5.28}$$

Therefore,

$$x(t) = \lim_{T_0\to\infty} x_{T_0}(t) = \lim_{\Delta\omega\to 0} \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(k\Delta\omega)\, e^{jk\Delta\omega t}\Delta\omega \tag{5.29}$$

The sum on the right-hand side of Eq. (5.29) can be viewed as the area under the function $X(\omega)\, e^{j\omega t}$, as shown in Fig. 5-2. Therefore, we obtain

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \tag{5.30}$$

which is the Fourier representation of a nonperiodic $x(t)$.

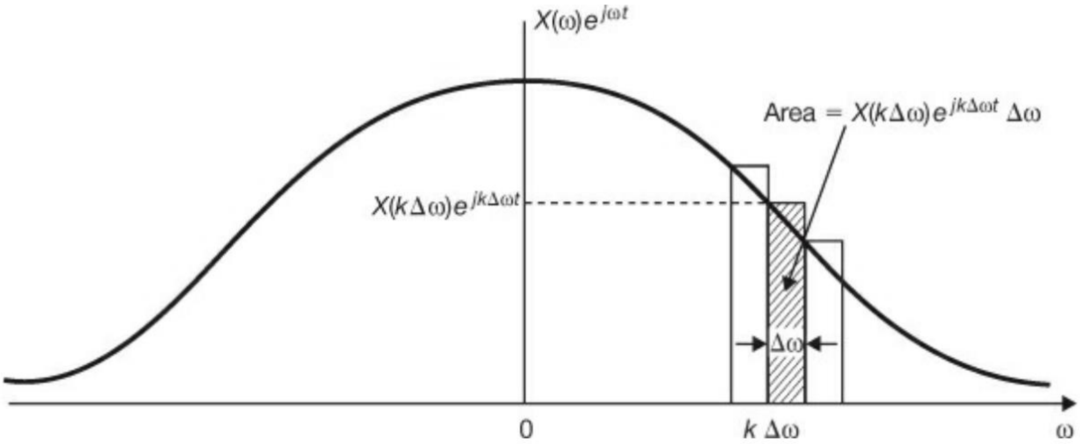

Fig. 5-2 Graphical interpretation of Eq. (5.29).

B. Fourier Transform Pair:

The function $X(\omega)$ defined by Eq. (5.25) is called the *Fourier transform* of $x(t)$, and Eq. (5.30) defines the *inverse Fourier transform of* $X(\omega)$. Symbolically they are denoted by

$$X(\omega) = \mathscr{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{j\omega t}\, dt \tag{5.31}$$

$$x(t) = \mathscr{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega \tag{5.32}$$

and we say that $x(t)$ and $X(\omega)$ form a Fourier transform pair denoted by

$$x(t) \leftrightarrow X(\omega) \tag{5.33}$$

C. Fourier Spectra:

The Fourier transform $X(\omega)$ of $x(t)$ is, in general, complex, and it can be expressed as

$$X(\omega) = |X(\omega)|\, e^{j\phi(\omega)} \tag{5.34}$$

By analogy with the terminology used for the complex Fourier coefficients of

a periodic signal $x(t)$, the Fourier transform $X(\omega)$ of a nonperiodic signal $x(t)$ is the frequency-domain specification of $x(t)$ and is referred to as the *spectrum* (or *Fourier spectrum*) of $x(t)$. The quantity $|X(\omega)|$ is called the *magnitude spectrum* of $x(t)$, and $\varphi(\omega)$ is called the *phase spectrum* of $x(t)$.

If $x(t)$ is a real signal, then from Eq. (5.31) we get

$$X(-\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t}\, dt \tag{5.35}$$

Then it follows that

$$|X(-\omega)| = |X(\omega)| \qquad \phi(-\omega) = -\phi(\omega) \tag{5.36b}$$

and

$$|X(-\omega)| = |X(\omega)| \qquad \phi(-\omega) = -\phi(\omega) \tag{5.36b}$$

Hence, as in the case of periodic signals, the amplitude spectrum $|X(\omega)|$ is an even function and the phase spectrum $\varphi(\omega)$ is an odd function of ω.

D. Convergence of Fourier Transforms:

Just as in the case of periodic signals, the sufficient conditions for the convergence of $X(\omega)$ are the following (again referred to as the Dirichlet conditions):

1. $x(t)$ is absolutely integrable; that is,

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty \tag{5.37}$$

2. $x(t)$ has a finite number of maxima and minima within any finite interval.
3. $x(t)$ has a finite number of discontinuities within any finite interval, and each of these discontinuities is finite.

E. Connection between the Fourier Transform and the Laplace Transform:

Equation (5.31) defines the Fourier transform of $x(t)$ as

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{5.38}$$

The bilateral Laplace transform of $x(t)$, as defined in Eq. (4.3), is given by

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{5.39}$$

Comparing Eqs. (5.38) and (5.39), we see that the Fourier transform is a special case of the Laplace transform in which $s = j\omega$; that is,

$$X(s)\big|_{s=j\omega} = \mathscr{F}\{x(t)\} \tag{5.40}$$

Setting $s = \sigma + j\omega$ in Eq. (5.39), we have

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma + j\omega)t}\,dt = \int_{-\infty}^{\infty} [x(t)e^{-\sigma t}]\,e^{-j\omega t}\,dt$$

or

$$X(\sigma + j\omega) = \mathscr{F}\{x(t)e^{-\sigma t}\} \tag{5.41}$$

which indicates that the bilateral Laplace transform of $x(t)$ can be interpreted as the Fourier transform of $x(t)\,e^{-\sigma t}$.

Since the Laplace transform may be considered a generalization of the Fourier transform in which the frequency is generalized from $j\omega$ to $s = \sigma + j\omega$, the complex variable s is often referred to as the *complex frequency*.

2. The Properties of Continuous Time Fourier Transform

A. Linearity:

$$a_1x_1(t) + a_2x_2(t) \leftrightarrow a_1X_1(\omega) + a_2X_2(\omega) \tag{5.49}$$

B. Time Shifting:

$$x(t - t_0) \leftrightarrow e^{-j\omega t_0}X(\omega) \tag{5.50}$$

Equation (5.50) shows that the effect of a shift in the time domain is simply to add a linear term $-\omega t_0$ to the original phase spectrum $\theta(\omega)$. This is known as a *linear phase shift* of the Fourier transform $X(\omega)$.

C. Frequency Shifting:

$$e^{j\omega_0 t}x(t) \leftrightarrow X(\omega - \omega_0) \tag{5.51}$$

The multiplication of $x(t)$ by a complex exponential signal $e^{j\omega_0 t}$ is sometimes called *complex modulation*. Thus, Eq. (5.51) shows that complex modulation in the time domain corresponds to a shift of $X(\omega)$ in the frequency domain. Note that the frequency-shifting property Eq. (5.51) is the dual of the time-shifting property Eq. (5.50).

D. Time Scaling:

$$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \tag{5.52}$$

where a is a real constant. This property follows directly from the definition of the Fourier transform. Equation (5.52) indicates that scaling the time variable t by the factor a causes an inverse scaling of the frequency variable ω by $1/a$, as well as an amplitude scaling of $X(\omega/a)$ by $1/|a|$. Thus, the scaling property (5.52) implies that time compression of a signal $(a > 1)$ results in its spectral expansion and that time expansion of the signal $(a < 1)$ results in its spectral compression.

E. Time Reversal:

$$x(-t) \leftrightarrow X(-\omega) \tag{5.53}$$

Thus, time reversal of $x(t)$ produces a like reversal of the frequency axis for $X(\omega)$. Equation (5.53) is readily obtained by setting $a = -1$ in Eq. (5.52).

F. Duality (or Symmetry):

$$X(t) \leftrightarrow 2\pi x(-\omega) \tag{5.54}$$

The duality property of the Fourier transform has significant implications. This property allows us to obtain both of these dual Fourier transform pairs from one evaluation of Eq. (5.31) (Probs. 5.20 and 5.22).

G. Differentiation in the Time Domain:

$$\frac{dx(t)}{dt} \leftrightarrow j\omega X(\omega) \tag{5.55}$$

Equation (5.55) shows that the effect of differentiation in the time domain is the multiplication of $X(\omega)$ by $j\omega$ in the frequency domain (Prob. 5.28).

H. Differentiation in the Frequency Domain:

$$(-jt)x(t) \leftrightarrow \frac{dX(\omega)}{d\omega} \tag{5.56}$$

Equation (5.56) is the dual property of Eq. (5.55).

I. Integration in the Time Domain:

$$\int_{-\infty}^{t} x(\tau)\, d\tau \leftrightarrow \pi X(0)\,\delta(\omega) + \frac{1}{j\omega} X(\omega) \tag{5.57}$$

Since integration is the inverse of differentiation, Eq. (5.57) shows that the frequency domain operation corresponding to time-domain integration is multiplication by $1/j\omega$, but an additional term is needed to account for a possible dc component in the integrator output. Hence, unless $X(0) = 0$, a dc component is produced by the integrator (Prob. 5.33).

J. Convolution:

$$x_1(t) * x_2(t) \leftrightarrow X_1(\omega)\, X_2(\omega) \tag{5.58}$$

Equation (5.58) is referred to as the *time convolution theorem*, and it states that convolution in the time domain becomes multiplication in the frequency domain (Prob. 5.31). As in the case of the Laplace transform, this convolution property plays an important role in the study of continuous-time LTI systems (Sec. 5.5) and also forms the basis for our discussion of filtering (Sec. 5.6).

K. Multiplication:

$$x_1(t)x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega) \tag{5.59}$$

The multiplication property (5.59) is the dual property of Eq. (5.58) and is often referred to as the *frequency convolution theorem*. Thus, multiplication in the time domain becomes convolution in the frequency domain (Prob. 5.35).

L. Additional Properties:

If $x(t)$ is real, let

$$x(t) = x_e(t) + x_o(t) \tag{5.60}$$

where $X_e(t)$ and $X_0(t)$ are the even and odd components of $x(t)$, respectively. Let

$$x(t) \leftrightarrow X(\omega) = A(\omega) + jB(\omega)$$

Then

$$X(-\omega) = X^*(\omega) \tag{5.61a}$$

$$x_e(t) \leftrightarrow \mathrm{Re}\{X(\omega)\} = A(\omega) \tag{5.61b}$$

$$x_o(t) \leftrightarrow j\,\mathrm{Im}\{X(\omega)\} = jB(\omega) \tag{5.61c}$$

Equation (5.61a) is the necessary and sufficient condition for $x(t)$ to be real (Prob. 5.39). Equations (5.61b) and (5.61c) show that the Fourier transform of an even signal is a real function of ω and that the Fourier transform of an odd signal is a pure imaginary function of ω.

M. Parseval's Relations:

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)\,d\lambda = \int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)\,d\lambda \tag{5.62}$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\,d\omega \tag{5.63}$$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega \tag{5.64}$$

Equation (5.64) is called *Parseval's identity* (or *Parseval's theorem*) for the Fourier transform. Note that the quantity on the left-hand side of Eq. (5.64) is the normalized energy content E of $x(t)$ [Eq. (1.14)]. Parseval's identity says that this energy content E can be computed by integrating $|X(\omega)|^2$ over all frequencies ω. For this reason, $|X(\omega)|^2$ is often referred to as the *energy-density spectrum* of $x(t)$, and Eq. (5.64) is also known as the *energy theorem*.

PROPERTY	SIGNAL	FOURIER TRANSFORM
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\pi X(0)\,\delta(\omega) + \frac{1}{j\omega} X(\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Real signal	$x(t) = x_e(t) + x_o(t)$	$X(\omega) = A(\omega) + jB(\omega)$ $X(-\omega) = X^*(\omega)$
Even component	$x_e(t)$	$\mathrm{Re}\{X(\omega)\} = A(\omega)$
Odd component	$x_o(t)$	$j\,\mathrm{Im}\{X(\omega)\} = jB(\omega)$
Parseval's relations		

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)\, d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\, d\omega$$

Properties of Fourier Transform

$x(t)$	$X(\omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$u(-t)$	$\pi\delta(\omega)-\frac{1}{j\omega}$
$e^{-at}u(t), a>0$	$\frac{1}{j\omega+a}$
$te^{-at}u(t), a>0$	$\frac{1}{(j\omega+a)^2}$
$e^{-a\|t\|}, a>0$	$\frac{2a}{a^2+\omega^2}$
$\frac{1}{a^2+t^2}$	$e^{-a\|\omega\|}$
$e^{-at^2}, a>0$	$\sqrt{\frac{\pi}{a}}\, e^{-\omega^2/4a}$
$p_a(t)=\begin{cases}1 & \|t\|<a \\ 0 & \|t\|>a\end{cases}$	$2a\frac{\sin\omega a}{\omega a}$
$\frac{\sin at}{\pi t}$	$p_a(\omega)=\begin{cases}1 & \|\omega\|<a \\ 0 & \|\omega\|>a\end{cases}$
$\operatorname{sgn} t$	$\frac{2}{j\omega}$
$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0), \omega_0=\frac{2\pi}{T}$

Some Common Signals and their Fourier Transform

3. Frequency Response of Continuous Time LTI Systems

A. Frequency Response:

In Sec. 2.2 we showed that the output $y(t)$ of a continuous-time LTI system equals the convolution of the input $x(t)$ with the impulse response $h(t)$; that is,

$$y(t) = x(t) * h(t) \tag{5.65}$$

Applying the convolution property (5.58), we obtain

$$Y(\omega) = X(\omega)H(\omega) \tag{5.66}$$

where $Y(\omega)$, $X(\omega)$, and $H(\omega)$ are the Fourier transforms of $y(t)$, $x(t)$, and $h(t)$, respectively. From Eq. (5.66) we have

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \tag{5.67}$$

The function $H(\omega)$ is called the *frequency response* of the system. Relationships represented by Eqs. (5.65) and (5.66) are depicted in Fig. 5-3. Let

$$H(\omega) = |H(\omega)|e^{j\theta_H(\omega)} \tag{5.68}$$

Then $|H(\omega)|$ is called the *magnitude response* of the system, and $\theta_H(\omega)$ the *phase response* of the system.

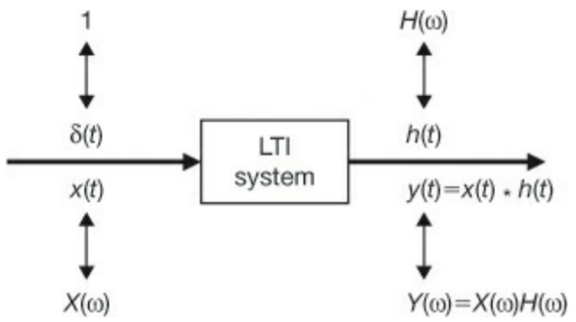

Fig. 5-3 Relationships between inputs and outputs in an LTI system.

Then from Eq. (5.66) we have

$$|Y(\omega)| = |X(\omega)||H(\omega)| \tag{5.78a}$$

$$\theta_Y(\omega) = \theta_X(\omega) + \theta_H(\omega) \tag{5.78b}$$

Hence, the magnitude spectrum $|X(\omega)|$ of the input is multiplied by the magnitude response $|H(\omega)|$ of the system to determine the magnitude spectrum $|Y(\omega)|$ of the output, and the phase response $\theta_H(\omega)$ is added to the phase spectrum $\theta_X(\omega)$ of the input to produce the phase spectrum $\theta_Y(\omega)$ of the output. The magnitude response $|H(\omega)|$ is sometimes referred to as the *gain* of the system.

B. Distortionless Transmission:

For distortionless transmission through an LTI system we require that the exact input signal shape be reproduced at the output, although its amplitude may be different and it may be delayed in time. Therefore, if $x(t)$ is the input signal, the required output is

$$y(t) = Kx(t - t_d) \tag{5.79}$$

where t_d is the *time delay* and $K(> 0)$ is a *gain constant*. This is illustrated in Figs. 5-4(*a*) and (*b*). Taking the Fourier transform of both sides of Eq. (5.79), we get

$$Y(\omega) = Ke^{-j\omega t_d} X(\omega) \tag{5.80}$$

Thus, from Eq. (5.66) we see that for distortionless transmission, the system must have

$$H(\omega) = |H(\omega)|e^{j\theta_H(\omega)} = Ke^{-j\omega t_d} \tag{5.81}$$

Thus,

$$|H(\omega)| = K \tag{5.82a}$$

$$\theta_H(\omega) = -j\omega t_d \tag{5.82b}$$

That is, the amplitude of $H(\omega)$ must be constant over the entire frequency range, and the phase of $H(\omega)$ must be linear with the frequency. This is illustrated in Figs. 5-4(*c*) and (*d*).

Amplitude Distortion and Phase Distortion:

When the amplitude spectrum $|H(\omega)|$ of the system is not constant within the frequency band of interest, the frequency components of the input signal are transmitted with a different amount of gain or attenuation. This effect is called *amplitude distortion*. When the phase spectrum $\theta_H(\omega)$ of the system is not linear with the frequency, the output signal has a different waveform than the input signal because of different delays in passing through the system for different frequency components of the input signal. This form of distortion is called *phase distortion*.

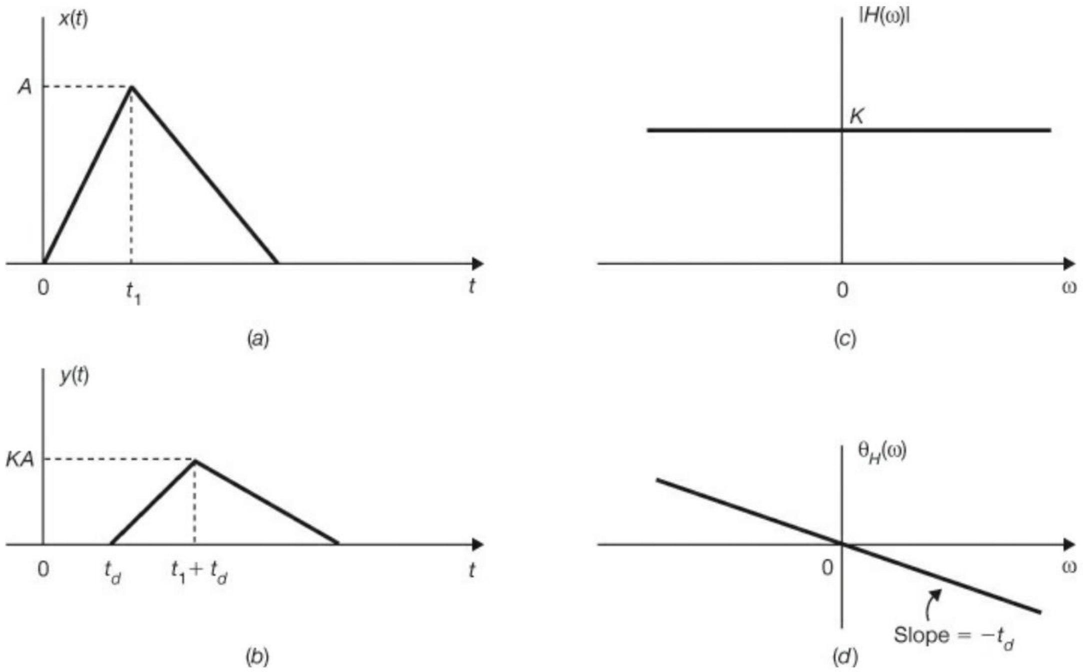

Fig. 5-4 Distortionless transmission.

4. Filtering

A. Ideal Frequency-Selective Filters:

An *ideal* frequency-selective filter is one that exactly passes signals at one set of frequencies and completely rejects the rest. The band of frequencies passed by the filter is referred to as the *pass band*, and the band of frequencies rejected by the filter is called the *stop band*.

The most common types of ideal frequency-selective filters are the following.

1. Ideal Low-Pass Filter:

An ideal low-pass filter (LPF) is specified by

$$|H(\omega)| = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases} \qquad (5.86)$$

which is shown in Fig. 5-5(a). The frequency ω_c is called the *cutoff* frequency.

2. Ideal High-Pass Filter:

An ideal high-pass filter (HPF) is specified by

$$|H(\omega)| = \begin{cases} 0 & |\omega| < \omega_c \\ 1 & |\omega| > \omega_c \end{cases} \tag{5.87}$$

which is shown in Fig. 5-5(*b*).

3. Ideal Bandpass Filter:

An ideal bandpass filter (BPF) is specified by

$$|H(\omega)| = \begin{cases} 1 & \omega_1 < |\omega| < \omega_2 \\ 0 & \text{otherwise} \end{cases} \tag{5.88}$$

which is shown in Fig. 5-5(*c*).

4. Ideal Bandstop Filter:

An ideal bandstop filter (BSF) is specified by

$$|H(\omega)| = \begin{cases} 0 & \omega_1 < |\omega| < \omega_2 \\ 1 & \text{otherwise} \end{cases} \tag{5.89}$$

which is shown in Fig. 5-5(*d*).

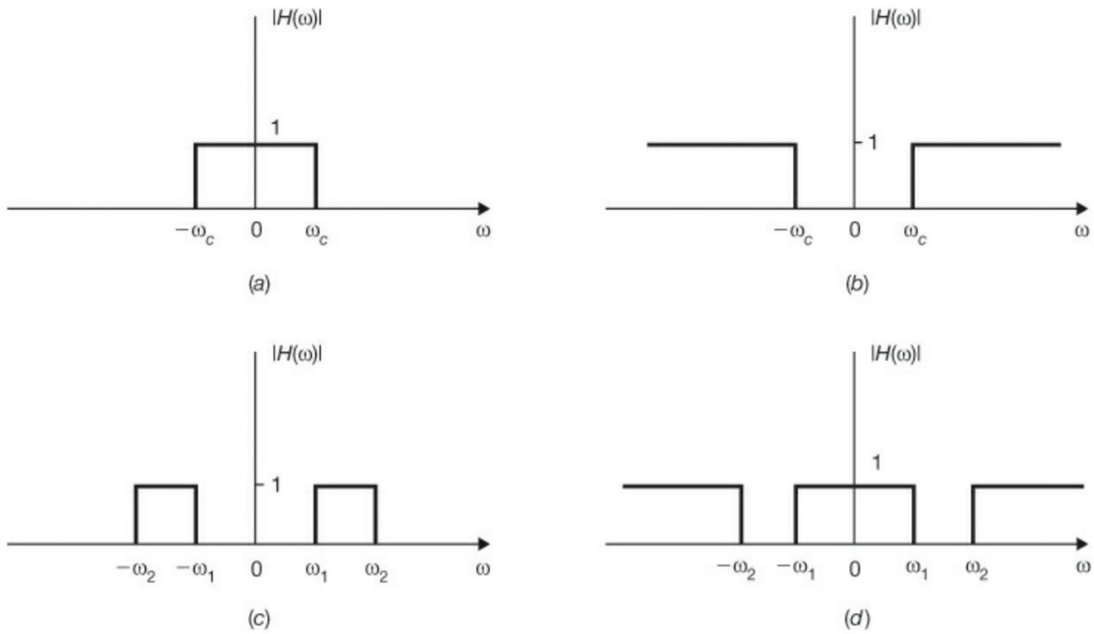

Fig. 5-5 Magnitude responses of ideal frequency-selective filters.

In the above discussion, we said nothing regarding the phase response of the filters. To avoid phase distortion in the filtering process, a filter should have a linear phase characteristic over the pass band of the filter; that is [Eq. (5.82b)],

$$\theta_H(\omega) = -\omega t_d \tag{5.90}$$

where t_d is a constant.

Note that all ideal frequency-selective filters are noncausal systems.

B. Nonideal Frequency-Selective Filters:

As an example of a simple continuous-time causal frequency-selective filter, we consider the *RC* filter shown in Fig. 5-6(*a*). The output *y*(*t*) and the input *x*(*t*) are related by (Prob. 1.32)

$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

Taking the Fourier transforms of both sides of the above equation, the frequency response $H(\omega)$ of the *RC* filter is given by

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega/\omega_0} \tag{5.91}$$

where $\omega_0 = 1/RC$. Thus, the amplitude response $|H(\omega)|$ and phase response $\theta_H(\omega)$ are given by

$$|H(\omega)| = \frac{1}{|1 + j\omega/\omega_0|} = \frac{1}{\left[1 + (\omega/\omega_0)^2\right]^{1/2}} \tag{5.92}$$

$$\theta_H(\omega) = -\tan^{-1}\frac{\omega}{\omega_0} \tag{5.93}$$

which are plotted in Fig. 5-6(*b*). From Fig. 5-6(*b*) we see that the *RC* network in Fig. 5-6(*a*) performs as a low-pass filter.

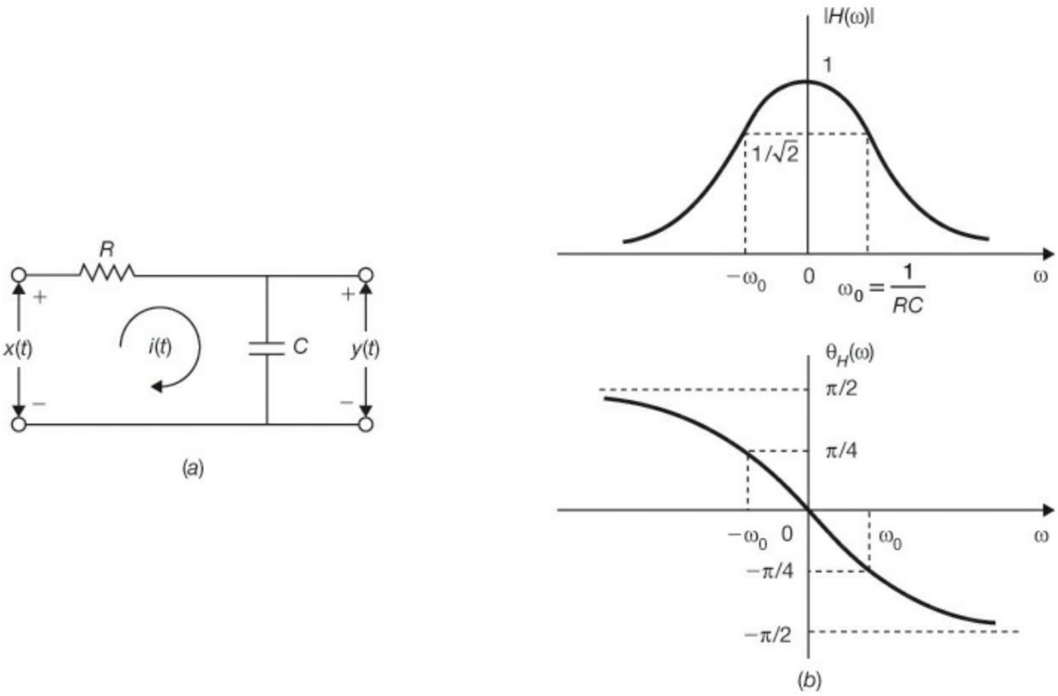

Fig. 5-6 *RC* filter and its frequency response.

3. Glossary – English/Chinese Translation

English	Chinese
Fourier Transform	傅里叶变换
continuous time	连续时间
filtering	滤波
fundamental period	基本时期
Complex Fourier Coefficients	复傅里叶系数
infinitesimal	渺小
nonperiodic	非周期性
Fourier spectra	傅里叶光谱
Fourier Transform Pair	傅里叶变换对
Convergence	收敛
Duality	二重性
Parseval's Theorem	帕斯瓦尔定理
Frequency response	频率响应
amplitude distortion	幅度失真
phase distortion	相位失真
Low pass filter	低通滤波器
High pass filter	高通滤波器
Band pass filter	带通滤波器
Band stop filter	带阻滤波器

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