Dr. Norbert Cheung's Series in Electrical Engineering

Level 2 Topic no: 29

PID Control and Tuning

Contents

Reference:

"Modern Control Engineering", Ogata, Prentice Hall

"Feedback Control of Dynamic Systems" Frankin & Powell, Addison Wesley

"Automatic Control Systems" BC Kuo, Prentice Hall

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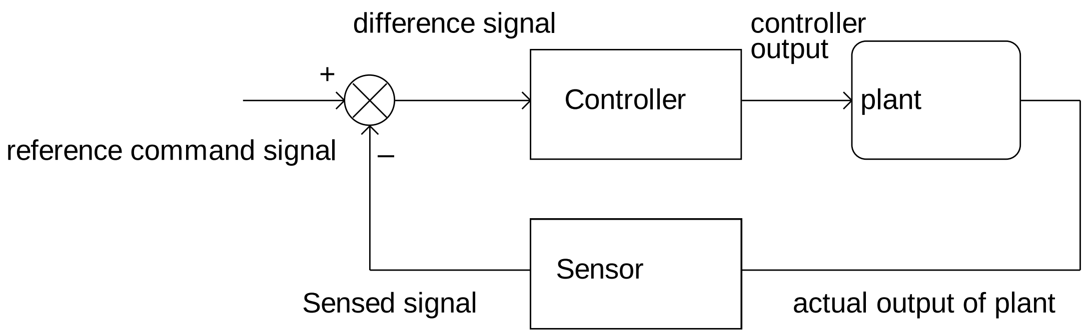
1. Introduction

Feedback Control System: A system that maintains a prescribed relationship between the output and the reference input by comparing them and the using the difference as a means of control.

Closed Loop System: Feedback control systems are called closed loop system. In the control field, the closed loop control always implies the use of feedback control action to reduce the error.

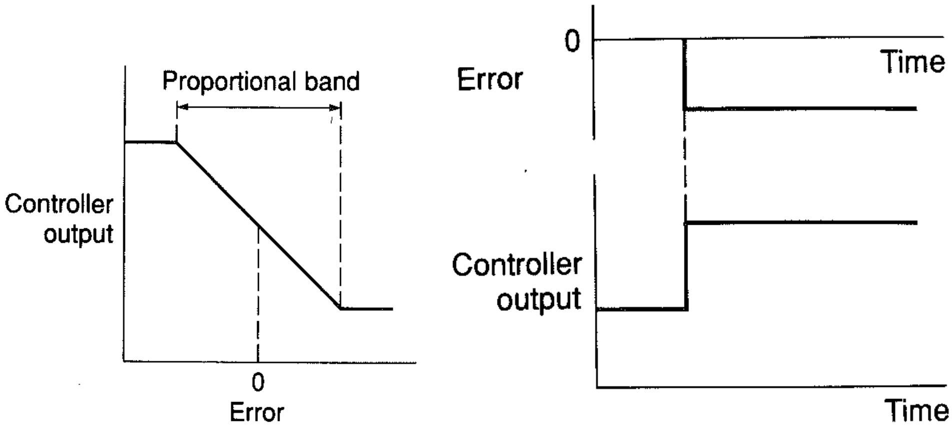
Open Loop Control: The control will not use the reference output signal. Open loop control can only be used if the input and output relationship is exactly known, and when there is no external disturbance.



2. The Basic PID Control Algorithm

Almost 80% of all controllers in the world use PID control, or variations of PID control. It is the easiest to use, and easiest to understand and visualize.

Proportional Control

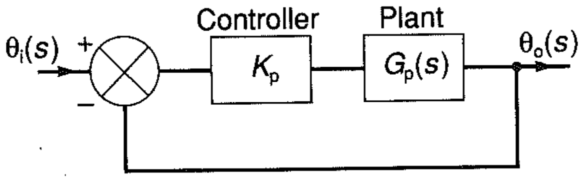


With proportional control, the output of the controller is directly proportional to its input, the input being the error signal *e* which is a function of time. Thus

$$\text{Output} = K_p e \quad [1]$$

where K_p is a constant called the *proportional gain*. The output from the controller depends only on the size of the error at the instant of time concerned. The transfer function $G_c(s)$ for the controller is thus

$$G_c(s) = K_p \quad [2]$$



$$G_o(s) = K_p G_p(s) \quad [4]$$

where $G_p(s)$ is the transfer function of the plant.

the closed-loop transfer function with the controller, and unity feedback, is

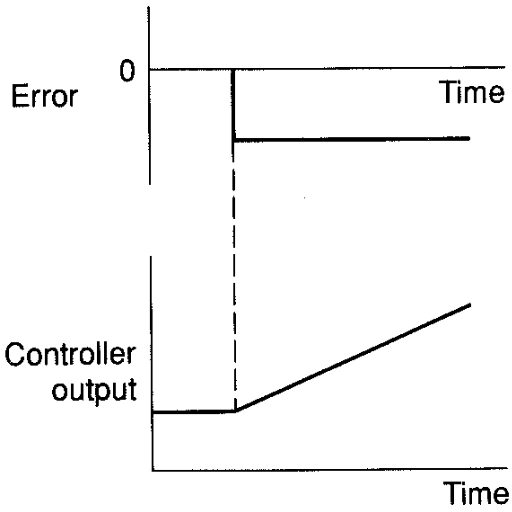
$$G(s) = \frac{K_p G_p(s)}{1 + K_p G_p(s)}$$

and so the characteristic equation of $(1 + K_p G_p(s))$ has its values of roots affected by the value of K_p.

Integral Control

With integral control the output of the controller is proportional to the integral of the error signal e with time, i.e.

$$\text{Output} = K_i \int_0^t e\,\mathrm{d}t \qquad [5]$$

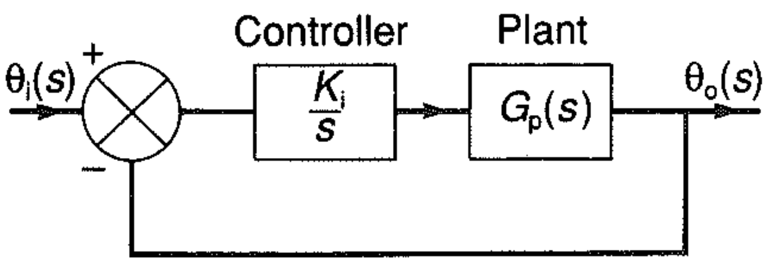


Taking the Laplace transform of equation [5] results in a transfer function, for the integral controller, of

$$G_c(s) = \frac{\text{output }(s)}{e(s)} = \frac{K_i}{s} \qquad [6]$$

Thus, for a system of the form shown in Fig. 10.5, integral control gives a forward-path transfer function of $(K_i/s)G_p(s)$ and hence an open-loop transfer function of

$$G_o(s) = \left(\frac{K_i}{s}\right) G_p(s) \qquad [7]$$



Integral plus Proportional Control

$$\text{Output} = K_p e + K_i \int_0^t e \, dt \qquad [8]$$

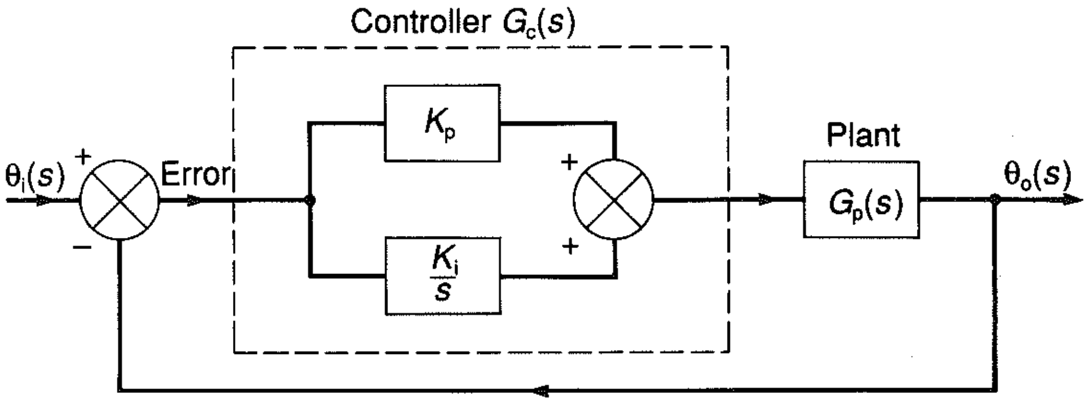


Figure 10.8 shows the type of controller output that occurs with such a system when there is an error step input.

Taking the Laplace transform of equation [8] gives a transfer function, output(s)/e(s), for the PI controller of

$$G_c(s) = K_p + \frac{K_i}{s}$$

$$= \frac{sK_p + K_i}{s}$$

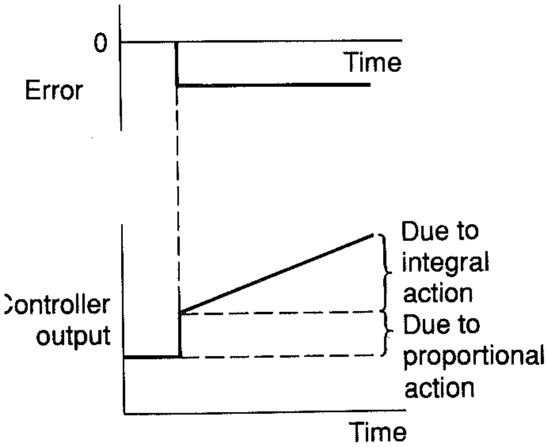
$$= \frac{K_p[s + (K_i/K_p)]}{s}$$

(K_p/K_i) is called the *integral time constant* τ_i. Thus

$$G_c(s) = \frac{K_p[s + (1/\tau_i)]}{s} \qquad [9]$$

Consequently, the forward-path transfer function for the Fig. 10.7 system is

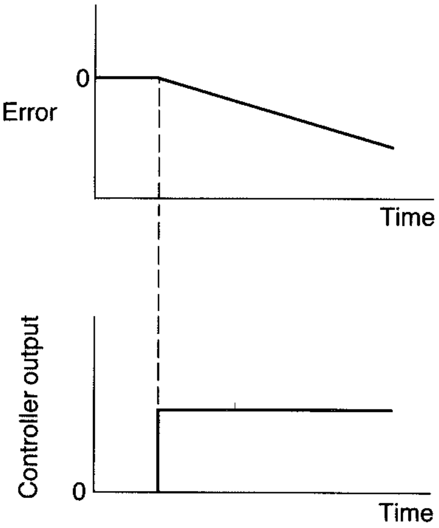
$$G_o(s) = \frac{K_p[s + (1/\tau_i)]G_p(s)}{s} \qquad [10]$$

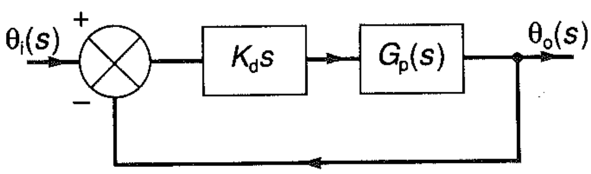


Derivative Control

With the derivative form of controller, the controller output is proportional to the rate of change of the error e with time, i.e.

$$\text{Output} = K_d \frac{de}{dt} \qquad [11]$$





Taking the Laplace transform of equation [11] gives, for derivative control, a transfer function output$(s)/e(s)$ of

$$G_c(s) = K_d s \qquad [12]$$

Hence for the closed-loop system shown in Fig. 10.11, the presence of the derivative controller results in an open-loop transfer function of

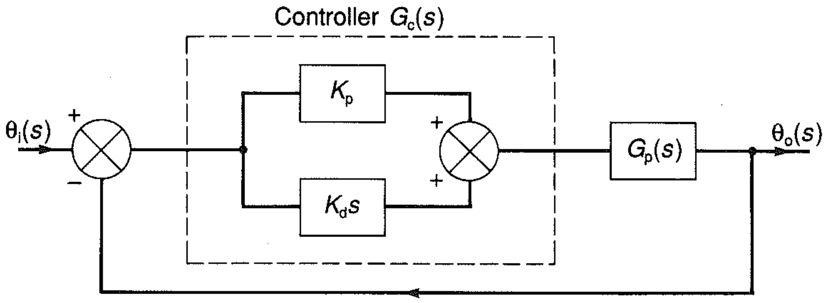
$$G_o(s) = \frac{K_d s G_p(s)}{1 + K_d s G_p(s)} \qquad [13]$$

Proportional plus Derivative Control

$$G_o(s) = (K_p + K_d s)G_p(s)$$

$$G_o(s) = K_d[(1/\tau_d) + s]G_p(s) \qquad [14]$$

where $\tau_d = K_p/K_d$ and is called the *derivative time constant.*



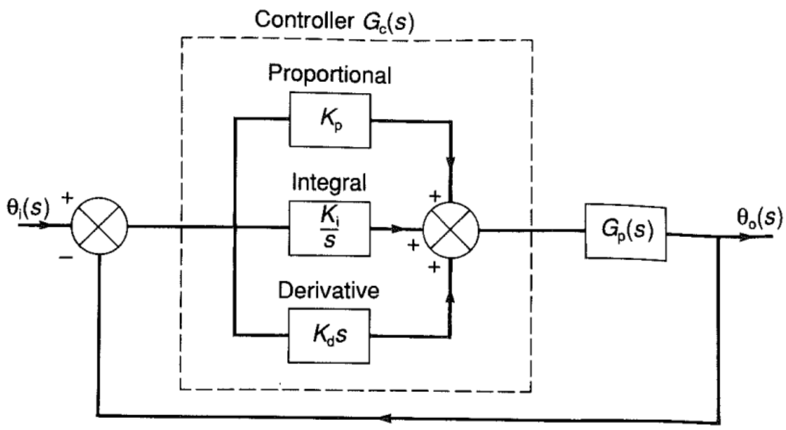
PID Control (the three term controller)

A proportional plus integral plus derivative (PID) controller, or so-called *three-term controller*, with a system of the form shown in Fig. 10.14 will give an output, for an input of an error e, of

$$\text{output} = K_p e + K_i \int_0^t e\,dt + K_d \frac{de}{dt} \qquad [15]$$

The transfer function, output $(s)/e(s)$, of the controller is thus

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \qquad [16]$$



Since the integral time constant τ_i is K_p/K_i and the derivative time constant τ_d is K_d/K_p equation [15] can be written as

$$G_c(s) = K_p\left(1 + \frac{K_i}{K_p s} + \frac{K_d s}{K_p}\right)$$

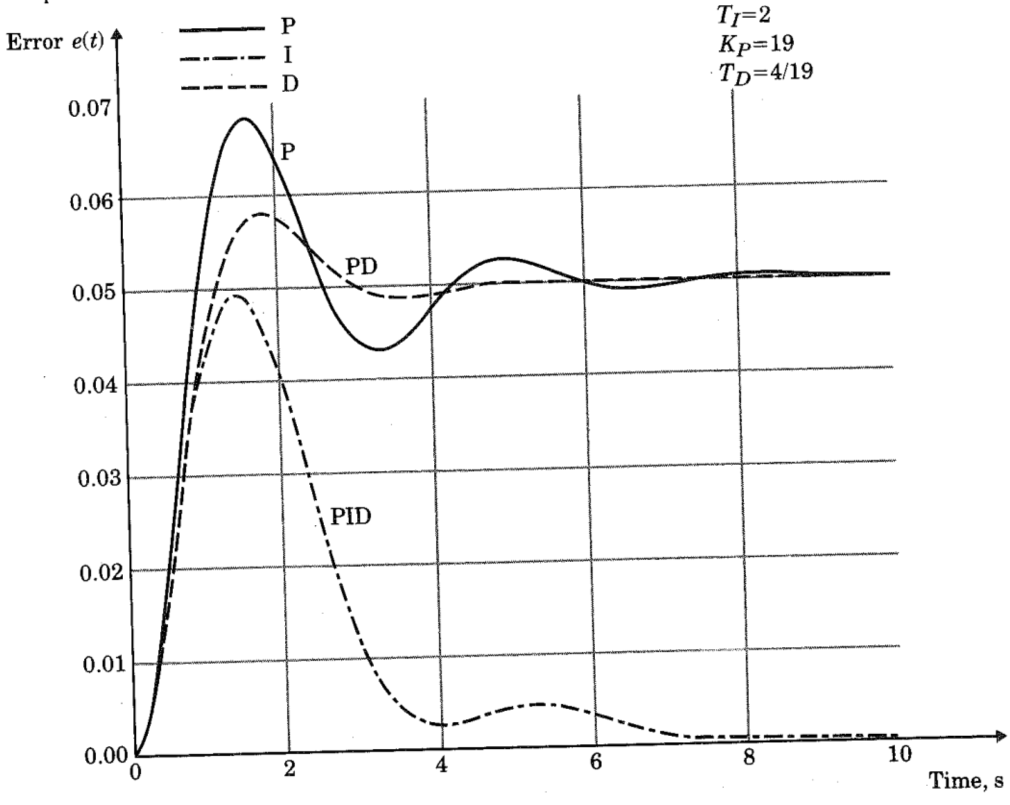
$$G_c(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \qquad [17]$$

The open-loop transfer function for the system shown in Fig. 10.14 is

$$G_o(s) = G_c(s)G_p(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)G_p(s)$$

$$G_o(s) = \frac{K_p(\tau_i s + 1 + \tau_i\tau_d s^2)G_p(s)}{\tau_i s} \qquad [18]$$

FIGURE 3.8
Response of a second-order system $G(s) = 1/[(s+1)(5s+1)]$ to a unit disturbance W.



3. Ziegler-Nichols Tuning for PID controller

Tuning of PID values

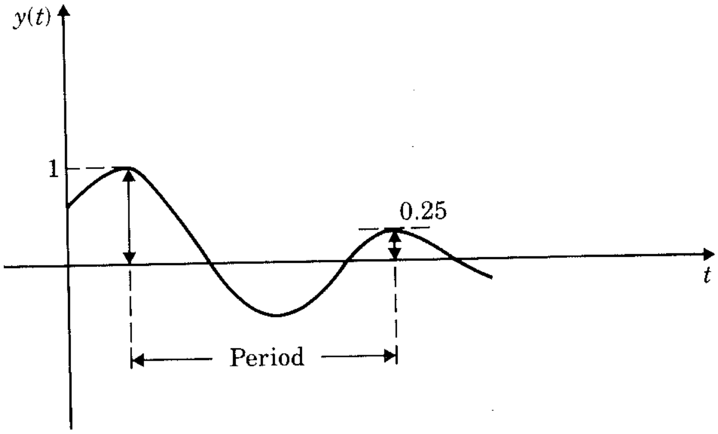
Need to tune PID values to get optimum results.

- Tuning one value will affect the other
- P is for the main feedback function
- I is to overcome the steady state error
- D is to improve the dynamic response.
- Over tune P will lead to oscillation
- Over tune I will lead to instability
- Over tune D will lead to noise increase
- PID is only good for a fixed system with a fixed load
- The disturbance should be small and limited

This is the simplest, quite systematic way of tuning the PID controller. The main advantage: there is no requirement on the plant model, or any knowledge of the plant. However, in most cases, Ziegler-Nichols Tuning will not lead to the most optimum tuning parameters. There are two types of Ziegler Nichols tuning methods:

1. Transient Response Method
2. Stability Limit Method

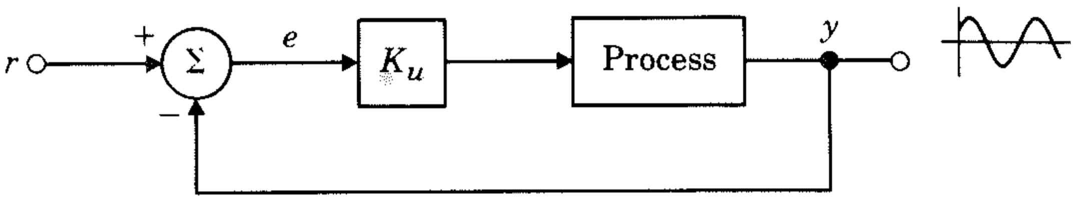
Ziegler and Nichols gave two methods for tuning the controller. In the first method, the choice of controller parameters is based on a decay ratio of approximately 0.25, which means that a dominant transient decays to a quarter of its value after one period of oscillation, as shown in Fig. 3.10. A quarter decay corresponds to $\zeta = 0.21$ and is a good compromise between quick response and adequate stability margins. The equations for the system were simulated on an analog computer, and the controller parameters were adjusted until the transients showed a decay of



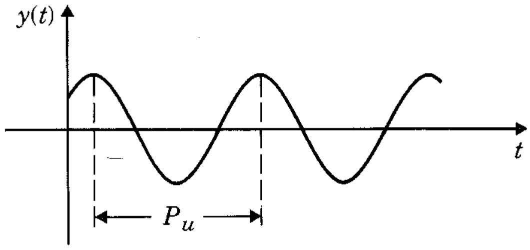
25% in one period. The regulator parameters suggested by Ziegler and Nichols are shown in Table 3.1.

In the second method, the criteria for adjusting the regulator parameters are based on evaluating the system at the limit of stability. Using only proportional control, the gain is increased until continuous oscillations are observed, that is, until the system becomes marginally stable. The corresponding gain K_u (also called the *ultimate gain*) and the period of oscillation P_u (also called the *ultimate period*) are determined as shown in Figs. 3.11 and 3.12. This period should be measured when the amplitude of oscillation is quite small. Then one "backs off" from this gain, as shown in Table 3.2. These parameters can be easily computed from a limit cycle under relay control (see Section 4.6.4).

Experience has shown that the controller settings according to Ziegler-Nichols rules provide a good closed-loop response for many systems. The final tuning of the controller can be done manually by the process operator to yield the "best" control.*



Determination of the ultimate gain and period.



Summary: Transient Response Method

1. Measure the open loop step response of the plant
2. Obtain L and R
3. Calculate the P, I, and D values according to the table below.

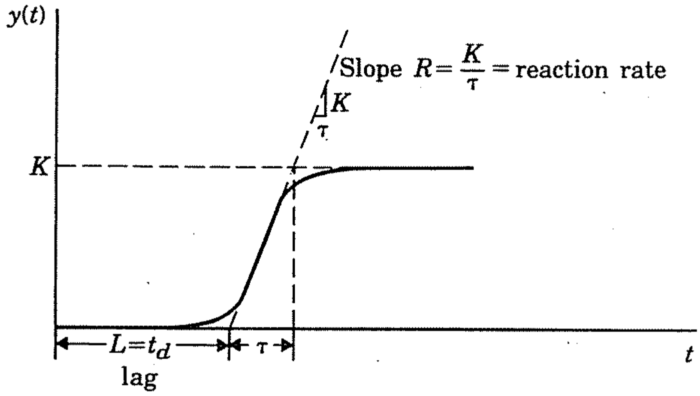


Figure 5.30 Process open-loop step response.

Table 5.2 Ziegler-Nichols tuning parameters using transient response.

	K_p	T_I	T_D
P	$1/RL$		
PI	$0.9/RL$	$3L$	
PID	$1.2/RL$	$2L$	$0.5L$

Summary: Stability Limit Method

1. Switch off I and D, increase P until it oscillates.
2. The P gain (K_u) and the oscillation period (P_u) are recorded
3. Calculate P, I, and D from the table below

Table 5.3 Ziegler-Nichols tuning parameters using stability limit.

	K_p	T_I	T_D
P	$0.5K_u$		
PI	$0.45K_u$	$P_u/1.2$	
PID	$0.6K_u$	$P_u/2$	$P_u/8$

----- END -----