Tutorial

Question 1

3.1. Find the Laplace transform of

(*a*) $x(t) = -e^{-at}u(-t)$

(*b*) $x(t) = e^{at}u(-t)$

Question 2

3.3. Let

$$x(t) = \begin{cases} e^{-at} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

Find the Laplace transform of x(t)

Question 3

3.5. Find the Laplace transform $X(s)$ and sketch the pole-zero plot with the ROC for the following signals $x(t)$:

(*a*) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(*b*) $x(t) = e^{-3t}u(t) + e^{2t}u(-t)$

(*c*) $x(t) = e^{2t}u(t) + e^{-3t}u(-t)$

SOLUTION

Question 1

(a) From Eq. (3.3)

$$X(s) = -\int_{-\infty}^{\infty} e^{-at}u(-t)e^{-st}\,dt = -\int_{-\infty}^{0^-} e^{-(s+a)t}\,dt$$
$$= \frac{1}{s+a}e^{-(s+a)t}\Big|_{-\infty}^{0-} = \frac{1}{s+a} \qquad \text{Re}(s) < -a$$

(b) Thus, we obtain

$$-e^{-at}u(-t) \leftrightarrow \frac{1}{s+a} \qquad \text{Re}(s) < -a \tag{3.54}$$

(b) Similarly,

$$X(s) = \int_{-\infty}^{\infty} e^{at}u(-t)e^{-st}\,dt = \int_{-\infty}^{0^-} e^{-(s-a)t}\,dt$$
$$= -\frac{1}{s-a}e^{-(s-a)t}\Big|_{-\infty}^{0-} = -\frac{1}{s-a} \qquad \text{Re}(s) < a$$

Thus, we obtain

$$e^{at}u(-t) \leftrightarrow -\frac{1}{s-a} \qquad \text{Re}(s) < a \tag{3.55}$$

Question 2

By Eq. (3.3)

$$X(s) = \int_0^T e^{-at}e^{-st}\,dt = \int_0^T e^{-(s+a)t}\,dt$$
$$= -\frac{1}{s+a}e^{-(s+a)t}\Big|_0^T = \frac{1}{s+a}[1 - e^{-(s+a)T}] \tag{3.58}$$

Since $x(t)$ is a finite-duration signal, the ROC of $X(s)$ is the entire s-plane. Note that from Eq. (3.58) it appears that $X(s)$ does not converge at $s = -a$. But this is not the case. Setting $s = -a$ in the integral in Eq. (3.58), we have

$$X(-a) = \int_0^T e^{-(a+a)t}\,dt = \int_0^T dt = T$$

Question 3

(a) From Table 3-1

$$e^{-2t}u(t) \leftrightarrow \frac{1}{s+2} \qquad \text{Re}(s) > -2 \tag{3.59}$$

$$e^{-3t}u(t) \leftrightarrow \frac{1}{s+3} \qquad \text{Re}(s) > -3 \tag{3.60}$$

We see that the ROCs in Eqs. (3.59) and (3.60) overlap, and thus,

$$X(s) = \frac{1}{s+2} + \frac{1}{s+3} = \frac{2\left(s+\frac{5}{2}\right)}{(s+2)(s+3)} \qquad \mathrm{Re}(s) > -2 \tag{3.61}$$

From Eq. (3.61) we see that $X(s)$ has one zero at $s = -\frac{5}{2}$ and two poles at $s = -2$ and $s = -3$ and that the ROC is $\mathrm{Re}(s) > -2$, as sketched in Fig. 3-11(*a*).

(*b*) From Table 3-1

$$e^{-3t}u(t) \leftrightarrow \frac{1}{s+3} \qquad \mathrm{Re}(s) > -3 \tag{3.62}$$

$$e^{2t}u(-t) \leftrightarrow -\frac{1}{s-2} \qquad \mathrm{Re}(s) < 2 \tag{3.63}$$

We see that the ROCs in Eqs. (3.62) and (3.63) overlap, and thus,

$$X(s) = \frac{1}{s+3} - \frac{1}{s-2} = \frac{-5}{(s-2)(s+3)} \qquad -3 < \mathrm{Re}(s) < 2 \tag{3.64}$$

From Eq. (3.64) we see that $X(s)$ has no zeros and two poles at $s = 2$ and $s = -3$ and that the ROC is $-3 < \mathrm{Re}(s) < 2$, as sketched in Fig. 3-11(*b*).

(*c*) From Table 3-1

$$e^{2t}u(t) \leftrightarrow \frac{1}{s-2} \qquad \mathrm{Re}(s) > 2 \tag{3.65}$$

$$e^{-3t}u(-t) \leftrightarrow -\frac{1}{s+3} \qquad \mathrm{Re}(s) < -3 \tag{3.66}$$

We see that the ROCs in Eqs. (3.65) and (3.66) do not overlap and that there is no common ROC; thus, $x(t)$ has no transform $X(s)$.

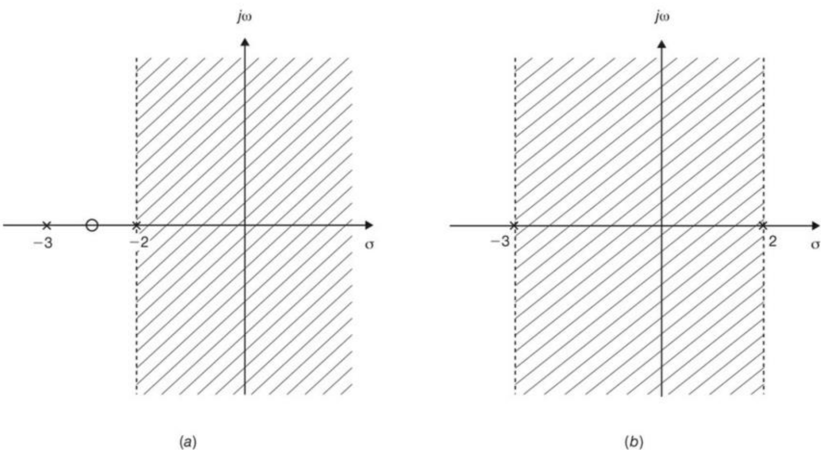

Fig. 3-11