Tutorial 1-03-f

Question 1

3.17. Find the inverse Laplace transform of the following $X(s)$:

$$(a)\quad X(s)=\frac{2s+4}{s^2+4s+3},\ \mathrm{Re}(s)>-1$$

$$(b)\quad X(s)=\frac{2s+4}{s^2+4s+3},\ \mathrm{Re}(s)<-3$$

$$(c)\quad X(s)=\frac{2s+4}{s^2+4s+3},\ -3<\mathrm{Re}(s)<-1$$

Question 2

3.19. Find the inverse Laplace transform of

$$X(s)=\frac{s^2+2s+5}{(s+3)(s+5)^2} \qquad \mathrm{Re}(s)>-3$$

Question 3

3.31. The feedback interconnection of two causal subsystems with system functions $F(s)$ and $G(s)$ is depicted in Fig. 3-13. Find the overall system function $H(s)$ for this feedback system.

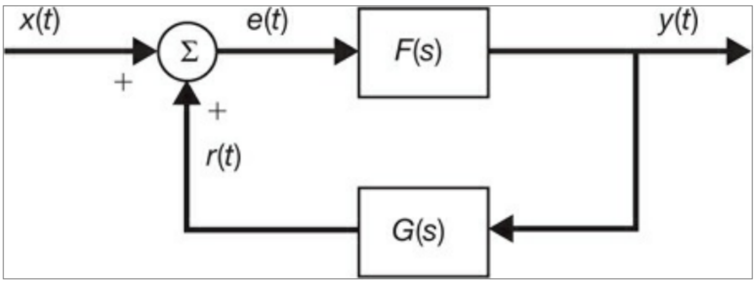

Fig. 3-13 Feedback system.

Question 4

3.38. Solve the second-order linear differential equation

$$y''(t)+5y'(t)+6y(t)=x(t) \qquad (3.105)$$

with the initial conditions $y(0)=2$, $y'(0)=1$, and $x(t)=e^{-t}u(t)$.

SOLUTION

Question 1

Expanding by partial fractions, we have

$$X(s) = \frac{2s+4}{s^2+4s+3} = 2\frac{s+2}{(s+1)(s+3)} = \frac{c_1}{s+1} + \frac{c_2}{s+3}$$

Using Eq. (3.30), we obtain

$$c_1 = (s+1)X(s)\big|_{s=-1} = 2\left.\frac{s+2}{s+3}\right|_{s=-1} = 1$$

$$c_2 = (s+3)X(s)\big|_{s=-3} = 2\left.\frac{s+2}{s+1}\right|_{s=-3} = 1$$

Hence,

$$X(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

(*a*) The ROC of $X(s)$ is $\text{Re}(s) > -1$. Thus, $x(t)$ is a right-sided signal and from Table 3-1 we obtain

$$x(t) = e^{-t}u(t) + e^{-3t}u(t) = (e^{-t} + e^{-3t})u(t)$$

(*b*) The ROC of $X(s)$ is $\text{Re}(s) < -3$. Thus, $x(t)$ is a left-sided signal and

from Table 3-1 we obtain

$$x(t) = -e^{-t}u(-t) - e^{-3t}u(-t) = -(e^{-t} + e^{-3t})u(-t)$$

(*c*) The ROC of $X(s)$ is $-3 < \text{Re}(s) < -1$. Thus, $x(t)$ is a double-sided signal and from Table 3-1 we obtain

$$x(t) = -e^{-t}u(-t) + e^{-3t}u(t)$$

Question 2

$$X(s)=\frac{c_1}{s+3}+\frac{\lambda_1}{s+5}+\frac{\lambda_2}{(s+5)^2} \qquad (3.84)$$

By Eqs. (3.30) and (3.32) we have

$$c_1=(s+3)X(s)\big|_{s=-3}=\left.\frac{s^2+2s+5}{(s+5)^2}\right|_{s=-3}=2$$

$$\lambda_2=(s+5)^2X(s)\big|_{s=-5}=\left.\frac{s^2+2s+5}{s+3}\right|_{s=-5}=-10$$

$$\lambda_1=\frac{d}{ds}\left[(s+5)^2X(s)\right]\Big|_{s=-5}=\frac{d}{ds}\left[\frac{s^2+2s+5}{s+3}\right]\Bigg|_{s=-5}$$

$$=\left.\frac{s^2+6s+1}{(s+3)^2}\right|_{s=-5}=-1$$

Hence,

$$X(s)=\frac{2}{s+3}-\frac{1}{s+5}-\frac{10}{(s+5)^2}$$

The ROC of $X(s)$ is $\text{Re}(s) > -3$. Thus, $x(t)$ is a right-sided signal and from Table 3-1 we obtain

$$x(t) = 2e^{-3t}\,u(t) - e^{-5t}\,u(t) - 10\,te^{-5t}\,u(t)$$

$$= [2e^{-3t} - e^{-5t}\,u(t) - 10\,te^{-5t}\,u(t)$$

Question 3

Let

$$x(t) \leftrightarrow X(s) \qquad y(t) \leftrightarrow Y(s) \qquad r(t) \leftrightarrow R(s) \qquad e(t) \leftrightarrow E(s)$$

Then,

$$Y(s) = E(s)F(s) \qquad (3.87)$$

$$R(s) = Y(s)G(s) \qquad (3.88)$$

Since

$$e(t) = x(t) + r(t)$$

we have

$$E(s) = X(s) + R(s) \qquad (3.89)$$

Substituting Eq. (3.88) into Eq. (3.89) and then substituting the result into Eq. (3.87), we obtain

$$Y(s) = [X(s) + Y(s)G(s)]\, F(s)$$

or

$$[1 - F(s)G(s)]\, Y(s) = F(s)X(s)$$

Thus, the overall system function is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{F(s)}{1 - F(s)G(s)} \qquad (3.90)$$

<u>Question 4</u>

$$y''(t) + 5y'(t) + 6y(t) = x(t) \qquad (3.105)$$

with the initial conditions $y(0) = 2$, $y'(0) = 1$, and $x(t) = e^{-t}u(t)$.

Assume that $y(0) = y(0^-)$ and $y'(0) = y(0^-)$. Let

$$y(t) \leftrightarrow Y_I(s)$$

Then from Eqs. (3.44) and (3.45)

$$y'(t) \leftrightarrow sY_I(s) - y(0^-) = sY_I(s) - 2$$
$$y'(t) \leftrightarrow s^2Y_I(s)\ sy(0^-)\ y'(0^-)\ s^2Y_I(s) - 2s - 1$$

From Table 3-1 we have

$$x(t) \leftrightarrow X_I(s) = \frac{1}{s+1}$$

Taking the unilateral Laplace transform of Eq. (3.105), we obtain

$$[s^2Y_I(s) - 2s - 1] + 5[sY_I(s) - 2] + 6Y_I(s) = \frac{1}{s+1}$$

or

$$(s^2 + 5s + 6)Y_I(s) = \frac{1}{s+1} + 2s + 11 = \frac{2s^2 + 13s + 12}{s+1}$$

Thus,

$$Y_I(s) = \frac{2s^2 + 13s + 12}{(s+1)(s^2 + 5s + 6)} = \frac{2s^2 + 13s + 12}{(s+1)(s+2)(s+3)}$$

Using partial-fraction expansions, we obtain

$$Y_I(s) = \frac{1}{2}\frac{1}{s+1} + 6\frac{1}{s+2} - \frac{9}{2}\frac{1}{s+3}$$

Taking the inverse Laplace transform of $Y_I(s)$, we have

$$y(t) = \left(\frac{1}{2}e^{-t} + 6e^{-2t} - \frac{9}{2}e^{-3t}\right)u(t)$$

Notice that $y(0^+) = 2 = y(0)$ and $y'(0^+) = 1 = y'(0)$; and we can write $y(t)$ as

$$y(t) = \frac{1}{2}e^{-t} + 6e^{-2t} - \frac{9}{2}e^{-3t} \qquad t \geq 0$$