Question 1 (20 marks)

Determine the range of K such that the system with the characteristic equation, $\Delta(s) = s^4 + 4s^3 + 3s^2 + 4s + K = 0$, is stable.

ANSWER

Question 2 (30 marks)

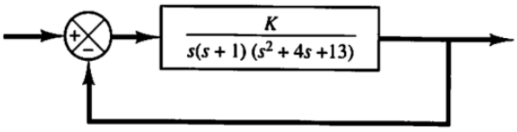
Plot the Bode diagram for the transfer function,

$$G(s) = \frac{10}{s + 20}$$

ANSWER

Question 3 (40 marks)

Sketch the root locus plot for the system shown in Fig. Q5 below:



Show all your working steps and calculations.

If there is not enough space, use the space below to answer…

SOLUTION

Question 1

$$
\begin{array}{c|lll}
s^4 & 1 & 3 & K \\
s^3 & 4 & 4 & \\
s^2 & \dfrac{(4)(3)-(1)(4)}{4} = 2 & \dfrac{(4)(K)-(1)(0)}{4} = K & \\
s^1 & \dfrac{(2)(4)-(4)(K)}{2} & & \\
s^0 & K & &
\end{array}
$$

$K > 0$

$$\frac{8-4K}{2} > 0, K < 2$$

$\therefore 0 < K < 2$

Question 2

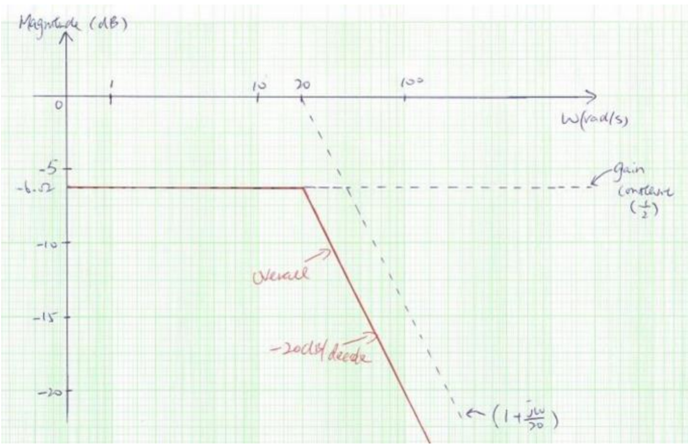
Step 1: Replacing s by $j\omega$ and rewrite the transfer function as a product of basic factors

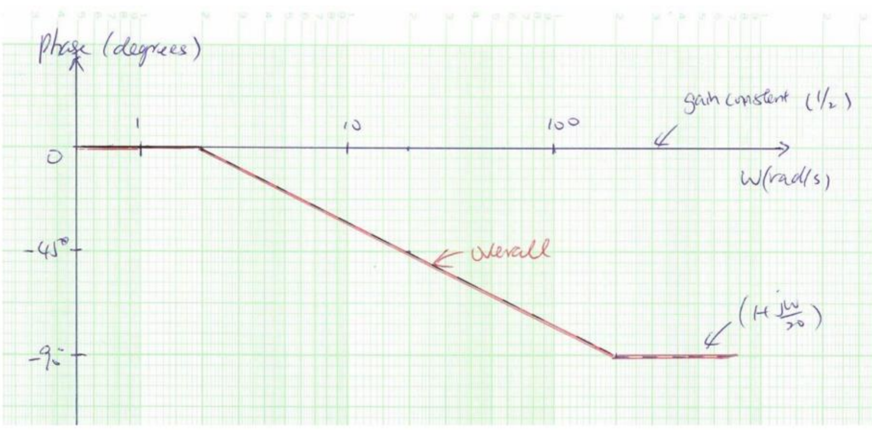
$$G(j\omega) = \frac{10}{j\omega + 20} = \frac{\frac{10}{20}}{\frac{j\omega + 20}{20}} = \frac{1/2}{\frac{j\omega}{20} + 1}$$

$$\text{Magnitude of the Gain constant} \quad = 20\log\left(\frac{1}{2}\right) = -6.01 \text{ dB}$$

Step 2: Identify the corner frequencies

Since there is only one first order factor (Pole), the corner frequency is, $\omega = 20$ rad/s





Question 3

Solution. The open-loop poles are located at $s = 0$, $s = -1$, $s = -2 + j3$, and $s = -2 - j3$. A root locus exists on the real axis between points $s = 0$ and $s = -1$. The asymptotes are found as follows:

$$\text{Angles of asymptotes} = \frac{\pm 180°(2k + 1)}{4} = 45°, -45°, 135°, -135°$$

The intersection of the asymptotes and the real axis is found from

$$\sigma_a = -\frac{0 + 1 + 2 + 2}{4} = -1.25$$

The breakaway and break-in points are found from $dK/ds = 0$. Noting that

$$K = -s(s + 1)(s^2 + 4s + 13) = -(s^4 + 5s^3 + 17s^2 + 13s)$$

we have

$$\frac{dK}{ds} = -(4s^3 + 15s^2 + 34s + 13) = 0$$

from which we get

$$s = -0.467, \qquad s = -1.642 + j2.067, \qquad s = -1.642 - j2.067$$

The point $s = -0.467$ is on a root locus. Therefore, it is an actual breakaway point. The gain values K corresponding to points $s = -1.642 \pm j2.067$ are complex quantities. Since the gain values are not real positive, these points are neither breakaway nor break-in points.

The angle of departure from the complex pole in the upper half s plane is

$$\theta = 180° - 123.69° - 108.44° - 90°$$

or

$$\theta = -142.13°$$

$$s^4 + 5s^3 + 17s^2 + 13s + K = 0$$

by substituting $s = j\omega$ into it we obtain

$$(j\omega)^4 + 5(j\omega)^3 + 17(j\omega)^2 + 13(j\omega) + K = 0$$

or

$$(K + \omega^4 - 17\omega^2) + j\omega(13 - 5\omega^2) = 0$$

from which we obtain

$$\omega = \pm 1.6125, \quad K = 37.44 \quad \text{or} \quad \omega = 0, \quad K = 0$$