THE HONG KONG POLYTECHNIC UNIVERSITY

DEPARTMENT OF ELECTRICAL ENGINEERING

This question paper has a total of $\frac{7}{2}$ pages (attachments included).

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO.

Q1. Find the solution x(t) of the differential equation with Laplace transform, and the initial condition is presented as: $x(0) = 0$, $\dot{x}(0) = 0$, $t \ge 0$. (12 marks)

 $\ddot{x} + 4\dot{x} - 4 = -3x$

Q2. With the zero initial condition in **Figure 1**, R=10Ω, C=100nF, please calculate the output response Vout(t) if the input Vin(t) is a unit ramp signal. After that, please draw the graph of Vout(t) in time domain ($t \ge 0$).

(12 marks)

Figure 1 RC circuit with ideal amplifier

Q3. Given the control system as follows:

Figure 2 Block diagram of the overall system

$$
G(s) = \frac{4}{(s+4)(s+5)}
$$

- (a) For $G_c(s) = 1$, find the steady-state error of the overall system for the unit step input. (6 marks)
- (b) If the controller is given by $G_c(s) = 1 + \frac{0.1}{s}$, find the steady-state error for the unit step input.

(3 marks)

(c) If the controller is given by $G_c(s) = \frac{s+4}{s}$, is the system stable, critically stable, or unstable? Please clarify the system stability and stretch the zero-pole location in s domain. (5 marks)

Q4. Simplify the block diagram shown in **Figure 3**, and calculate the closed loop transfer function $E_o(s)/E_i(s)$. (8 marks)

Figure 3 Block diagram of the overall system

Q5. The characteristic equation of a given system is shown below. (10 marks) $s^4 + 6s^3 + 11s^2 + 6s + K = 0$ Please find the range of K stability using the Routh-Hurwitz stability criterion.

Q6. Consider the following first-order transfer function:

$$
G(s) = \frac{1}{(s+10)}
$$

(a) Sketch a polar plot of this transfer function. (3 marks) (b) Please obtain the steady-state output in time domain if the input is $x(t) = \cos(10t \frac{\pi}{3}$ (t≥0). (7 marks)

Q7. The system is a unit negative feedback system, and **Figure 4** is the Bode Plot of the open-loop system. Please answer the following questions and give detailed reasons.

Figure 4 Bode Plot of the open-loop system

Q8. Please answer the following questions regarding the closed-loop and open-loop control systems.

- (a) Give one real-life example of the application for the closed-loop and open-loop control systems in the lecture theatre. You must explain your answer properly. (2 marks)
- (b) Compare the major differences between closed-loop and open-loop control systems. You should analyze the disturbance rejection and measurement noises suppression of the two systems by giving a detailed scientific analysis. (6 marks)

Q9. A fuzzy controller is defined by two inputs T (Temperature) and H (Humidity), the output is F(Speed of the fan). Since the fan is controlled by PWM, the output is the duty cycle of the PWM signal. The fuzzy memberships for T, H and F are depicted in **Figure 5**, **6** and **7**, and the fuzzy rules are given in **Table 1**. The rules are based on the format IF temperature AND humidity THEN Fan speed. Now if $T = 45$ and $H = 60$, determine the corresponding Fan Speed based on the Min-Max and CoG method. Should fuzzy control be applied in speed control of an autonomous vehicle? Explain your answer in detail. (10 marks)

 $VC - very cold$; $C - cold$; $M - moderate$; $H - hot$; $VH - very hot$ VD – very dry; D – dry; M – medium; H – humid; VH – very humid VS – very slow; S – slow; M – medium; F – fast; VF – very fast; NM – no movement

Figure 5 Fuzzy set for temperature **Figure Figure** 6 Fuzzy set for humidity

Q10. The position control of the robotic arm is shown in **Figure 8**. The input position signal is a unit step function.

Figure 8 Block diagram of the robotic arm control

- (a) If the plant model $G_p(s) = \frac{6}{(2s+6)}$, please design an optimum PID controller $(G_c(s))$ to make sure the output position signal can be easily controlled while the designed closed-loop system must be a first-order system and there is no steady-state error. Please also analyze how to adjust the dynamic response of the robotic arm position via PID parameters. Note: the values of K_p , K_i , and K_d can be zero. (2 marks)
- (b) If the plant model $G_p(s) = \frac{1}{s}$, please design an optimum PID controller $(G_c(s))$ while the designed closed-loop system must be a *second-order system* and there is no steady-state error. Please also analyze how to control the dynamic response of the robotic arm position via PID parameters. Note: the values of K_p , K_i , and K_d can be zero. (2 marks)
- (c) If the plant model $G_p(s) = \frac{10}{s+10}$, please roughly design a model predictive controller to achieve a fast position tracking ability. (2 marks)
- (d) If the feedback path in **Figure 8** is removed and the controller is designed as $G_c(s) = G_p^{-1}(s)$, please explicitly illustrate the unrealistic reason in practical digital systems (*The answer must be less than 40 words*). (2 marks)
- (e) If the robotic arm input is a rather complex command e.g. $\theta_i(s) = \frac{5s^5 + 3s + 6}{s^{50} + 12s^{30} + 23}$ and there exists disturbances in real systems, please give a suitable approach to handle each issue. (2 marks)

------ End of Paper ------

Time function $f(t)$		Laplace transform $L[f(t)] = F(s)$
	Unit impulse $\delta(t)$	
	Unit step 1	1/s
$\frac{2}{3}$	Unit ramp t	$1/s^2$
$\overline{4}$.п	n! $\frac{1}{s^{n+1}}$
5	e^{-at}	$s + a$
6	$1-e^{-at}$	$s(s+a)$
7	$\sin \omega t$	$\overline{s^2 + \omega^2}$
8	$\cos \omega t$	$s^2 + \omega^2$
9	e^{-at} sin ωt	$\frac{(s+a)^2+\omega^2}{s+a}$
10	e^{-at} $\cos \omega t$	$(s+a)^2 + \omega^2$

Reference Formulae

Derivatives: The Laplace transform of a time derivative is

Where $f(0)$, $f'(0)$ are the initial conditions, or the values of $f(t)$, $d/dt f(t)$ etc. at $t = 0$

