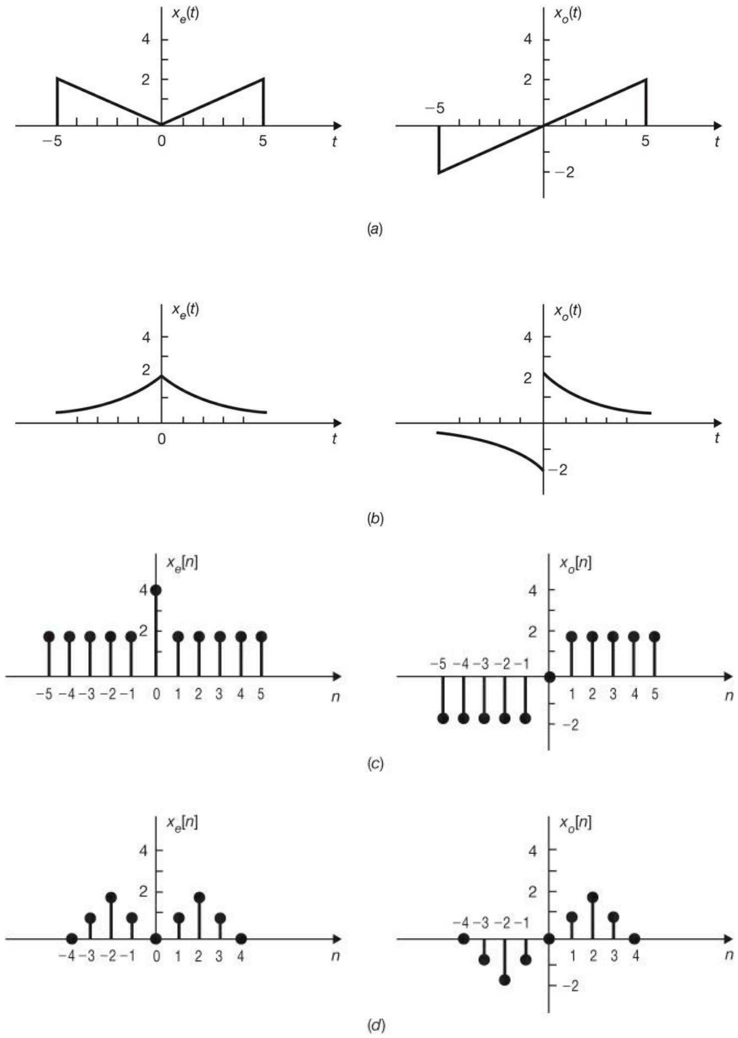
1-03-b – Tutorial – Solution

Question 1



Question 2

Let $x(t) = x_1(t)x_2(t)$. If $x_1(t)$ and $x_2(t)$ are both even, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ and $x_2(t)$ are both odd, then

$$x(-t) = x_1(-t)x_2(-t) = -x_1(t)\,[-x_2(t)] = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ is even and $x_2(t)$ is odd, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)[-\,x_2(t)] = -\,x_1(t)x_2(t) = -x(t)$$

and $x(t)$ is odd. Note that in the above proof, variable t represents either a continuous or a discrete variable.

Question 3

(a) $x(t) = \cos\left(t + \frac{\pi}{4}\right) = \cos\left(\omega_0 t + \frac{\pi}{4}\right) \rightarrow \omega_0 = 1$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 2\pi$.

(b) $x(t) = \sin\frac{2\pi}{3}t \rightarrow \omega_0 = \frac{2\pi}{3}$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 3$.

(c) $x(t) = \cos\frac{\pi}{3}t + \sin\frac{\pi}{4}t = x_1(t) + x_2(t)$

where $x_1(t) = \cos(\pi/3)t = \cos\omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 6$ and $x_2(t) = \sin(\pi/4)t = \sin\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = 8$. Since

$T_1/T_2 = \frac{6}{8} = \frac{3}{4}$ is a rational number, $x(t)$ is periodic with fundamental period $T_0 = 4T_1 = 3T_2 = 24$.

(d) $x(t) = \cos t + \sin\sqrt{2}\,t = x_1(t) + x_2(t)$

where $x_1(t) = \cos t = \cos\omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 2\pi$ and $x_2(t) = \sin\sqrt{2}\,t = \sin\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = \sqrt{2}\,\pi$. Since $T_1/T_2 = \sqrt{2}$ is an irrational number, $x(t)$ is nonperiodic.

(e) Using the trigonometric identity $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$, we can write

$$x(t) = \sin^2 t = \frac{1}{2} - \frac{1}{2}\cos 2t = x_1(t) + x_2(t)$$

where $x_1(t) = \frac{1}{2}$ is a dc signal with an arbitrary period and $x_2(t) = -\frac{1}{2}\cos 2t = -\frac{1}{2}\cos\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = \pi$. Thus, $x(t)$ is periodic with fundamental period $T_0 = \pi$.

(f) $x(t) = e^{j[(\pi/2)t-1]} = e^{-j}e^{j(\pi/2)t} = e^{-j}e^{j\omega_0 t} \rightarrow \omega_0 = \frac{\pi}{2}$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 4$.

Question 4

(a) By definition (1.19)

$$u(1-t)=\begin{cases}1 & t<1\\0 & t>1\end{cases}$$

and $x(t)u(1-t)$ is sketched in Fig. 1-28(a).

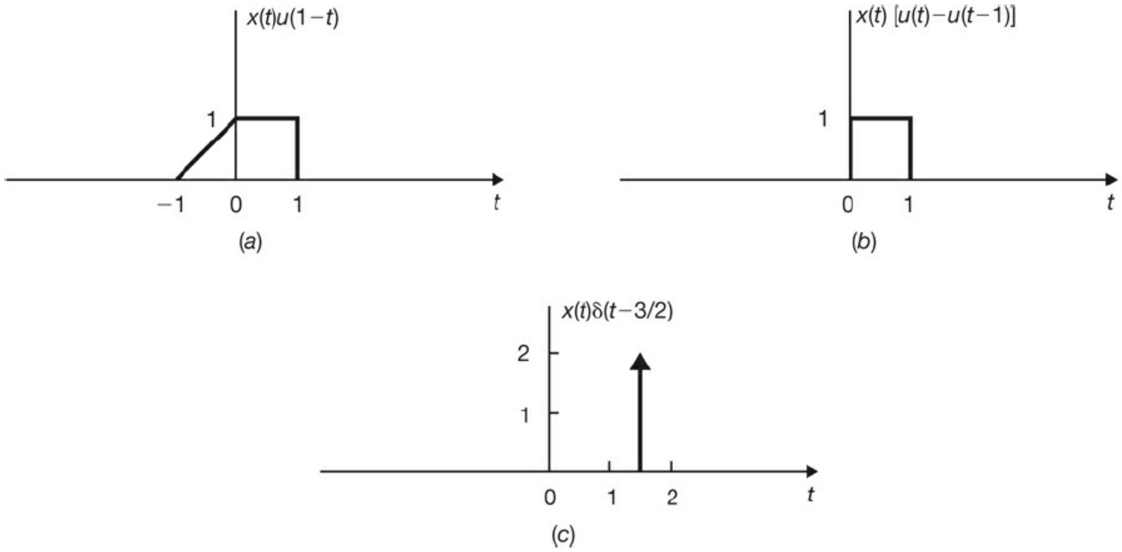


Fig. 1-28

(b) By definitions (1.18) and (1.19)

$$u(t)-u(t-1)=\begin{cases}1 & 0<t\le 1\\0 & \text{otherwise}\end{cases}$$

and $x(t)[u(t)-u(t-1)]$ is sketched in Fig. 1-28(b).

(c) By Eq. (1.26)

$$x(t)\delta\left(t-\frac{3}{2}\right)=x\left(\frac{3}{2}\right)\delta\left(t-\frac{3}{2}\right)=2\delta\left(t-\frac{3}{2}\right)$$

which is sketched in Fig. 1-28(c).