Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 03-b

Signal Types and Systems Classification

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Reference:
Signals and Systems – Schaum's Outline Series

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1. Basic Continuous Time Signals

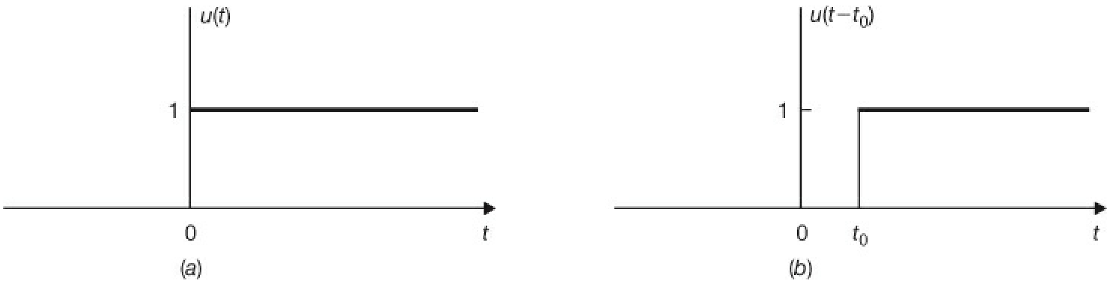
Unit Step Function

The unit step function *u(t),* also known as the Heaviside unit function, is defined as

$$u(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases} \tag{1.18}$$

Note that it is discontinuous at $t = 0$ and that the value at $t = 0$ is undefined. Similarly, the shifted unit step function $u(t - t_0)$ is defined as

$$u(t-t_0)=\begin{cases}1 & t>t_0\\ 0 & t<t_0\end{cases} \tag{1.19}$$



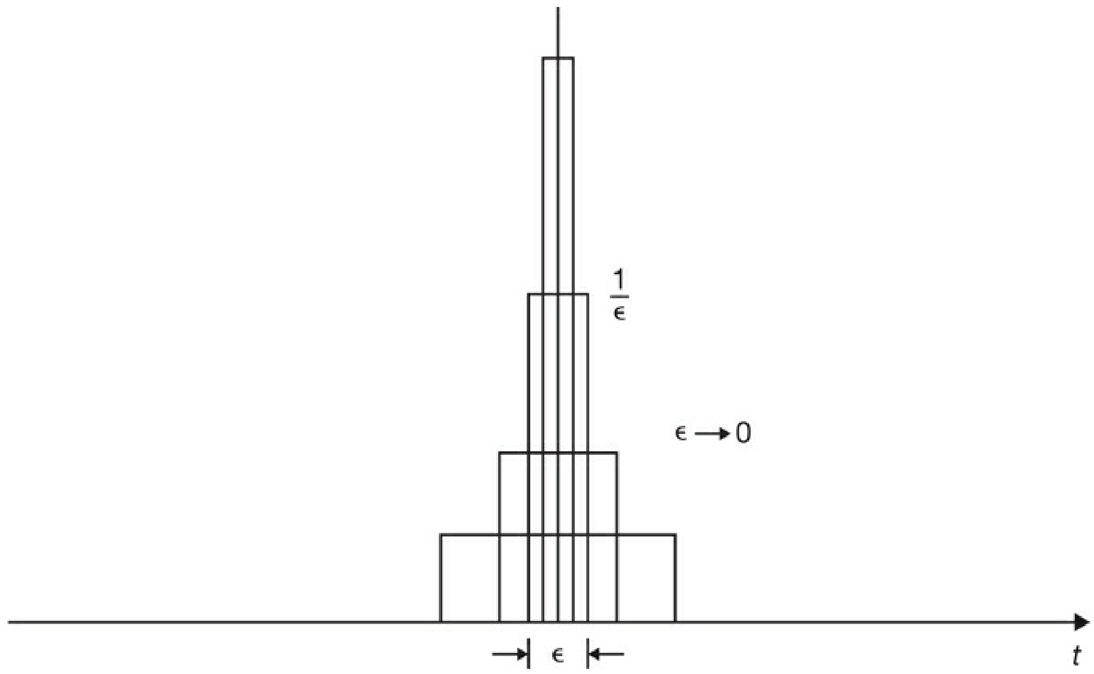
(a) Unit step function; (b) shifted unit step function

Unit Impulse Function

The unit impulse function *δ(t),* also known as the Dirac delta function, and it possesses the following properties:

$$\delta(t)=\begin{cases}0 & t\neq 0\\ \infty & t=0\end{cases}$$

$$\int_{-\varepsilon}^{\varepsilon}\delta(t)\,dt=1$$



$\delta(t)$ cannot be an ordinary function and mathematically it is defined by:

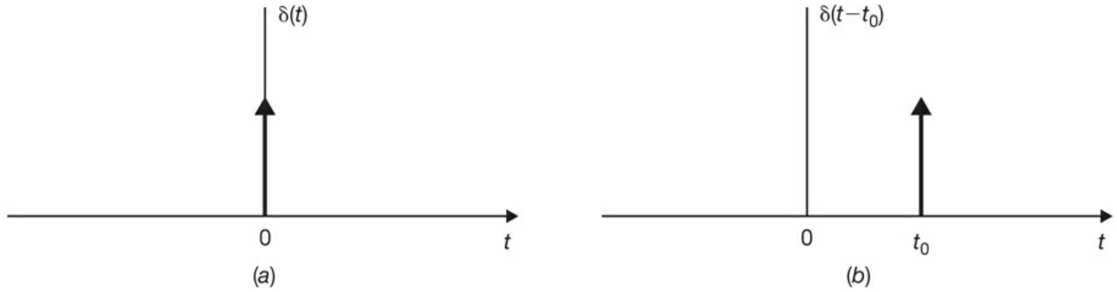
$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\, dt = \phi(0) \tag{1.20}$$

where $\phi(t)$ is any regular function continuous at $t = 0$.

$\delta(t)$ is often called a generalized function and $\phi(t)$ is known as a testing function. Similarly, the delayed delta function $\delta(t - t_0)$ is defined by:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)\, dt = \phi(t_0) \tag{1.22}$$

where $\phi(t)$ is any regular function continuous at $t = t_0$.



(a) Unit impulse function; (b) shifted unit impulse function.

Some additional properties of $\delta(t)$ are

$$\delta(at) = \frac{1}{|a|}\delta(t) \tag{1.23}$$

$$\delta(-t) = \delta(t) \tag{1.24}$$

$$x(t)\delta(t) = x(0)\delta(t) \tag{1.25}$$

if $x(t)$ is continuous at $t = 0$; and

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0) \tag{1.26}$$

Using Eqs. (1.22) and (1.24), any continuous-time signal $x(t)$ can be expressed as:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)\, d\tau \tag{1.27}$$

Complex Exponential Signals

The complex exponential signal

$$x(t) = e^{j\omega_0 t} \tag{1.32}$$

is an important example of a complex signal. Using Euler's formula, this signal can be defined as

$$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t \tag{1.33}$$

Thus, $x(t)$ is a complex signal whose real part is $\cos \omega_0 t$ and imaginary part is $\sin \omega_0 t$. An important property of the complex exponential signal $x(t)$ in Eq.(1.32) is that it is periodic. The fundamental period T_0 of $x(t)$ is given by:

$$T_0 = \frac{2\pi}{\omega_0} \tag{1.34}$$

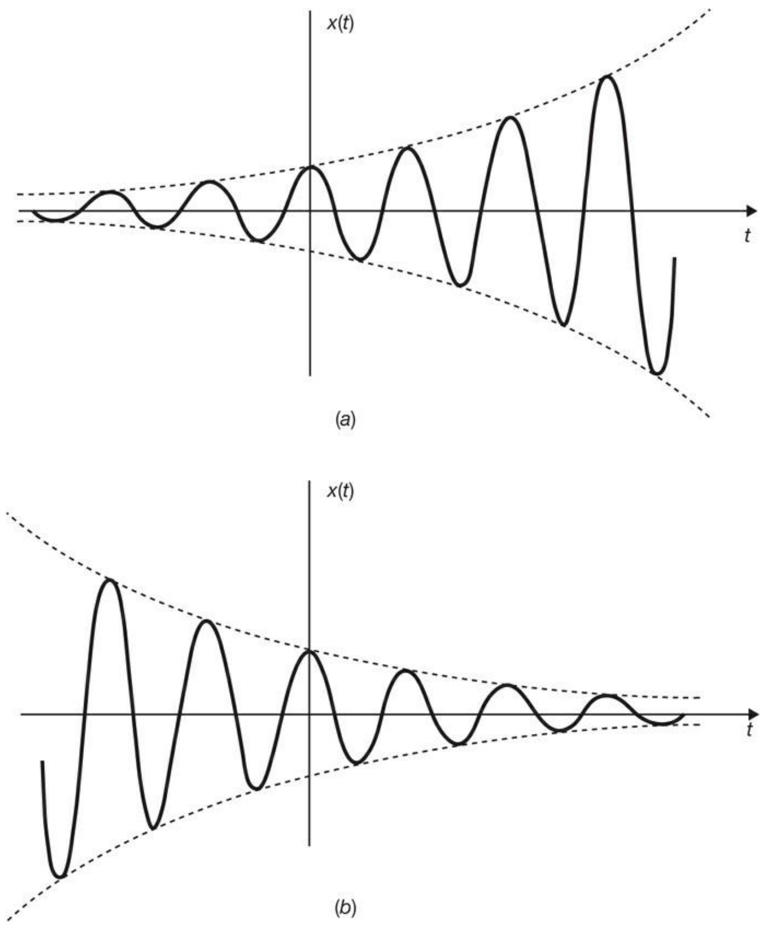
Note that $x(t)$ is periodic for any value of ω_0.

General Complex Exponential Signals

Let $s = \sigma + j\omega$ be a complex number. We define *x(t)* as:

$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}(\cos \omega t + j \sin \omega t) \tag{1.35}$$

Then signal *x(t)* in Eq. (1.35) is known as a general complex exponential signal whose real part $e^{\sigma t} \cos \omega t$ and imaginary part $e^{\sigma t} \sin \omega t$ are exponentially increasing ($\sigma > 0$) or decreasing ($\sigma < 0$) sinusoidal signals.

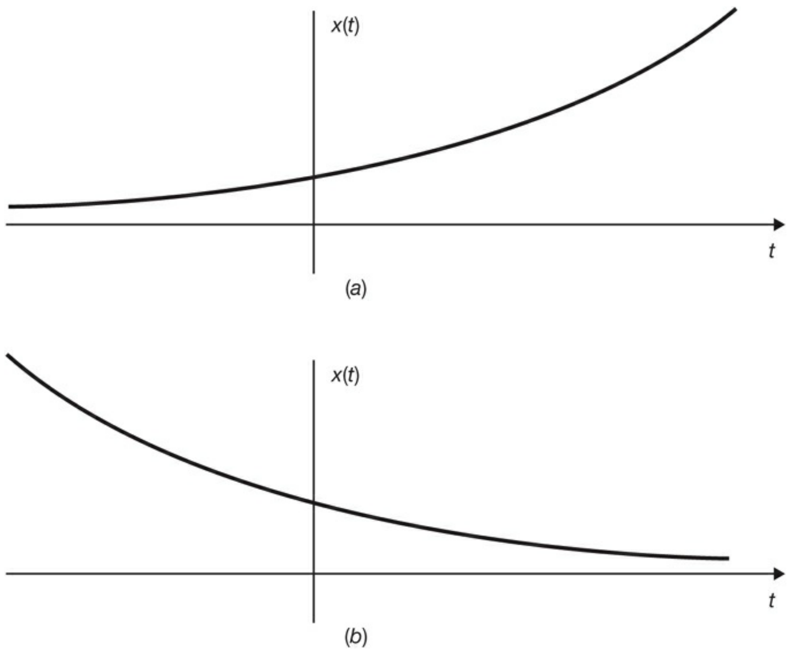


(a) Exponentially increasing sinusoidal signal; (b) exponentially decreasing sinusoidal signal

Real Exponential Signals

Note that if $s = \sigma$ (a real number), then Eq. (1.35) reduces to a real exponential signal. $\sigma > 0,$ then $x(t)$ is a growing exponential; and if $\sigma < 0$, then $x(t)$ is a decaying exponential.

$$x(t) = e^{\sigma t} \tag{1.36}$$



Continuous-time real exponential signals. (a) σ > 0; (b) σ < 0.

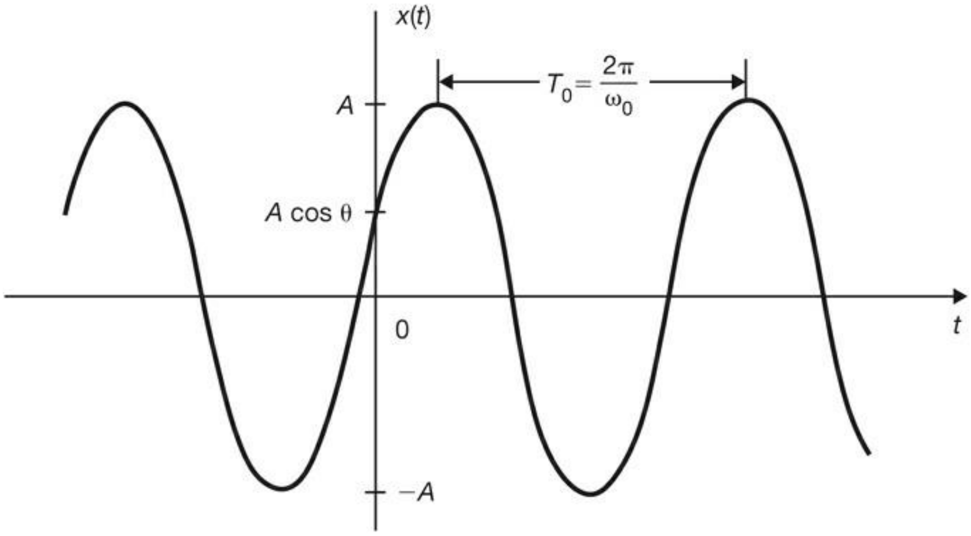
Sinusoidal Signals

A continuous-time sinusoidal signal can be expressed as:

$$x(t) = A \cos(\omega_0 t + \theta) \tag{1.37}$$

where A is the amplitude (real), ω_0 is the radian frequency in radians per second, and θ is the phase angle in radians.

$$T_0 = \frac{2\pi}{\omega_0} \tag{1.38}$$



Continuous-time sinusoidal signal

fundamental period T_0; fundamental frequency f_0:

$$f_0 = \frac{1}{T_0} \quad \text{hertz (Hz)} \tag{1.39}$$

$$\omega_0 = 2\pi f_0 \tag{1.40}$$

Using Euler's formula:

$$A\cos(\omega_0 t + \theta) = A\,\mathrm{Re}\{e^{j(\omega_0 t + \theta)}\} \tag{1.41}$$

$$A\,\mathrm{Im}\{e^{j(\omega_0 t + \theta)}\} = A\sin(\omega_0 t + \theta) \tag{1.42}$$

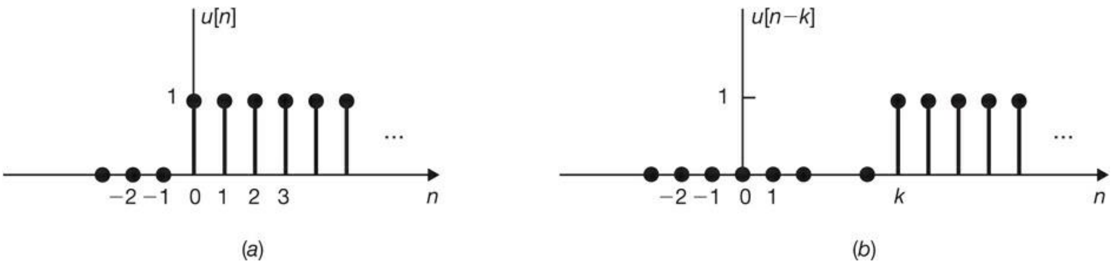
Re: real part of
Im: imaginary part of

2. Basic Discrete Time Signals

Unit Step Sequence

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.43}$$

Note that the value of $u[n]$ at $n = 0$ is defined [unlike the continuous-time step function $u(t)$ at $t = 0$] and equals unity.



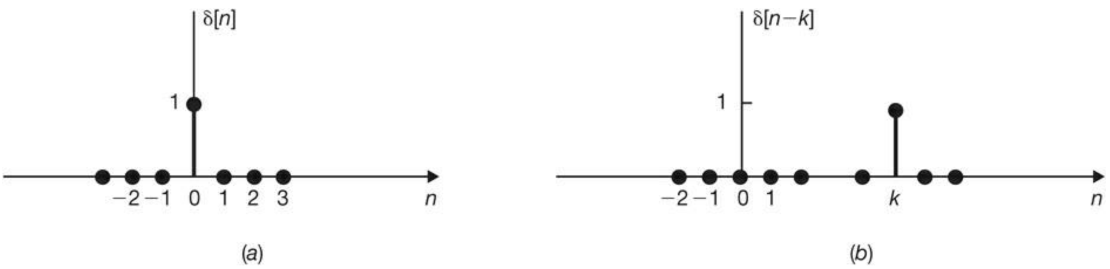
(a) unit step sequence; (b) shifted unit step sequence

Unit Impulse Sequence

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{1.45}$$

Shifted unit impulse (or sample) sequence:

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases} \tag{1.46}$$



Unlike the continuous-time unit impulse function $\delta(t)$,$\delta[n]$ is defined without mathematical complication or difficulty. From definitions (1.45) and (1.46) it is readily seen that

$$x[n]\,\delta[n] = x[0]\,\delta[n] \tag{1.47}$$

$$x[n]\,\delta[n-k] = x[k]\,\delta[n-k] \tag{1.48}$$

which are the discrete-time counterparts of Eqs. (1.25) and (1.26)

$$\delta[n] = u[n] - u[n-1] \tag{1.49}$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k] \tag{1.50}$$

which are the discrete-time counterparts of Eqs. (1.30) and (1.31)

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{1.51}$$

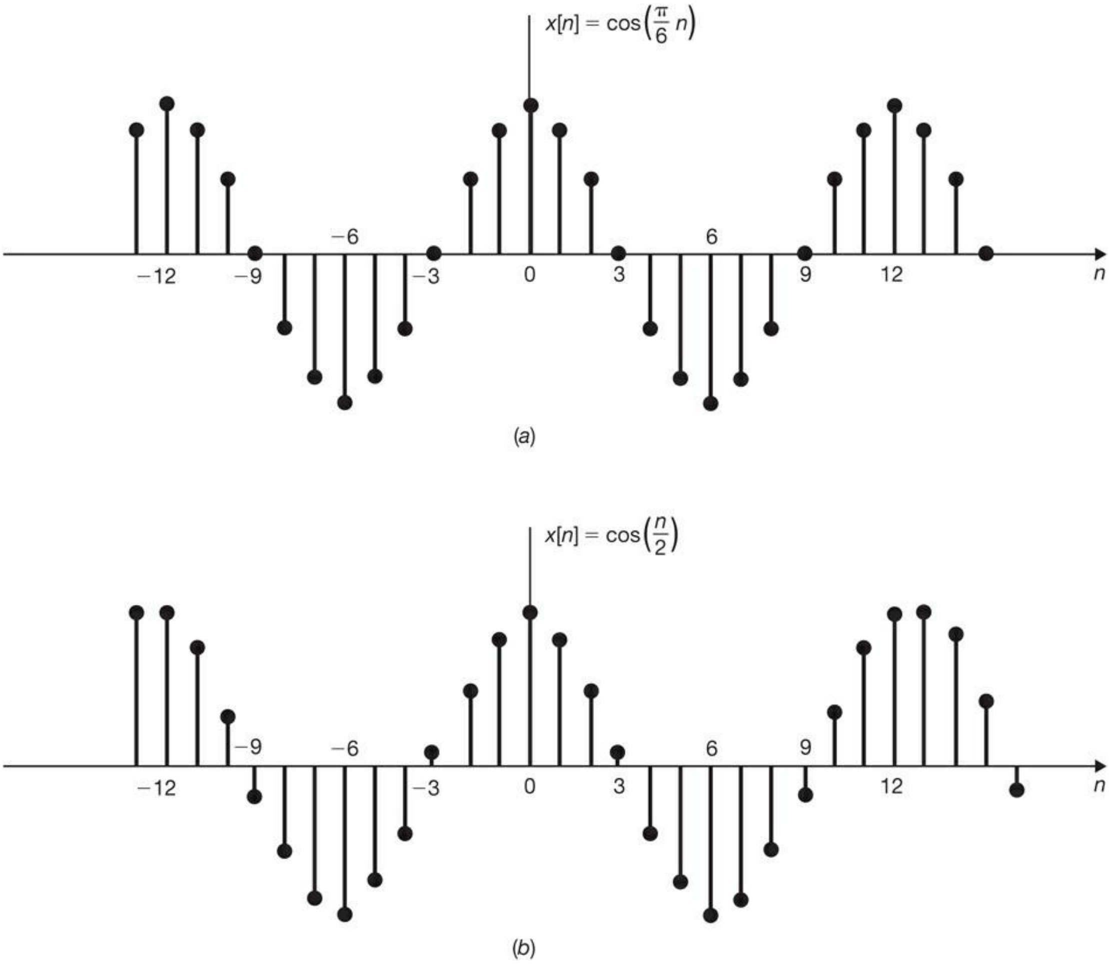
which corresponds to Eq. (1.27) in the continuous-time signal case.

Sinusoidal Sequences

$$x[n] = A\cos(\Omega_0 n + \theta) \tag{1.58}$$

If *n* is dimensionless, then both Ω_0 and θ have units of radians. Two examples of sinusoidal sequences are shown in Fig. 1-13. As before, the sinusoidal sequence in Eq. (1.58) can be expressed as

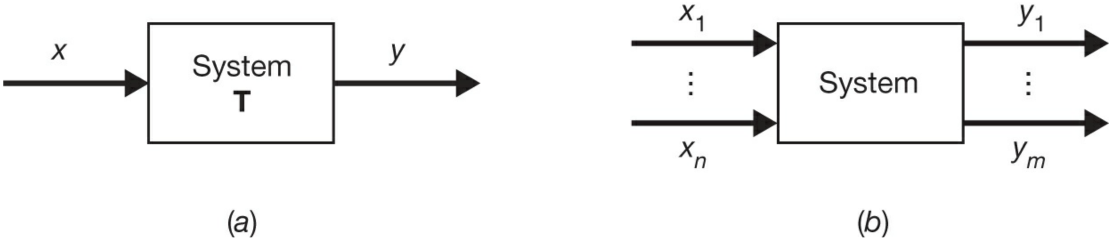
$$A\cos(\Omega_0 n + \theta) = A\,\mathrm{Re}\{e^{j(\Omega_0 n + \theta)}\} \tag{1.59}$$



Sinusoidal sequences. (a) $x[n] = \cos(\pi n/6)$; (b) $x[n] = \cos(n/2)$.

3. Systems and Classification of Systems

System Representation



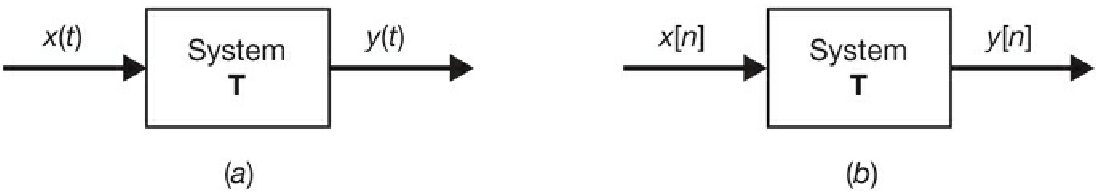
$$y = \mathbf{T}x \tag{1.60}$$

y: output   *x: input*   *T: operator*

Deterministic and Stochastic Systems

If the input and output signals x and y are deterministic signals, then the system is called a deterministic system. If the input and output signals x and y are random signals, then the system is called a stochastic system.

Continuous-Time and Discrete-Time Systems



If the input and output signals x and y are continuous-time signals, then the system is called a continuous-time system (left). If the input and output signals are discrete-time signals or sequences, then the system is called a discrete-time system (right).

Systems with Memory and without Memory

A system is said to be memoryless if the output at any time depends on only the input at that same time. Otherwise, the system is said to have memory. An example of a memoryless system is a resistor R with the input *x(t)* taken as the current and the voltage taken as the output *y(t)*. The input-output relationship (Ohm's law) of a resistor is

$$y(t) = Rx(t) \tag{1.61}$$

An example of a system with memory is a capacitor *C* with the current as the input *x(t)* and the voltage as the output *y(t)*; then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau \tag{1.62}$$

A second example of a system with memory is a discrete-time system whose input and output sequences are related by:

$$y[n] = \sum_{k=-\infty}^{n} x[k] \tag{1.63}$$

Linear Systems and Nonlinear Systems

If the operator T satisfies the following conditions, then T is called a linear operator and the system represented by a linear operator T is called a linear system:

Additivity:

$$\mathbf{T}\{x_1 + x_2\} = y_1 + y_2 \tag{1.66}$$

Homogeneity (or Scaling):

$$\mathbf{T}\{\alpha x\} = \alpha y \tag{1.67}$$

Eqs. (1.66) and (1.67) can be combined into a single known as superposition property

$$\mathbf{T}\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2 \tag{1.68}$$

Any other systems are classified as nonlinear system.

Time-Invariant and Time-Varying Systems

A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, for a continuous-time system, the system is time-invariant if

$$\mathbf{T}\{x(t-\tau)\} = y(t-\tau) \tag{1.71}$$

for any real value of τ. For a discrete-time system, the system is time invariant (or shift-invariant) if

$$\mathbf{T}\{x[n-k]\} = y[n-k] \tag{1.72}$$

Linear Time-Invariant Systems

If the system is linear and also time-invariant, then it is called a linear time invariant (LTI) system.

Stable Systems

A system is bounded-input/bounded-output (BIBO) stable if for any bounded input x defined by

$$|x| \leq k_1 \quad (1.73)$$

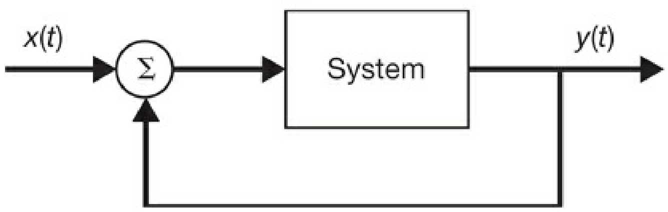
the corresponding output y is also bounded defined by

$$|y| \leq k_2 \quad (1.74)$$

where *k1* and *k2* are finite real constants. An unstable system is one in which not all bounded inputs lead to bounded output.

Feedback Systems

A special class of systems of great importance consists of systems having feedback. In a feedback system, the output signal is fed back and added to the input to the system as shown below.



Glossary – English/Chinese Translation

English	Chinese
Unit Step Function	单位步进函数
Heaviside Unit Function	Heaviside 单元功能
Unit Impulse Function	单位脉冲函数
Dirac Delta Function	狄拉克 Delta 函数
Time Shifted	时移
Exponential Function	指数函数
Euler's Formula	欧拉公式
Real Signal	真实信号
Complex Signal	复杂信号
Periodic Signal	周期性信号
Fundamental Period	基本面
Fundamental Frequency	基频
Real Part and Imaginary Part	实部和虚部
Radian	弧度
Discrete Time Signal	离散时间信号
Continuous Time Signal	连续时间信号
Unit Step Sequence	单位步长序列
Unit Impulse Sequence	单位脉冲序列
Sinusoidal Sequence	正弦序列
Operator	算子
Deterministic System	确定性系统
Stochastic System	随机系统
Nonlinear System	非线性系统
Time Invariant System	时间不变系统
Time Varying System	时变系统
Bounded Input	有界输入
Bounded Output	有界输出