**DEPARTMENT OF ELECTRICAL ENGINEERING**

**SOLUTION & MARKING SCHEME**

**(Semester 1, 2022/23)**

| | |
|---|---|
| SUBJECT (Code & Title) | EE3005/EE3005A/EE3005B Systems and Control |

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|---|---|
| SUBJECT EXAMINER | Dr Xin Yuan<br>Dr Fung Yu-fai |
| SUBJECT MODERATOR | Prof. Alan P.T. Lau |

| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|
| 1 | 1. Solve: Differential equation using Laplace transform<br>2. Get the relationship between the output and input in s domain<br>3. Get the x(t) in time domain | 5<br>4<br>3 |

| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|

Q1

$\ddot{x} + 4\dot{x} - 4 = -3x$

By writing $\mathcal{L}[x(t)] = X(s)$,

We obtain $\mathcal{L}[\dot{x}] = sX(s) - x(0)$

$= sX(s)$ 1

$\mathcal{L}[\ddot{x}] = s^2X(s) - s\dot{x}(0) - x(0)$

$= s^2X(s)$ 1

$\Rightarrow s^2X(s) + 4sX(s) - \frac{4}{s} = -3X(s)$ 3

$(s^2+4s+3)X(s) = \frac{4}{s}$

$X(s) = \frac{4}{s(s+1)(s+3)}$

$= \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+3}$

$= \frac{as^2+4as+3a+bs^2+3bs+cs^2+cs}{s(s+1)(s+3)}$

$\begin{cases} a+b+c=0 \\ 4a+3b+c=0 \\ 3a=4 \end{cases}$

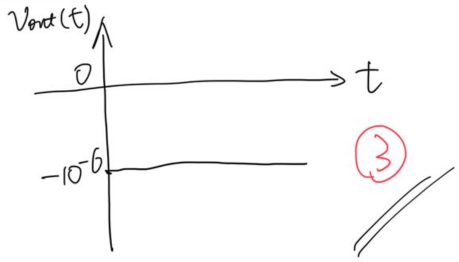
$\Rightarrow a = \frac{4}{3} \Rightarrow \begin{cases} b+c = -\frac{4}{3} \\ 3b+c = -\frac{16}{3} \end{cases}$

$\Rightarrow 2b = -\frac{12}{3}, \quad b = -\frac{6}{3} = -2$

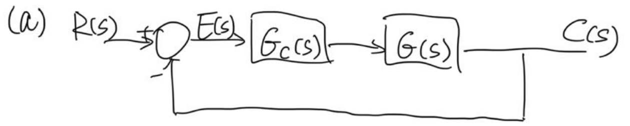
$\Rightarrow c = -\frac{4}{3} + \frac{6}{3} = \frac{2}{3}$

$\Rightarrow X(s) = \frac{\frac{4}{3}}{s} + \frac{-2}{s+1} + \frac{\frac{2}{3}}{s+3}$

$x(t) = \frac{4}{3} - 2e^{-t} + \frac{2}{3}e^{-3t}$

| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|
| 2 | 1. Get the relationship between the Vout(s) and Vin(s) in s domain<br>Note: If they use jw format, it is correct.<br>2. Get the Vout(t) in time domain using inverse laplace transform<br>3. Draw the graft of Vout(t).<br>method 1 $C\frac{dV_{in}}{dt} = -\frac{V_{out}}{R}$ 3<br>$\Rightarrow V_{out} = -RC\frac{dV_{in}}{dt}$ 3<br>$= -10\times100\times10^{-9}\times1 = -10^{-6}$ 3 $, t\geq0$<br>method 2<br>$\frac{V_{in}(s)}{1/sC} = -\frac{V_{out}(s)}{R}$ 3<br>$\Rightarrow V_{out}(s) = -sCRV_{in} = -\frac{10^{-6}s}{s^2} = -\frac{10^{-6}}{s}$ 3<br>$\Rightarrow V_{out}(t) = -10^{-6}$ 3 $, t\geq0$<br>graft<br><br> | 6<br><br>3<br>3 |
| 3(a) | 1. Obtain the closed-loop transfer function<br>2. Obtain the steady-state error of the overall system for the unit step input | 3<br>3 |
| 3(b) | 1. Obtain the closed-loop transfer function<br>2. Obtain the steady-state error | 1<br>2 |
| 3(c) | 1. The system is stable<br>2. Stretch the zero-pole location in s domain | 2<br>3 |

(a)




$$E(s) = R(s) - C(s) = R(s) - E(s)G_c(s)G(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G_c(s)G(s)} \quad (2)$$

$$= \frac{R(s)}{1+\frac{4}{(s+4)(s+5)}} = \frac{s^2+9s+20}{s^2+9s+24} R(s) \quad (1)$$

$$= \frac{s^2+9s+20}{s(s^2+9s+24)} \quad (1)$$

$$e(\infty) = \lim_{s\to 0} sE(s) = \frac{s^2+9s+20}{s^2+9s+24} = \frac{20}{24} = \frac{5}{6} \quad (2)$$

(b)

$$G_c(s)G(s) = \left(1+\frac{0.1}{s}\right)\frac{4}{(s+4)(s+5)} = \frac{4s+0.4}{s(s+4)(s+5)}$$

$$E(s) = \frac{R(s)}{1+G_c(s)G(s)} = \frac{\frac{1}{s}}{1+\frac{4s+0.4}{s(s+4)(s+5)}} \qquad \begin{array}{c} \parallel \\ \frac{4s+0.4}{s^3+9s^2+20s} \end{array}$$

$$= \frac{s^2+9s+20}{s^3+9s^2+24s+0.4} \quad (1)$$

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s(s^2+9s+20)}{s^3+9s^2+24s+0.4} = 0 \quad (2)$$

(c)

$$G_c(s)G(s) = \frac{s+4}{s} \cdot \frac{4}{(s+4)(s+5)} = \frac{4}{s(s+5)}$$

closed loop transfer function:

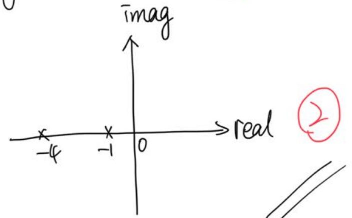
$$G_{closed}(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s^2+5s}}$$

$$= \frac{4}{s^2+5s+4}$$

$$= \frac{4}{(s+4)(s+1)} \quad (1)$$

The two poles $\begin{cases} s=-4 \\ s=-1 \end{cases}$ (1) are both at left half-plane.

so the system is stable. (1)




(2)

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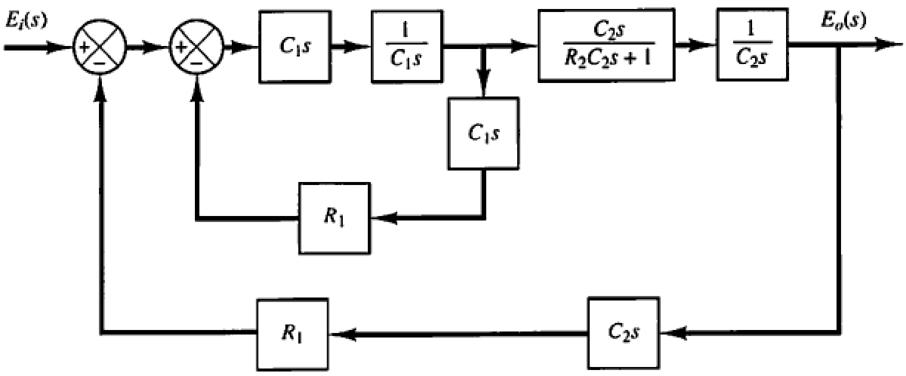

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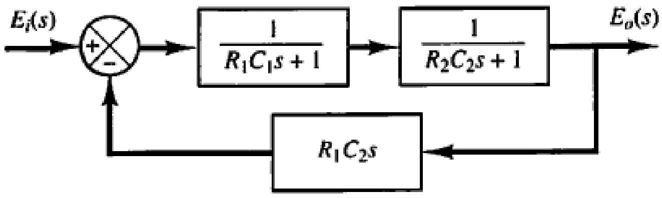

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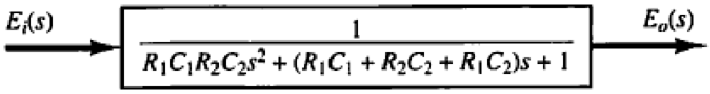

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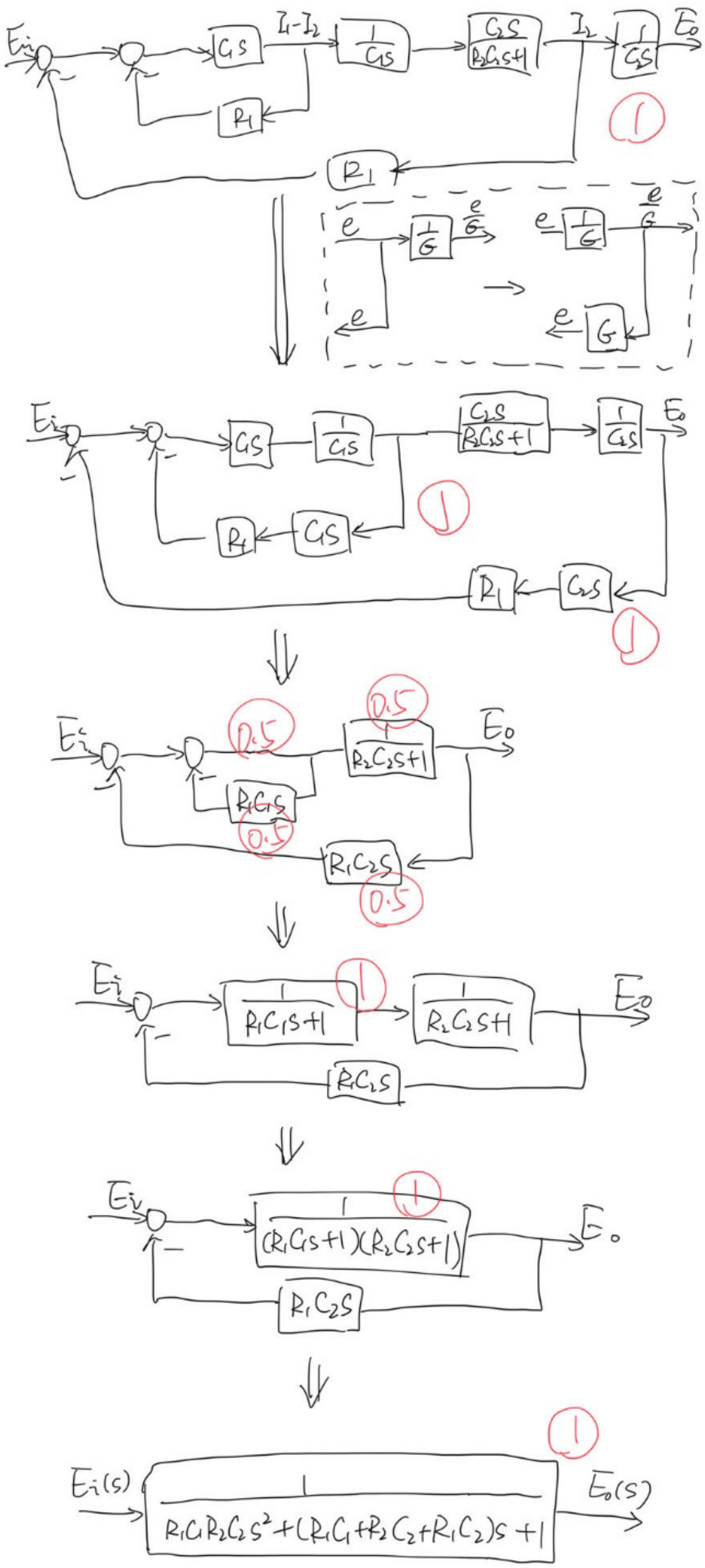
| |   | |

| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|
| 5 | 1. List the Routh table<br>2. Using the correct routh-Hurwitz criterion<br>3. Getting the correct K value | 4<br>4<br>2 |

**5.11. The characteristic equation of a given system is**

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

**What restrictions must be placed upon the parameter $K$ in order to insure that the system is stable?**

**The Routh table for this system is**

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 11 | $K$ |
| $s^3$ | 6 | 6 | 0 |
| $s^2$ | 10 | $K$ | 0 |
| $s^1$ | $\dfrac{60-6K}{10}$ | 0 | |
| $s^0$ | $K$ | | |

**For the system to be stable, the following restrictions must be placed upon the parameter $K$: $60 - 6K > 0$ or $K < 10$, and $K > 0$. Thus $K$ must be greater than zero and less than 10.**

| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|
| 6(a) | 1. Getting the frequency response of the transfer function(jw)<br>2. Sketching a polar plot | 2<br><br>1 |

Q6(a)

- $G(j\omega) = \dfrac{1}{j\omega+10} = \dfrac{10-j\omega}{(j\omega+10)(10-j\omega)} = \dfrac{10}{100+\omega^2} - j\dfrac{\omega}{100+\omega^2}$

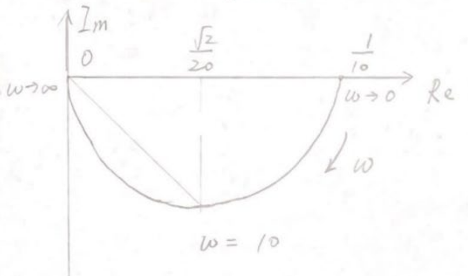
- Magnitude:

$$A(\omega) = \sqrt{\left(\frac{10}{100+\omega^2}\right)^2 + \left(-\frac{\omega}{100+\omega^2}\right)^2} = \frac{1}{\sqrt{100+\omega^2}}$$

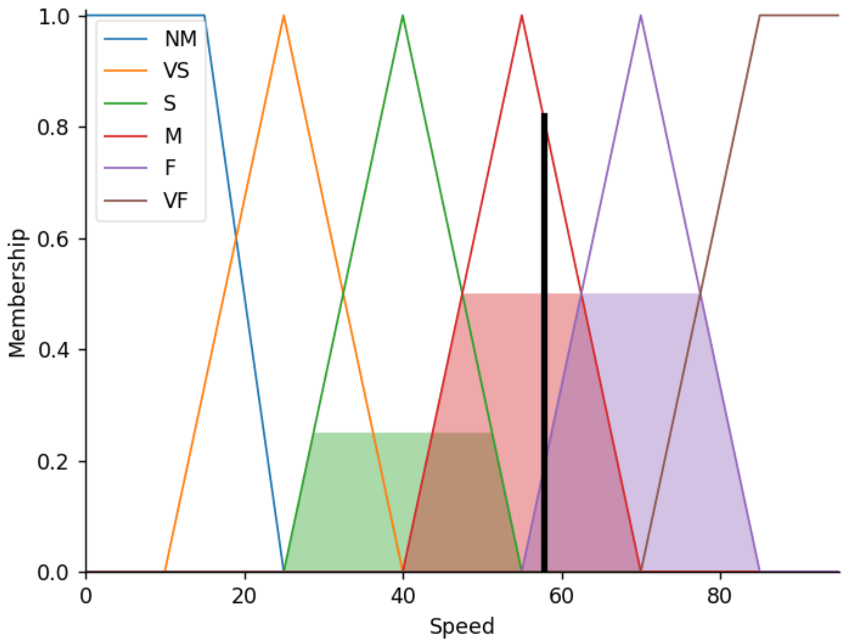
- Phase:

$$\varphi(\omega) = \angle\left(\frac{10}{100+\omega^2} - j\frac{\omega}{100+\omega^2}\right) = \tan^{-1}\left(\frac{-\frac{\omega}{100+\omega^2}}{\frac{10}{100+\omega^2}}\right) = -\tan^{-1}\left(\frac{\omega}{10}\right)$$

| $\omega$ | 0 | 10 | $\infty$ |
|---|---|---|---|
| $A(\omega)$ | $\frac{1}{10}$ | $\frac{\sqrt{2}}{20}$ | 0 |
| $\varphi(\omega)$ | $0^\circ$ | $-45^\circ$ | $-90^\circ$ |



| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|
| 6(b) | 1. Obtain the frequency response under w=10 for G(s)<br>2. Obtain the steady-state output in time-domain<br><br>Q6(b)<br>· $A(10) = \frac{1}{\sqrt{100+10^2}} = \frac{\sqrt{2}}{20}$<br>· $\varphi(10) = -\tan^{-1}(\frac{10}{10}) = -45^\circ$<br>· Steady-state output for $x(t) = \cos(10t - \frac{\pi}{3})$ :<br>$y(t) = A(10)\cos(10t - \frac{\pi}{3} + \varphi(10))$<br>$= \frac{\sqrt{2}}{20}\cos(10t - \frac{\pi}{3} - \frac{\pi}{4})$<br>$= \frac{\sqrt{2}}{20}\cos(10t - \frac{7}{12}\pi)$ | 3<br>4 |
| 7(a)<br><br><br><br><br>7(b) | 1. Is the overall system stable or unstable<br>2. Provide a detailed reason<br><br>Answer: The overall system is stable, because open loop gain is less than 0db at open loop angle of -180 degrees.<br><br>1. The system is a low-pass filter<br>2. Provide a detailed reason<br><br>Answer: The system is a low-pass filter. Because when the frequency converges to gain is greater than 1 (or 0db), the signal can pass. But when the frequency conver infinite, the gain converges to 0, the signal cannot pass. | 2<br>1<br><br><br>2<br>1 |
| 8(a)<br><br><br>8(b) | 1. Must be a real-life example in the control and system perspective. (Objective quiz)<br><br><br>1. The major differences between the closed-loop system and open-loop system is that the closed-loop system has certain level to suppress the parameter disturbances and external disturban<br>2. Furthermore, the closed-loop system can suppress the measurement noises. | 2<br><br><br>3<br><br>3 |
| 9 | Temperature at 45 then it has memberships in C (0.2) and M (0.6)<br>Humidity at 60, it has membership in M (0.5) and H (0.5)<br>So 4 rules will be fired<br>The output has S = 0.2; M (0.5 & 0.2) so M (0.5) and F (0.5)<br>The result of output is | 10 |

| QUESTION NO. | SOLUTION | MARKS |
|---|---|---|
| |  <br>The speed is about 57.7 (duty-cycle for the PWM) control<br>As fuzzy control is based on fuzzy set so the output is not 'exact' so it may not be suitable for speed control of an autonomous vehicle if the exact speed under certain conditions is important. For example if you want to keep the vehicle running at 30 Km per hour then the fuzzy controller may output signal to keep the vehicle in close to 30 Km per hour only. | |
| 10(a) | 1. Obtain the pid controller transfer function using the closed-loop first order system<br>2. Analyse the dynamic response by changing the system bandwidth via PID controller | 1<br>1 |
| 10(b) | 1. Design a pid controller transfer function using the closed-loop second order system<br>2. Dynamic response by zero and pole of the second order system. | 1<br>1 |
| 10(c) | 1. Design a model predictive controller, which can achieve deadbeat control. | 2 |
| 10(d) | 1. There is high measurement noise in output.<br>2. The differential part in digital system cannot be exactly equal to the real part. | 1<br>1 |
| 10(e) | 1. Create a high-order system to handle<br>2. Use a high-order pid controller or use a model predictive control with disturbance observe | 1<br>1 |