Dr. Norbert Cheung's Series in Electrical Engineering

Level 2 Topic no: 14

Bode Plots

Contents

Reference:

Chapter 6 Frequency Response and Systems Concepts; G. Rizzoni, "Principles and Applications of Electrical Engineering," 5th Edition, McGraw Hill International Edition.

Chapter 8 "Frequency Response Analysis; K. Ogata, "Modern Control Engineering"

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1. Introduction to Bode Plots

Bode plots consist of two graphs: the magnitude of $GH(j\omega)$, and the phase angle of $GH(j\omega)$, both plotted as a function of frequency ω. Logarithmic scales are usually used for the frequency axes and for $|GH(j\omega)|$.

Bode Plot displays the transfer function of a system or a circuit, in terms of frequency response. Normally, two plots are required:

1. Amplitude Gain (in dB) against frequency (log scale)
2. Phase Gain (in degrees) against frequency (log scale)

The decibels scale is $\left|\frac{A_o}{A_i}\right|_{dB} = 20\log_{10}\frac{A_o}{A_i}$

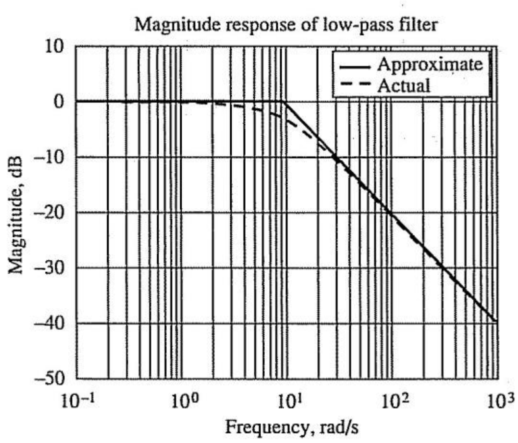
Phase scale is linear scale in degree

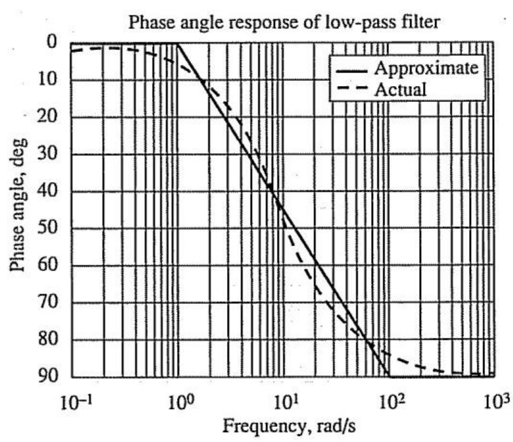
The frequency scale is in $\log_{10}$, or decade.
(e.g. 100Hz, 1kHz, 10kHz….)

2. Bode Plots for Low and High Pass Filter

For low pass filter:

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{j\omega/\omega_0 + 1} = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}} \angle -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

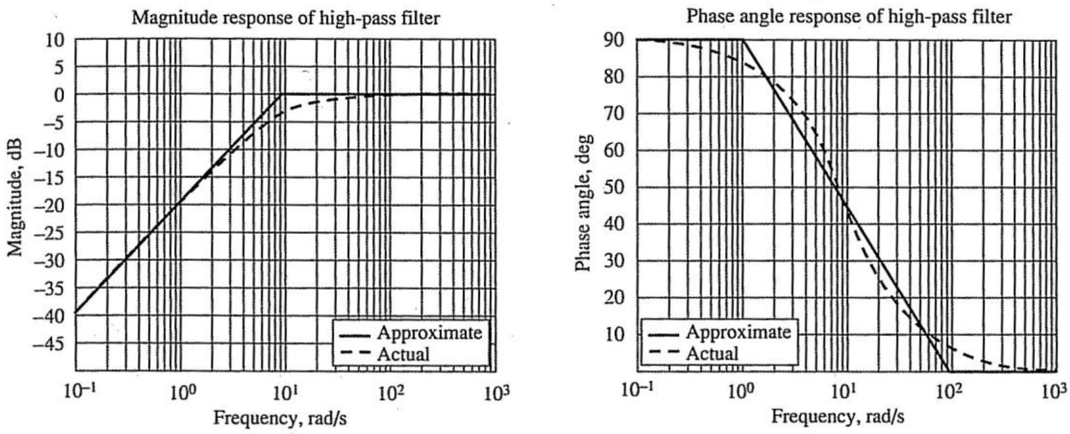




Magnitude: -20dB/decade, turn at cut-off frequency
Phase: -45°/decade; start to turn at lower one decade, flattens at higher one decade

For high pass filter:

$$\frac{V_o}{V_i}(j\omega)=\frac{j(\omega/\omega_0)}{1+j(\omega/\omega_0)}=\frac{\frac{\omega}{\omega_0}\angle\frac{\pi}{2}}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\angle\tan^{-1}\left(\frac{\omega}{\omega_0}\right)}=\frac{\frac{\omega}{\omega_0}}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\angle\left(\frac{\pi}{2}-\tan^{-1}\frac{\omega}{\omega_0}\right)$$



Magnitude: increase at +20dB/decade, flattens at cut-off frequency
Phase: -45°/decade; from 90° start to turn at lower one decade, flattens at higher one decade

Bode Form and the Bode Gain

It is convenient to use the so-called *Bode form* of a frequency response function for constructing Bode plots.

The **Bode form** for the function

$$\frac{K(j\omega+z_1)(j\omega+z_2)\cdots(j\omega+z_m)}{(j\omega)^l(j\omega+p_1)(j\omega+p_2)\cdots(j\omega+p_n)}$$

where l is a nonnegative integer, is obtained by factoring out all z_i and p_i and rearranging it in the form

$$\frac{\left[K\prod_{i=1}^{m}z_i\Big/\prod_{i=1}^{n}p_i\right](1+j\omega/z_1)(1+j\omega/z_2)\cdots(1+j\omega/z_m)}{(j\omega)^l(1+j\omega/p_1)(1+j\omega/p_2)\cdots(1+j\omega/p_n)} \tag{15.2}$$

The **Bode gain** K_B is defined as the coefficient of the numerator in (*15.2*):

$$K_B \equiv \frac{K\prod_{i=1}^{m}z_i}{\prod_{i=1}^{n}p_i} \tag{15.3}$$

Constant Gain

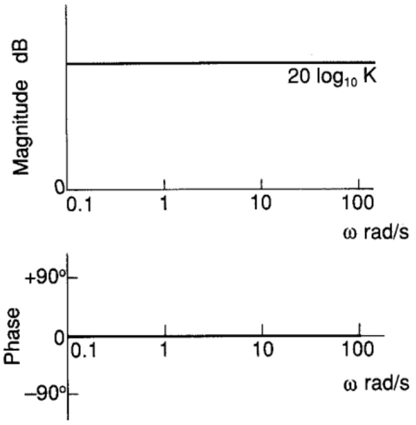


Fig. 11.5 Bode plot for constant gain

This is where

$$G(s) = K$$

and thus

$$G(j\omega) = K$$

For such a system the magnitude is, in decibels,

$$|G(j\omega)| = 20\lg K$$

A Pole at Origin

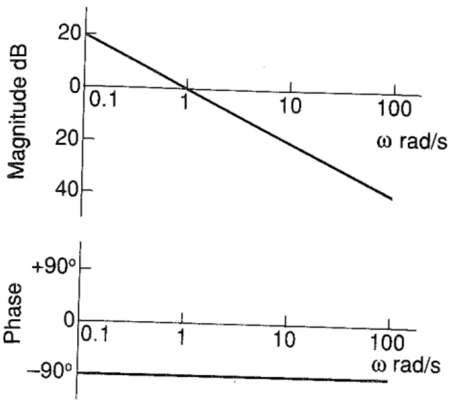


Fig. 11.6 Bode plot for pole at origin

This is where

$$G(s) = \frac{1}{s}$$

and so

$$G(j\omega) = \frac{1}{j\omega} = -\frac{j}{\omega}$$

A Zero at Origin

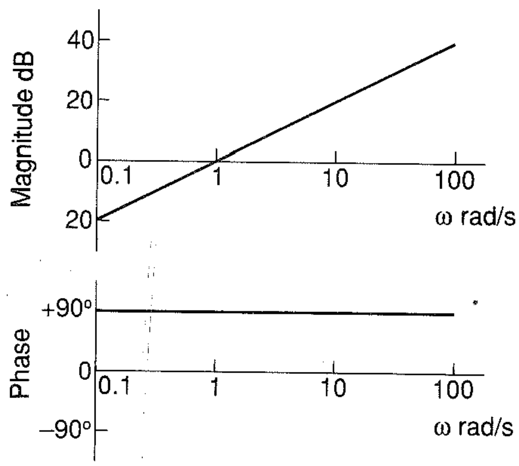


Fig. 11.7 Bode plot for zero at origin

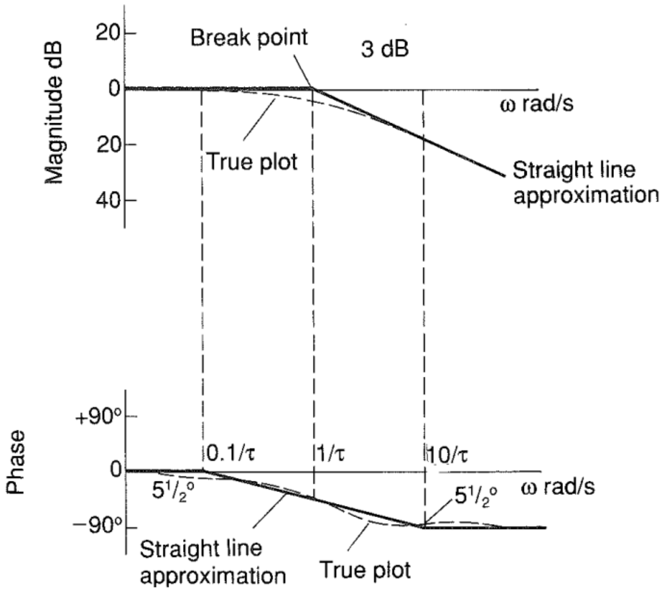
This is where

$$G(s) = s$$

and thus

$$G(j\omega) = j\omega$$

A Real Pole



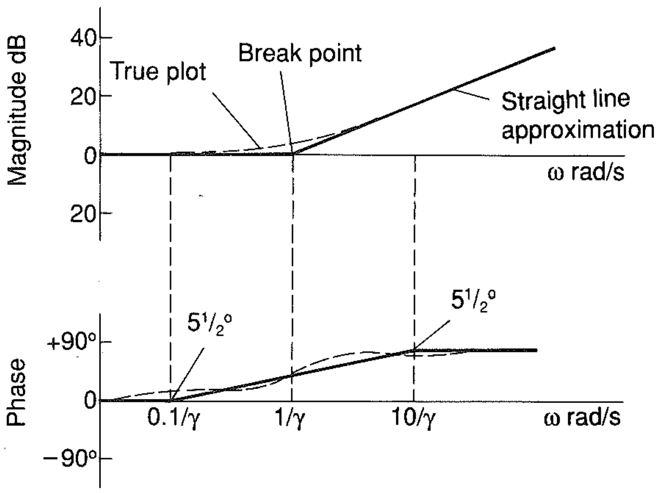
This means a first-order lag system, where

$$G(s) = \frac{1}{\tau s + 1}$$

and thus

$$G(j\omega) = \frac{1}{j\omega\tau + 1} = \frac{1 - j\omega\tau}{1 + \omega^2\tau^2}$$

A Real Zero



This means a first-order lead system where

$$G(s) = 1 + \tau s$$

and thus

$$G(j\omega) = 1 + j\omega\tau$$

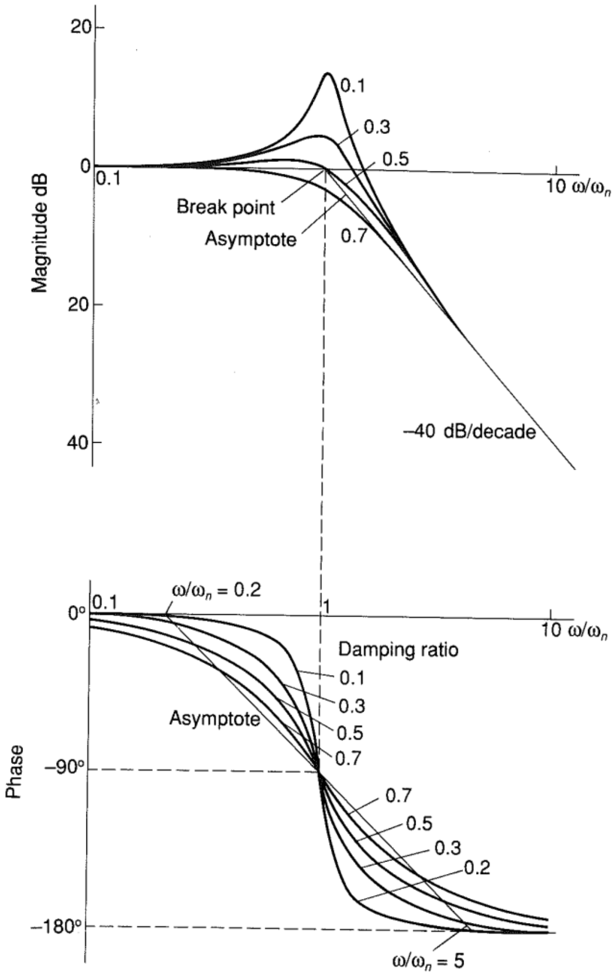
The magnitude, in decibels, is thus

$$20\lg\sqrt{(1 - \omega^2\tau^2)}$$

and the phase

$$\tan\phi = \omega\tau$$

A Pair of Complex Poles



This is where

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

and thus

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n + \omega_n^2}$$

$$G(j\omega) = \frac{1}{[1 - (\omega/\omega_n)^2] + j[2\zeta(\omega/\omega_n)]}$$

3. Bode Plot of Higher Order Filters

Step 1: Express in $H(j\omega) = \frac{K\left(\frac{j\omega}{\omega_1}+1\right)\cdots\left(\frac{j\omega}{\omega_m}+1\right)}{\left(\frac{j\omega}{\omega_{m+1}}+1\right)\cdots\left(\frac{j\omega}{\omega_m}+1\right)}$

Step 2: Select the appropriate scale for the Bode Plot

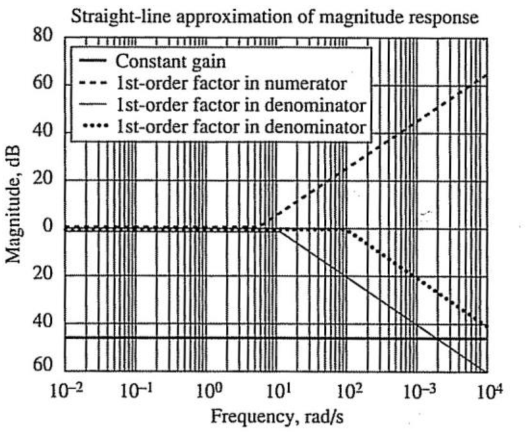
Step 3: Sketch the asymptotic approximations for each factor

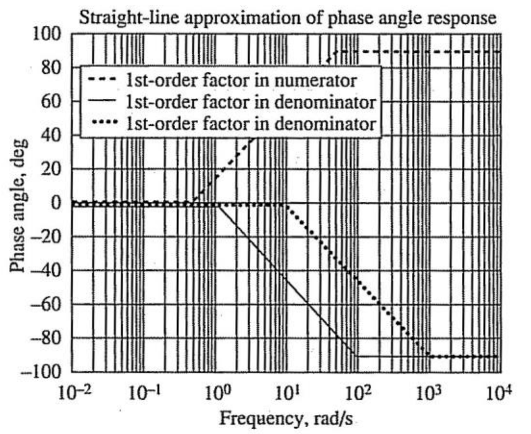
Step 4: Add the graphs graphically

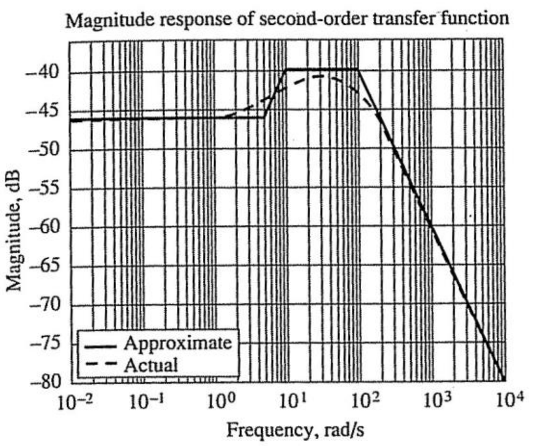
Step 5: Smooth out the clines if required

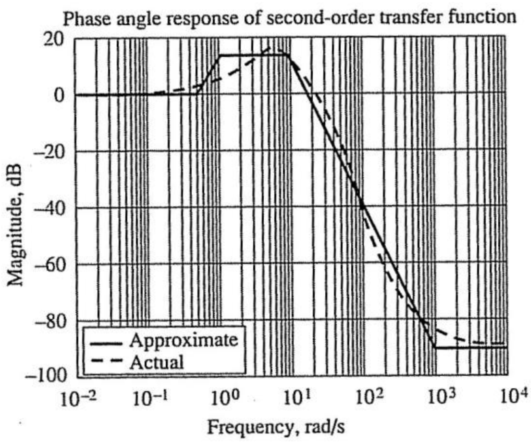
How to add the various lines together?

1. Find the starting point on the left of graph
2. Find the overall trend of the graph (going up or down)
3. From the starting point, draw the overall trend until you meet a turning point.
4. From the turning point, evaluate whether the trend is upward or downward
5. Change the forward direction according to 4, until you scan through all part of graph.





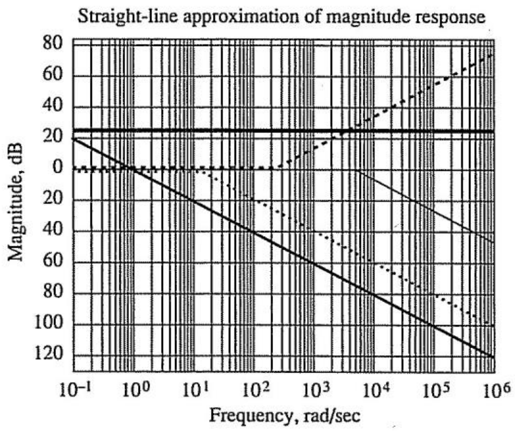


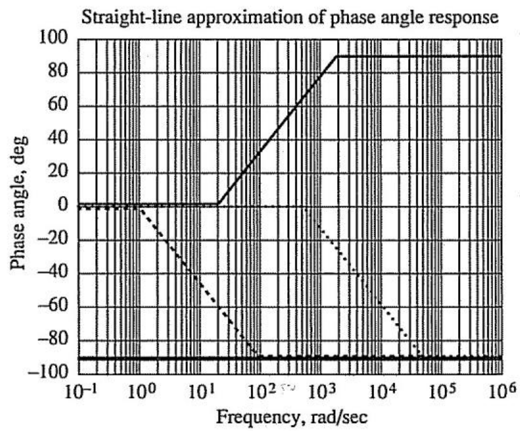


<u>4. Worked Examples</u>

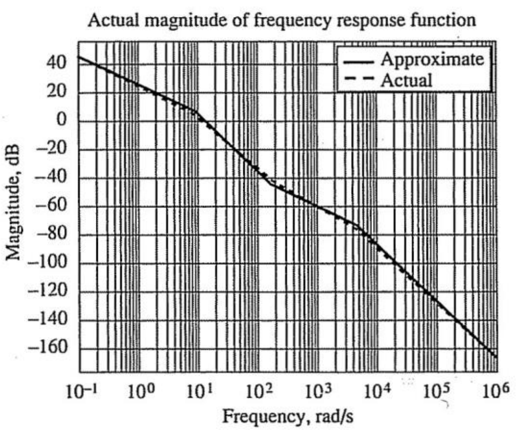
Example 1: $$H(j\omega)=\frac{20\left(\dfrac{j\omega}{200}+1\right)}{j\omega\left(\dfrac{j\omega}{10}+1\right)\left(\dfrac{j\omega}{5000}+1\right)}$$

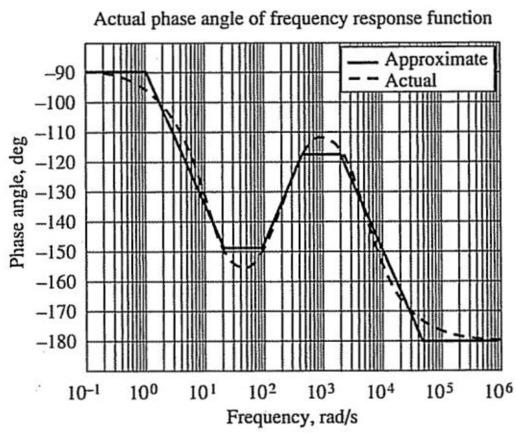
Draw the individual lines first





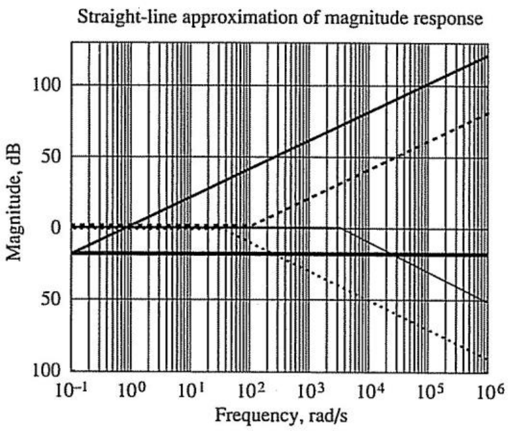
Then add the line together

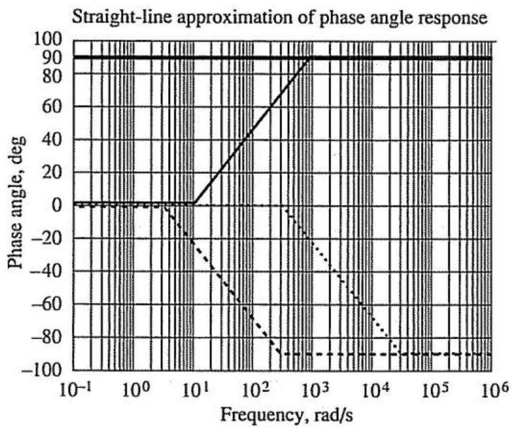




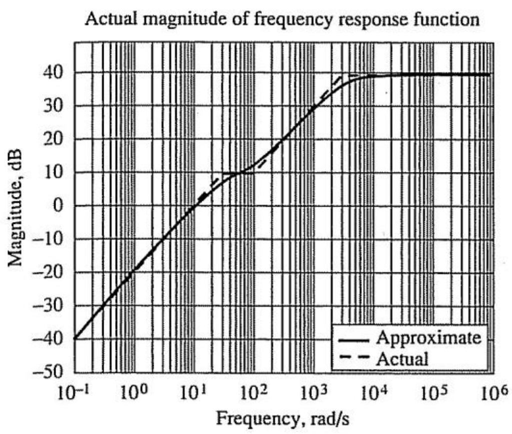
Example 2: $$H(j\omega)=\frac{0.1j\omega\left(\frac{j\omega}{100}+1\right)}{\left(\frac{j\omega}{30}+1\right)\left(\frac{j\omega}{3000}+1\right)}$$

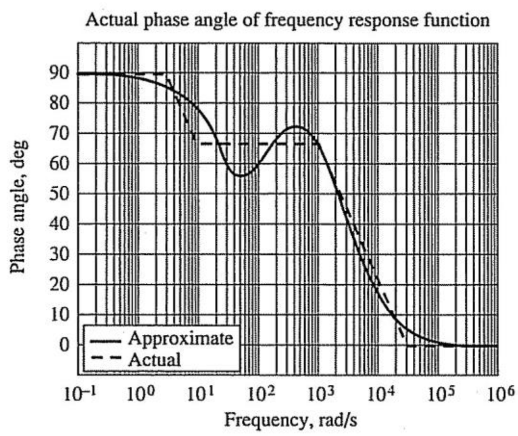
Draw the individual lines…..

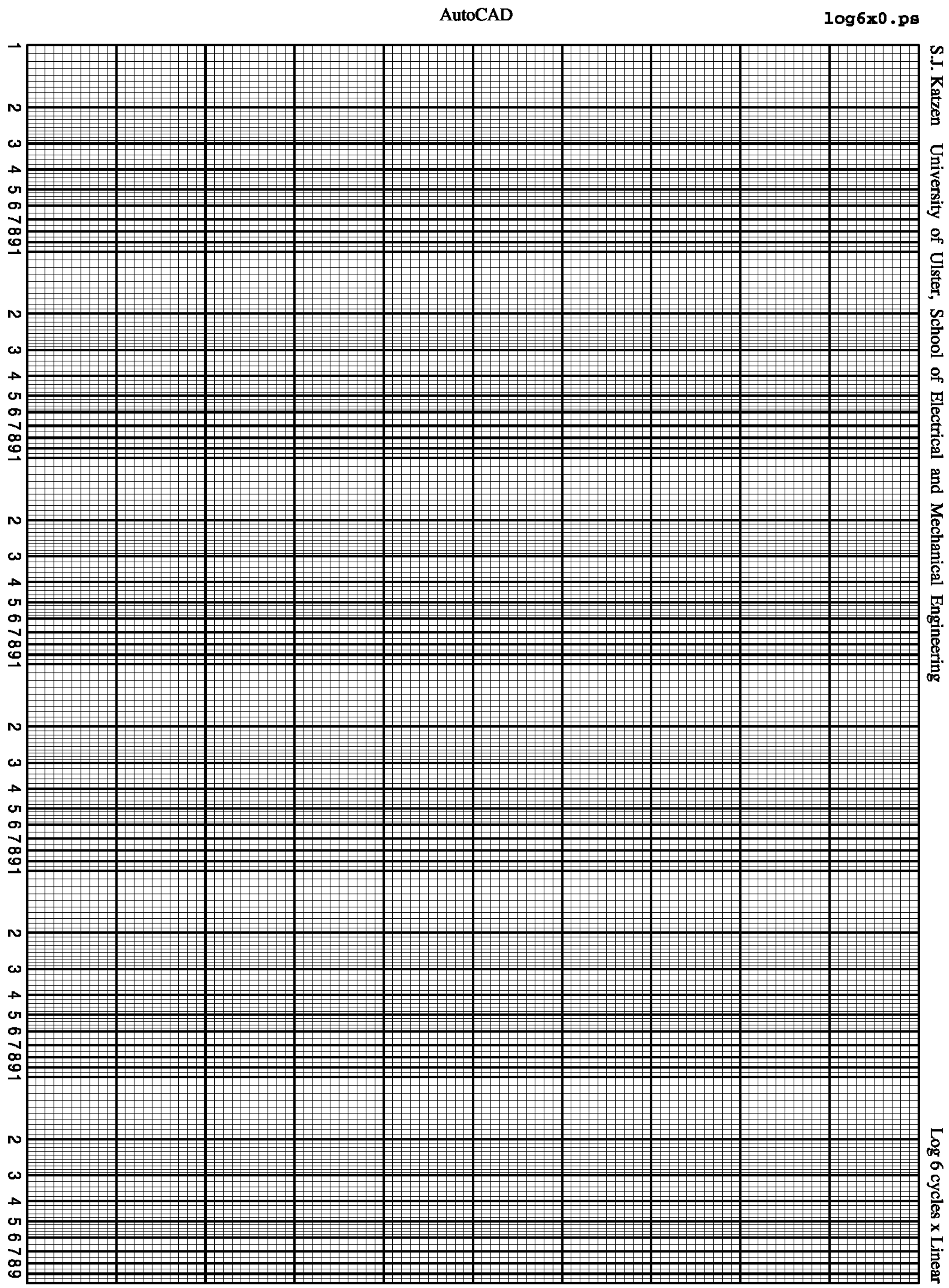




Then add them together…..






AutoCAD
log6x0.ps
S.J. Katzen University of Ulster, School of Electrical and Mechanical Engineering
Log 6 cycles x Linear

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