# Dr. Norbert Cheung's Lecture Series

# Level 2z      Topic no: 03

# Introduction to Signal and Systems

**Contents**



**Reference:**
xxx – Schaum's Outline Series

**Email:** norbertcheung@szu.edu.cn
**Web Site:** http://norbert.idv.hk
**Last Updated:** 2024-02

## 8. **Classification of Signals**

A signal is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon.

For instance, in an RC circuit the signal may represent the voltage across the capacitor or the current flowing in the resistor.

### 2. Continuous-Time and Discrete-Time Signals

A signal x(t) is a continuous-time signal if t is a continuous variable. If $t$ is a discrete variable—that is, $x(t)$ is defined at discrete times—then $x(t)$ is a discrete-time signal. Since a discrete-time signal is defined at discrete times, a discrete-time signal is often identified as a sequence of numbers, denoted by $\{xn\}$ or $x[n]$, where $n$ = integer.

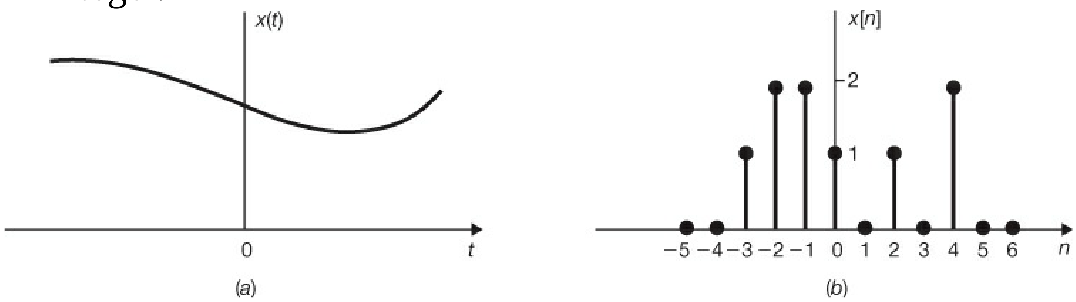



Fig. 1-1 Graphical representation of (a) continuous-time and (b) discrete-time signals.

### 3. Sampled Discrete Time Signals

A discrete-time signal $x[n]$ may be obtained by sampling a continuous-time signal $x(t)$ such as:

$$x(t0), x(t1), \ldots, x(tn), \ldots \qquad \text{or} \qquad x_n = x[n] = x(t_n)$$

and $x_n$'s are called samples and the time interval between them is called the sampling interval. When the sampling intervals are equal (uniform sampling), then

$$x_n = x[n] = x(nT_s)$$

where the constant $T_s$ is the sampling interval.

A discrete-time signal x[n] can be defined in two ways:

8. We can specify a rule for calculating the nth value of the sequence.

$$x[n] = x_n = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

or

$$\{x_n\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \left(\frac{1}{2}\right)^n, \ldots\right\}$$

2. We can also explicitly list the values of the sequence.

$$\{x_n\} = \{\ldots, 0, 0, 1, 2, 2, 1, 0, 1, 0, 2, 0, 0, \ldots\}$$
$$\uparrow$$

or

$$\{x_n\} = \{1, 2, 2, 1, 0, 1, 0, 2\}$$
$$\uparrow$$

We use the arrow to denote the *n=0* term. We shall use the convention that if no arrow is indicated, then the first term is *n=0* and all the values of the sequence are zero for *n<0*.

The sum and product of two sequences are defined as follows:

$$\begin{aligned} \{c_n\} &= \{a_n\} + \{b_n\} \rightarrow c_n = a_n + b_n \\ \{c_n\} &= \{a_n\}\{b_n\} \quad \rightarrow c_n = a_n\, b_n \\ \{c_n\} &= \alpha\{a_n\} \qquad \rightarrow c_n = \alpha\, a_n \qquad \alpha = \text{constant} \end{aligned}$$

## 4. Real and Complex Signals

A signal *x(t)* is a real signal if its value is a real number, and a signal *x(t)* is a complex signal if its value is a complex number. A general complex signal *x(t)* is a function of the form

$$x(t) = x_1(t) + jx_2(t) \tag{1.1}$$

where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$

## 5. Deterministic and Random Signals

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time *t*.

Random signals are those signals that take random values at any given time

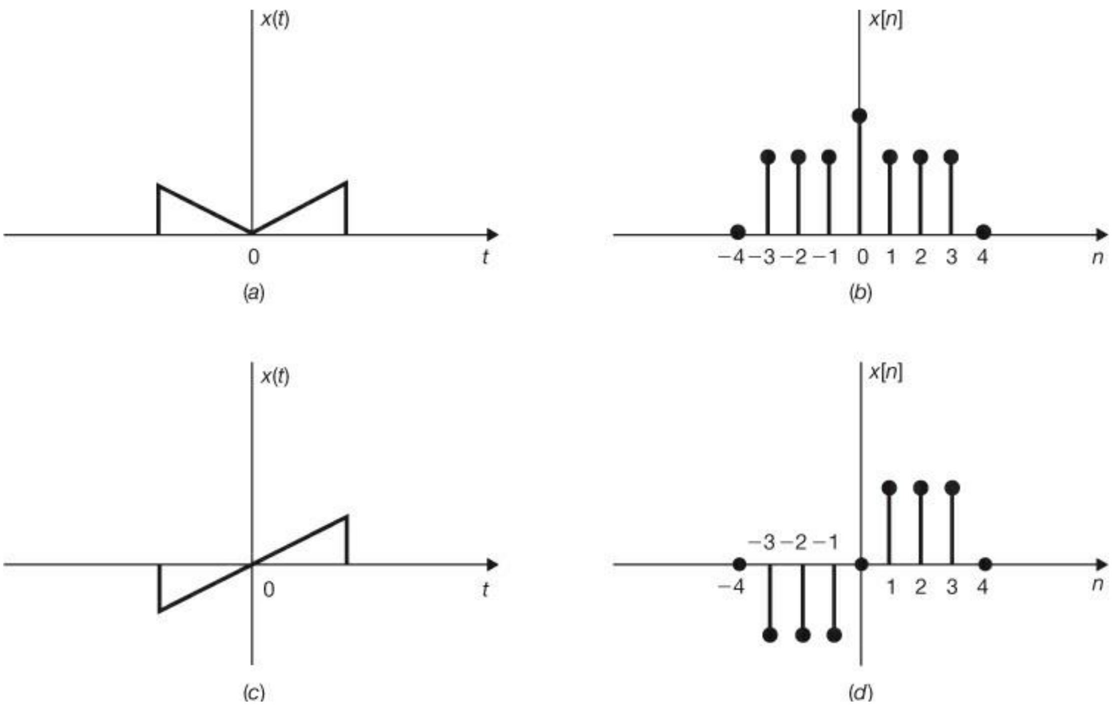
## 6. Even and Odd Signals

A signal *x(t)* or *x[n]* is referred to as an even signal if:

$$
\begin{aligned}
x(-t) &= x(t) \\
x[-n] &= x[n]
\end{aligned}
\tag{1.2}
$$

A signal *x(t)* or *x[n]* is referred to as an odd signal if

$$
\begin{aligned}
x(-t) &= -x(t) \\
x[-n] &= -x[n]
\end{aligned}
\tag{1.3}
$$




Examples of even signals (a and b) and odd signals (c and d).

Any signal *x(t)* or *x[n]* can be expressed as a sum of two signals, one of which is even and one of which is odd.

$$x(t) = x_e(t) + x_o(t)$$
$$x[n] = x_e[n] + x_o[n] \qquad (1.4)$$

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} \qquad \text{even part of } x(t)$$
$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\} \qquad \text{even part of } x[n] \qquad (1.5)$$

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} \qquad \text{odd part of } x(t)$$
$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\} \qquad \text{odd part of } x[n] \qquad (1.6)$$

Note that:

1. The product of two even signals or two odd signals is an even signal.
2. The product of an even signal and an odd signal is an odd signal.

## <u>7. Periodic and Non-periodic Signals</u>

A continuous-time signal x(t) is said to be periodic with period T if there is a positive nonzero value of T for which

For analogue signals:

$$x(t + T) = x(t) \qquad \text{all } t \qquad (1.7)$$
$$x(t + mT) = x(t) \qquad (1.8)$$

for all *t* and any integer *m*

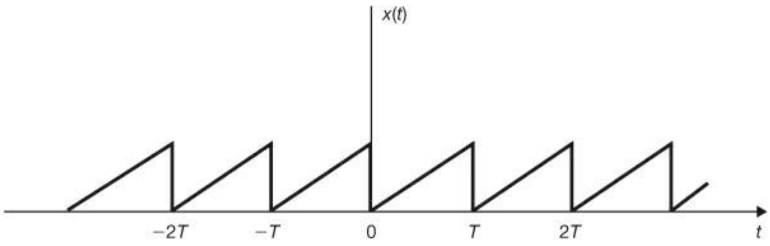
The <u>fundamental period</u> $T_0$ of *x(t)* is the smallest positive value of *T* for which Eq. (1.7) holds

For digital signals:

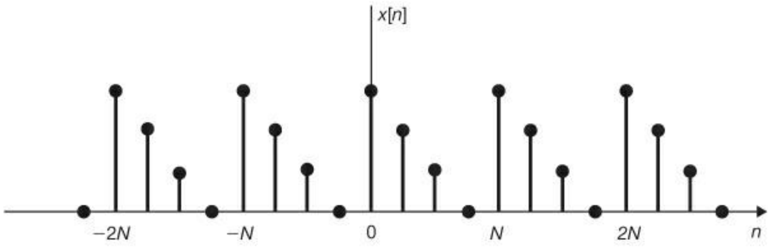
$$x[n + N] = x[n] \qquad \text{all } n \qquad (1.9)$$
$$x[n + mN] = x[n] \qquad (1.10)$$

for all *n* and any integer *m*

The <u>fundamental period</u> $N_0$ of *x[n]* is the smallest positive integer *N* for which Eq. (1.9) holds.




(a)




(b)

## 8. Energy and Power Signals

Consider *v(t)* to be the voltage across a resistor *R* producing a current *i(t).* The instantaneous power *p(t)* per ohm is defined as:

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t) \qquad (1.11)$$

Total energy E and average power P on a per-ohm basis are

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules} \qquad (1.12)$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts} \qquad (1.13)$$

Similarly, for a discrete-time signal *x[n]*, the normalized energy content *E* of *x[n]* is defined as:

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad (1.16)$$

The normalized average power P of x[n] is defined as:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \qquad (1.17)$$

--- END ---

# Glossary – English/Chinese Translation

| English | Chinese |
|---|---|
| signals and systems | 信号和系统 |
| continuous time signal | 连续时间信号 |
| discrete time signal | 离散时间信号 |
| sampled discrete time signal | 采样离散时间信号 |
| real and complex signals | 真实信号和复杂信号 |
| deterministic signal | 确定性信号 |
| random signal | 随机信号 |
| even signal | 均匀信号 |
| odd signal | 奇数信号 |
| periodic signal | 周期性信号 |
| non-periodic signal | 非周期性信号 |
| energy and power | 能源和电力 |
| physical quantity | 物理量 |
| variable | 变量 |
| phenomenon | 现象 |
| RC circuit | RC 电路 |
| capacitor | 电容器 |
| samples | 样品 |
| sampling interval | 采样间隔 |
| sequence | 序列 |
| real and complex number | 实数和复数 |
| analogue signal | 模拟信号 |
| digital signal | 数字信号 |
| fundamental period | 基本周期 |
| instantaneous power | 瞬时功率 |
| average power | 平均功率 |
| normalized energy | 归一化能量 |
| normalized average power | 归一化平均功率 |

## **Your Notes:**