Dr. Norbert Cheung's Lecture Series

Level 5 Topic no: 38

Quantitative Methods and Experimental Research

Contents

Reference

Engineering Research: Design Methods and Publication, Herman Tang, Wiley, 2021.

1. Quantitative Research

Among many quantitative methods, we often do calculation and analysis based on statistical principles. Bear in mind when we discuss a statistical analysis, there are other types of quantitative analysis in engineering and technology, such as a mathematical model, algorithm, differential equations, and dynamic neural network. In addition, we may jointly use multiple quantitative methods in a study.

Descriptive Statistical Analysis

There are two major types of statistical analysis: descriptive statistics and inferential statistics, refer to Figure 5.1. Briefly, descriptive statistics is to summarize and reveal the information about a given data set, present the data in a meaningful way, and come with interpretations. The conclusions from descriptive statistics stay within the data or only applicable to the sample data set.

On the other hand, inferential statistics uses the statistics to analyze a sample of data obtained from a population to make inferences about the population. The conclusions based on inferential statistics can be interesting and useful, which may better fit many research goals.

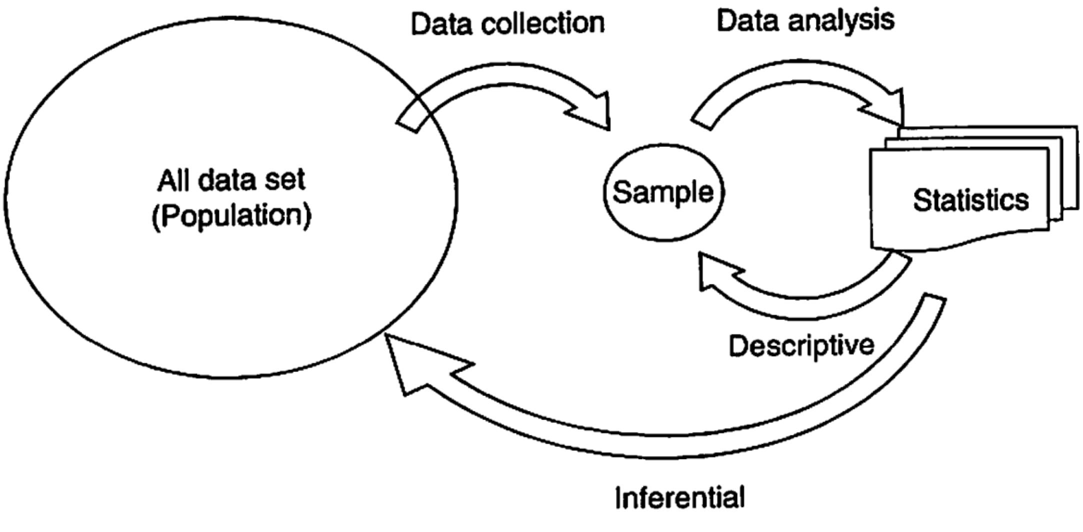

Figure 5.1 Descriptive and inferential statistical analysis.

With descriptive statistics, there is no uncertainty because we describe only the data measured. So we can verify the descriptive statements from the information provided. However, we should not use the statements to generalize and apply for any other groups or the entire population.

We may use central tendency to describe and compare data sets.

- *Mode* is the single number that occurs most frequently in a data set.
- *Median* is the "middle" number of a set of an ordered data set. If a data set is an odd number, the median is the middle value. For an even number, the median of a data set is the average of the two middle values.
- *Mean* $(\overline{x})$ is usually referred to an arithmetic average $\left(\overline{x} = \frac{x_1+x_2+\cdots+x_n}{n} = \frac{\sum x_i}{n}\right)$ of a data set with n samples. For an entire population, we often use a Greek letter μ for the mean parameter.
- A geometric mean $\left(\overline{x}_g = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n} = \sqrt[n]{\prod x_i}\right)$ is used for some situations, such as numbers that are in different ranges and the differences among data points that are logarithmic.

Variability of data set:

Data variability shows that data points cluster around the point of central tendency and comes from data sources and measurements. Some parts of a variability are the nature of random noise since all measurement instruments and procedures are subject to random noise.

- *Range (r)* is a simple way to reflect the spread of a data set from its lowest value to its highest value. That is, $r = \max\{x_i\} - \min\{x_i\}$, where, $i = 1, \ldots, n$.
- *Standard deviation (S)* is the most common way to measure of the variability of a data set. The standard deviation is $S = \sqrt{\frac{\sum (x_i-\mu)^2}{n-1}}$. For an entire data population, we normally use Greek letter σ to present the variance parameter.
- *Average deviation* is also called the mean absolute deviation. It is the average of differences of each value and the mean average for a sample data set is $\frac{\sum |x_i-\mu|}{n}$.

Inferential Statistical Analysis

In inferential statistics, we use similar calculations as in descriptive analysis, for example, the mean and standard deviation. However, for inferential analysis, we aim to generalize the parameters and characteristics along with a predefined degree of confidence. That is, we use a sample data set to make a generalization about the parameters of a given population.

Data Association

Correlation analysis is an important approach for the analysis of data association. For example, we may do a correlation analysis to identify a linear relationship between two variables. The result of a correlation analysis is a single number called correlation coefficient (r). The coefficient r indicates mathematical correlation direction varying from -1 to $+1$, as shown in Figure 5.2. The two variables are r^2% related with each other mathematically. For example, if r is $|0.6|$, then the linear correlation is a small, as $0.6^2 = 36\%$, between the two data sets. Sometimes, there is a possible nonlinear interrelationship for a low r, which may need an additional analysis.

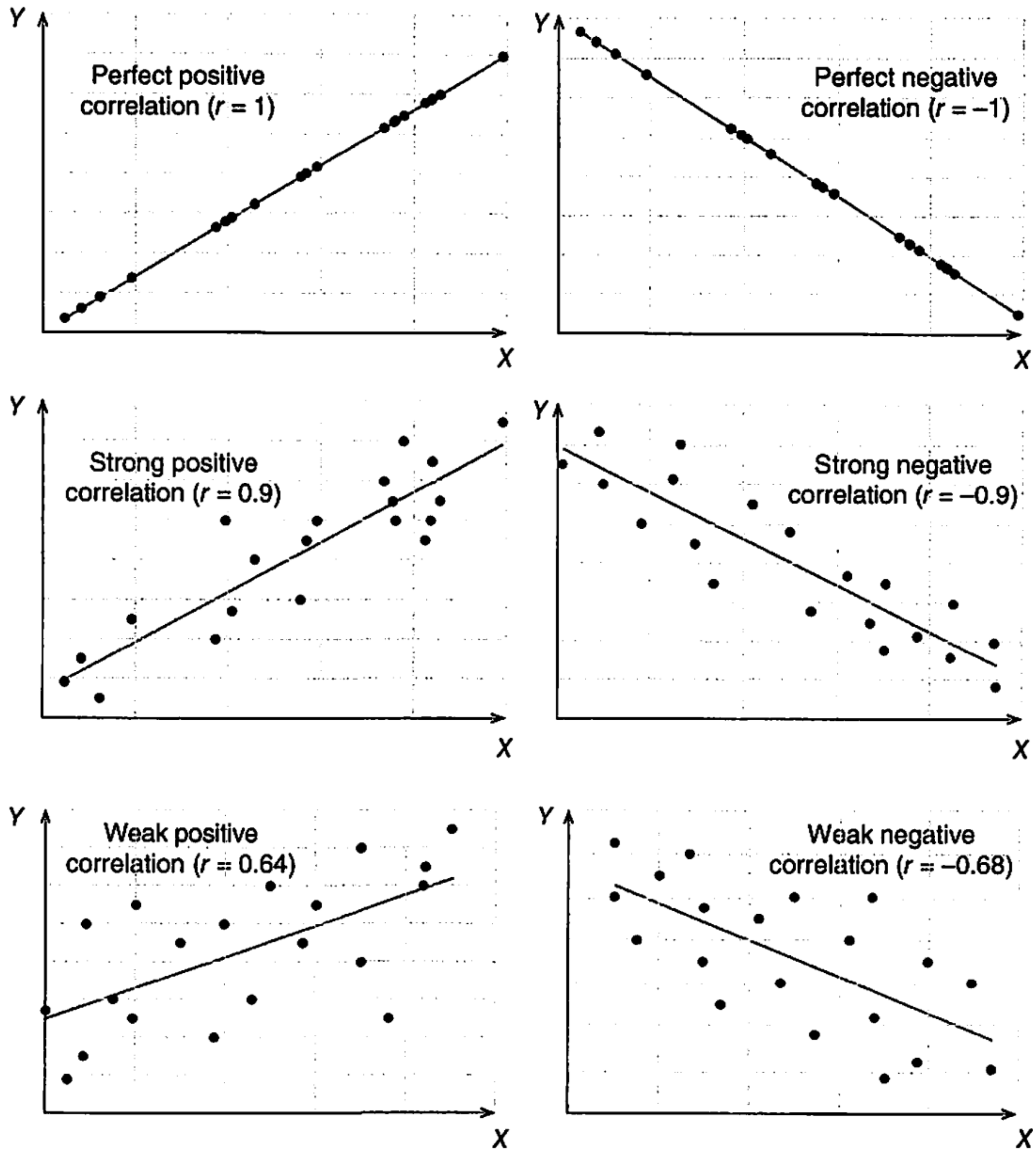

Figure 5.2 Correlations between two sets of data.

Analysis of Variance

To analyze the differences between the data collected under two (or more) different conditions, we can use analysis of variance (ANOVA), which is an effective analysis tool based on the linearity, independence, and normality of the data

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Method

Null hypothesis          All means are equal
Alternative hypothesis   At least one mean is different
Significance level       α = 0.05

Equal variances were assumed for the analysis.


Factor Information

Factor   Levels   Values
Factor        5   15.00%, 20.00%, 25.00%, 30.00%, 35.00%


Analysis of Variance

Source   DF   Adj SS    Adj MS   F-Value   P-Value
Factor    4    475.8   118.940     14.76     0.000
Error    20    161.2     8.060
Total    24    637.0
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Figure 5.3 An example of ANOVA result.

collected. We can use ANOVA to test whether the means of several groups are equal. Figure 5.3 shows an example of an analysis output using Minitab software.

To determine whether the difference between two means is significant, we compare the p-value to the predetermined significance level that is often 0.05. A linear model is statistically significant only when the p-value is smaller than the significance level. More discussion on p-value is in the next subsection.

Regression Analysis

There are other types of statistical analysis for the interrelationship among variables. For example, a regression analysis is used to estimate both linear and nonlinear relationships among variables for a prediction purpose.

For example, a linear regression function is $y = \beta_1 x + \beta_0 + \varepsilon$, where, y represents an outcome variable, x represents its corresponding predictor variable, ε is the error, and β_1 and β_0 are parameters. Figure 5.4 shows an example of regression analysis in a form of $y = \beta_1 x^2 + \beta_2 x + \beta_0$, where x is a quadratic variable. It is still called a linear regression as expressed in a linear combination of the βs.

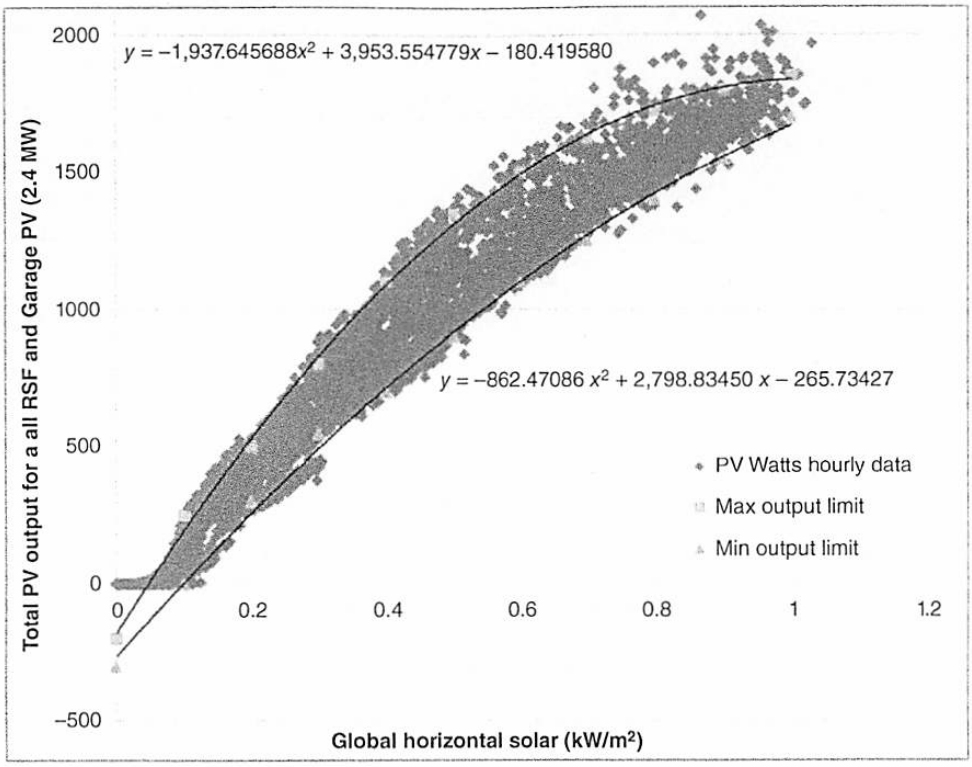


Figure 5.4 An example of nonlinear regression analysis. Source: Henze et al. (2014).

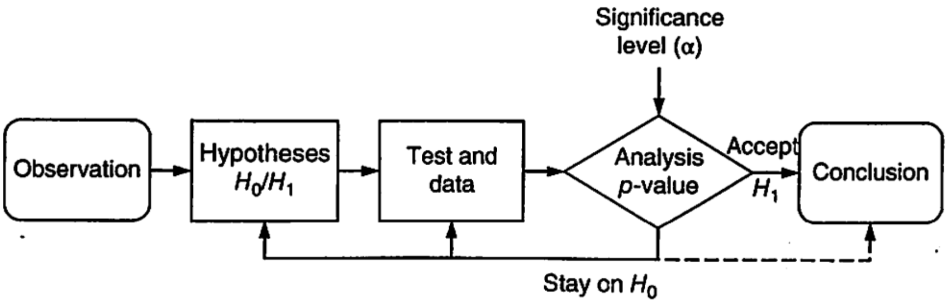
Interpretation of Analysis Results

After obtaining analysis results, we need to understand and interpret them.

 # Hypothesis Testing Process

Here are the key elements of hypothesis analysis.

- Establish the null hypotheses H_0 and alternative hypotheses H_1, where H_0 is often set at the status quo, based on the problem to solve and observation
- Select a required level of significance (α)
- Randomly collect data and analyze the data to get a p-value
- Determine to stay on or reject H_0 based on the p-value
- Interpret and discuss the analysis results
- Draw the conclusions for a new understanding, including the condition and limitations



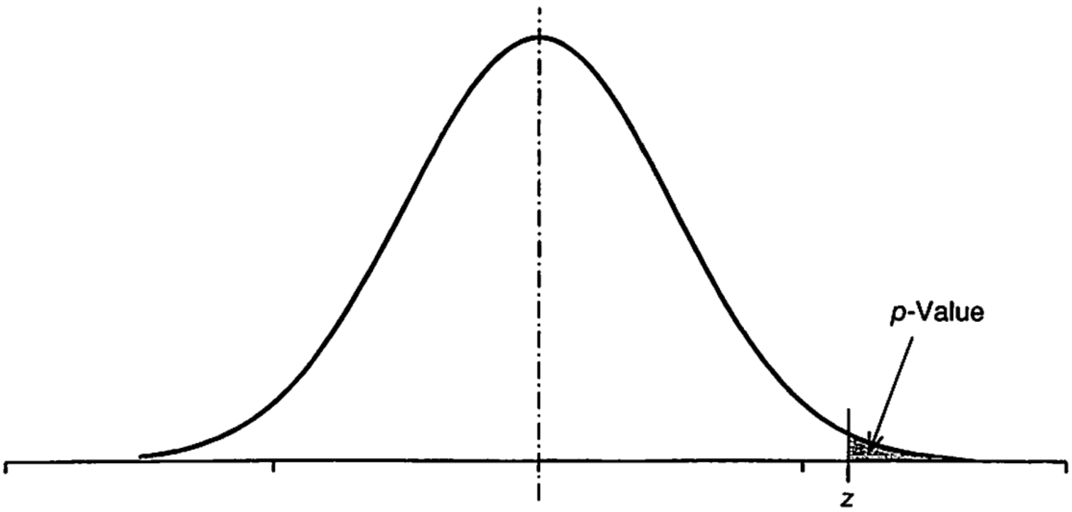


Figure 5.7 An illustration of p-value.

We often use a statistic p-value in hypothesis-based research. The p-value is the calculated probability of finding the observed results when the null hypothesis (H_0) of a research question is true. Figure 5.7 shows as an example of one-side p-value. In the figure, the p-value is the outside area of the normal distribution

2. Quantitative Research

Quantitative analysis is often known as statistical analysis in many cases, and there are numerous quantitative methods in engineering R&D. In this section, we discuss three types: mathematical modeling, optimization, and computer simulation.

Mathematical Modelling

Modeling is an approach and a process of using various scientific principles and terms to present real-world situations, as illustrated in Figure 5.8. Researchers using modeling techniques to describe the different aspects of the real world, advance scientific understandings, and solve problems.

A simple mathematical model may be a set of equations. For an example of a linear regression model discussed earlier, it may be in the form of $y = \beta_1 x + \beta_0 + \varepsilon$. The regression model determines the specific relationship between the input variable x and the outcome variable y.

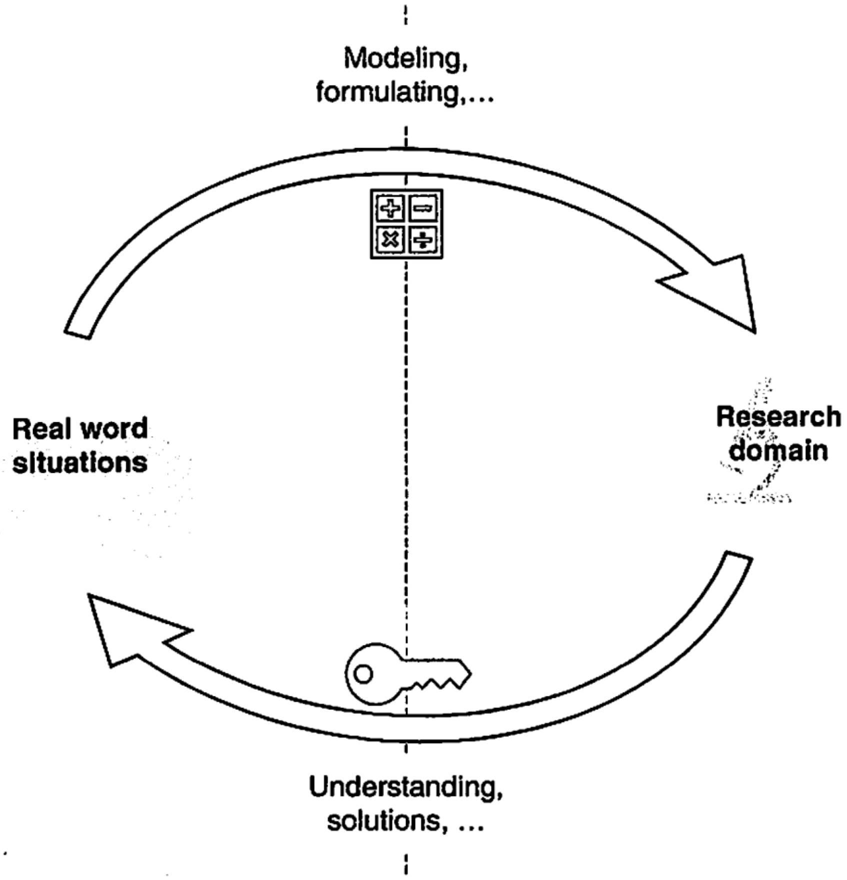


Figure 5.8 Modeling to bridge division between real word and research.

Optimization

In theory, optimization is based on the modeling of scientific principles. The concept of optimization started from mathematics. For a general case, a mathematical function may be expressed by:

$$y = f(x_1, x_2, x_3, \ldots)$$

where, y is the output of a function $f(\bullet)$; x_1, x_2, and x_3, etc., are input variables and the optimization task is about either maximizing or minimizing the output y by methodologically choosing input values of x_1, x_2, and x_3, etc., using various optimization algorithms and considering certain constraints. In other words, optimization aims to find the best solution when considering inputs and constraints.

Table 5.2 Examples of mathematical modeling in engineering research.

Engineering	Method	Example topic
Mechanical	Dimensionless governing equations	The development of mathematical modeling for nanofluid as a porous media in heat transfer technology (Tongkratoke et al. 2016)
Computer	Extensible network service model	The modeling and analysis of the extensible network service model (Ji et al. 2018)
Electrical	State-space modeling	State-space modeling and reachability analysis for a DC microgrid (Ghanbari et al. 2019)
Industrial	Energy consumption of stereolithography	Energy consumption modeling of stereolithography-based additive manufacturing toward environmental sustainability (Yang et al. 2017)
Material	Synchronous generator model in Park–Gorev's equations	The synthesis of precise rotating machine mathematical model, operating natural signals and virtual data (Zhilenkov and Kapitonov 2017)
Civil	Mualem–Van Genuchten method	Mathematical modeling of hydrophysical properties of soils in engineering and reclamation surveys (Terleev et al. 2016)
Manufacturing	Economic manufacturing quantity model	Mathematical modeling for exploring the effects of overtime option, rework, and discontinuous inventory issuing policy on EMQ model (Chiu et al. 2017)
Chemical	Math models based on binary gas separation	Mathematical modeling and investigation on the temperature and pressure dependency of permeation and membrane separation performance for natural gas treatment (Hosseini et al. 2016)
Nuclear	Nondimensional model	Mathematical modeling of orifice downstream flow under flow-accelerated corrosion (Sanama and Sharifpur 2018)

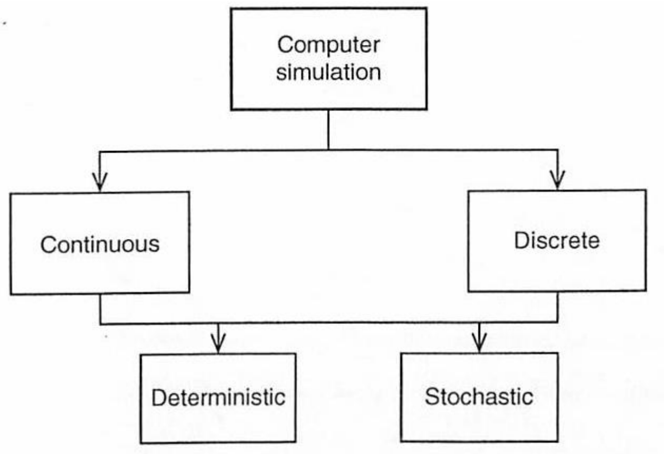
Table 5.3 Examples of engineering optimization research.

Engineering	Method	Example topic
Mechanical	Empirical model	Optimization of mechanical properties of printed acrylonitrile butadiene styrene using RSM design (Abid et al. 2019)
Computer	Process regression	Gaussian process regression tuned by Bayesian optimization for seawater intrusion prediction (Kopsiaftis et al. 2019)
Electrical	Surrogate-assisted robust algorithm	A new surrogate-assisted robust multiobjective optimization algorithm for an electrical machine design (Lim and Woo 2019)
Industrial	Multiobjective integer nonlinear programming model	Optimization of multiperiod three-echelon citrus supply chain problem (Sahebjamnia et al. 2019)
Material	Finite element analysis and neural networks	Development of an elastic material model for bcc lattice cell structures using finite element analysis and neural networks approaches (Alwattar and Mian 2019)
Civil	Teaching learning based	Time-cost trade-off optimization of construction projects using teaching learning-based optimization (Toğan and Eirgash 2019)
Manufacturing	3D kinematic	Optimization of centerless through-feed grinding using 3D kinematic simulation (Otaghvar et al. 2019)
Chemical	Multi-objective	Multi-objective optimization method for enhancing chemical reaction process (Cao et al. 2018)
Nuclear	Genetic algorithm	Module layout optimization using a genetic algorithm in light water modular nuclear reactor power plants (Wrigley et al. 2019)

Computer Simulation

To make a simulation study feasible, a simplification of real-world situations is necessary; some assumptions and parameters (including input data) are determined based on the assumptions. Therefore, a simulation may not be 100% representative to the corresponding real world, but hopefully close to. We should check the output of a simulation for its validity against the real situations if possible. In many cases, we can also perform sensitivity analyses for the parameters to improve the accuracy of the simulation results.

Based on a deterministic model, a computer simulation can generate the output that is fully determined by the parameter and the initial conditions. Due to the complexity of the real world, deterministic models may be used as an approximation of reality with simplified inputs and assumptions. For example, a deterministic model was used to study the electron transport for electron probe microanalysis



In contrast, the outputs of a stochastic simulation are different with the same parameter values and initial conditions. That is, the behaviors of a stochastic model cannot be entirely predictable. Thus, a stochastic simulation may be used to trace the evolution of variables that can change randomly. Comparatively, a stochastic simulation can be more complicated and closer to the real world than a deterministic simulation. For instance, stochastic method was used to solve a complex problem in production (Moheb-Alizadeh and Handfield 2017).

FEA, which is based on the principles of engineering, physics, mathematics, and statistics, can be either deterministic or stochastic depending on the principles used (Roirand et al. 2017). FEA has been widely used in various engineering studies. Originally started on the problems of mechanical structural analysis, FEA has successfully been applied on other areas, such as heat transfer, fluid flow, acoustics, electromagnetic fields, and electrical-chemical process. Figure 5.10 shows two FEA models (NASA 2017).

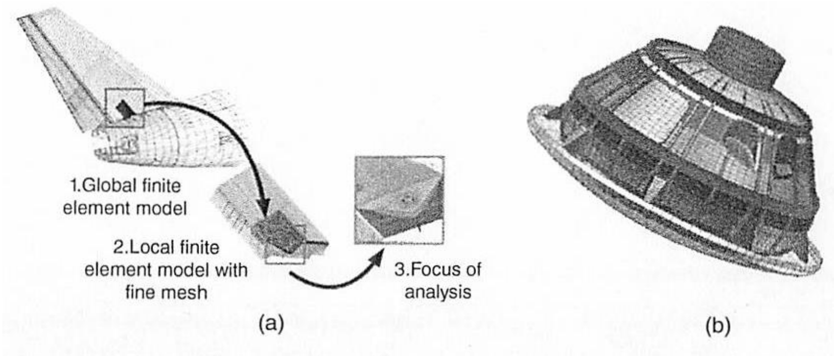


Figure 5.10 Two FEA examples. (a) Airplane empennage. (b) Orion crew module

Table 5.4 Examples of simulation in engineering research.

Engineering	Method	Example topic
Mechanical	FEA and computational fluid dynamics	Simulated transcatheter aortic valve flow: implications of elliptical deployment and underexpansion at the aortic annulus (Sirois et al. 2018)
Computer	Annealing algorithm	Training ANFIS structure using simulated annealing algorithm for dynamic systems identification (Haznedar and Kalinli 2018)
Electrical	Adaptive electro-kinematical model	Electrokinematical simulation for flexible energetic studies of railway systems (Mayet et al. 2018)
Industrial	Agent-based, discrete event, and system dynamics	Centralizing the admission process in a German hospital (Reuter-Oppermann et al. 2019)
Material	FEA	Simulation and experimental tests of ballistic impact on composite laminate armor (Soydan et al. 2018)
Civil	Agent-based	Advances in probabilistic and parallel agent-based simulation: Modeling climate change adaptation in agriculture (Troost and Berger 2016)
Manufacturing	Discrete event simulation	Effects analysis of internal buffers in serial manufacturing systems for optimal throughput (Imseitif and Tang 2019)
Chemical	Linear viscoelasticity	Micromechanics and rheology of colloidal gels via dynamic simulation (Johnson 2018)
Nuclear	Heat transfer model	Coupled thermochemical, isotopic evolution and heat transfer simulations in highly irradiated UO_2 nuclear fuel (Piro et al. 2016)

New Technologies

One remarkable characteristic of new technologies is that they are cross-disciplinary. Most research conducted in such emerging areas is related to new approaches, modeling, and computer simulation. The most popular research subjects include

- Virtual reality (VR) and augmented reality (AR)
- Industry 4.0 (cyber-physical systems, etc.)
- Blockchain (sensing and digital ledger)
- Autonomous vehicles
- Cybersecurity engineering
- 3D metal printing

3. Experimental Studies

Sometimes, it can be difficult to characterize a subject theoretically or in a mathematical model, so one possible way to study them is through experiments. In many fields, scientists and researchers start their new investigations with experiments. In addition, experimental studies may be more cost-effective compared with real operations out in the field, since it may be easier to control the environment of the experimental runs to obtain required data.

Overview of Experimental Studies

#1 Basic Elements of Experiments

An experiment design plays a key role in experimental research's success. When designing an experimental plan, we must consider several aspects of a study. We must select a proper experimental approach; for example, using a simple comparative design vs. a factorial experimental design. It is also important we have reliable instruments and means to measure the input factors as well as response variables in an experimental study. For a large experiment, we should use project management approaches for experiment planning and execution phases.

#2 Influencing Factors

A design of an experimental study includes two parts about its factors:

1. Having a good control for all known factors.
2. Estimating influence of unknown factors and holding them fixed to ensure that the extraneous conditions are not influencing the response to be measured.

#3 Other Considerations

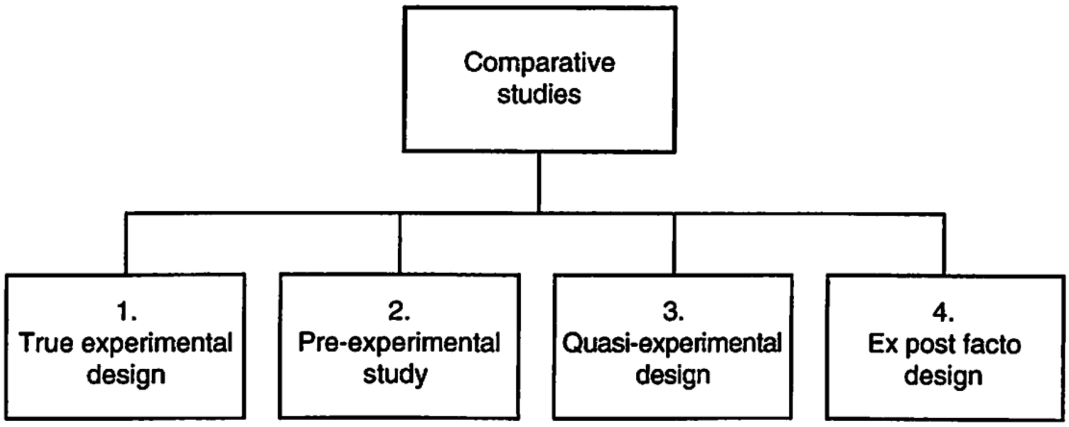
The randomness of a data collection in an experiment design is critical for the validity of an experiment. For example, when doing a comparative experiment with two groups (a treatment and a control), we must assign the treatments by a random process to eliminate potential biases. There are various ways to obtain randomness, including but not limited to randomization tables and computerized random number generators.

Replication is another essential factor. The repeated experimental results may have a significant within-treatment variation. In addition, during an experimental study, we may achieve a better estimation or prediction through replication. However, replication may not be always practical or economical.

Comparative Studies

A comparison study is a process of comparing two or more things to discover their differences or prove the same about them. In general, there are four types of comparative designs (see Figure 5.11).

Essentially, an experiment has two types of variables, independent x and dependent y. By designing and conducting an experiment, we study their relationship $y = f(x)$. When the objective is to find a causation relationship, the independent and dependent variables are called cause and effect, respectively. For example, we may try to know how using different methods (an independent variable) affect the research effectiveness (a dependent variable) on a subject.



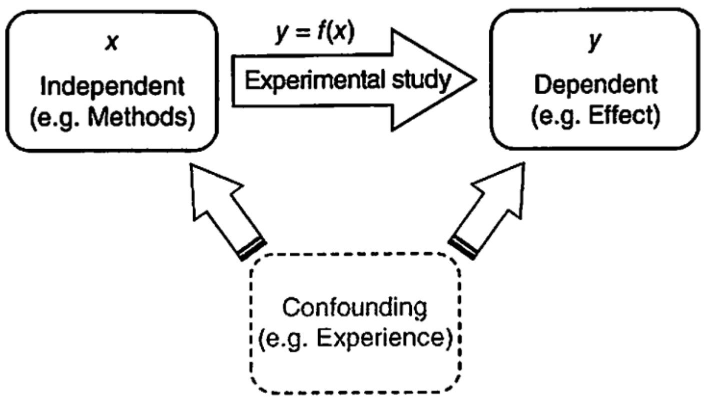


Figure 5.12 Independent, dependent, and confounding variables.

When setting experiments, be careful of the following:

- Keep certain things or parameters constant
- Include a control group with a placebo treatment or without a treatment
- Randomly assign participants or members to groups
- Increase diversity to reduce the likelihood of systematic confound

#1 True Experimental Design

Some comparative experimental studies do not include a control group, which can be a problem if two treatments yield similar results. A true experimental design is to compare two (or more) groups or situations, with a control group or situation. The true experimental designs are characterized by the random selection of all the cases to be tested and the use of a control group parallel to the experimental group(s). Normally, we select only one variable to manipulate

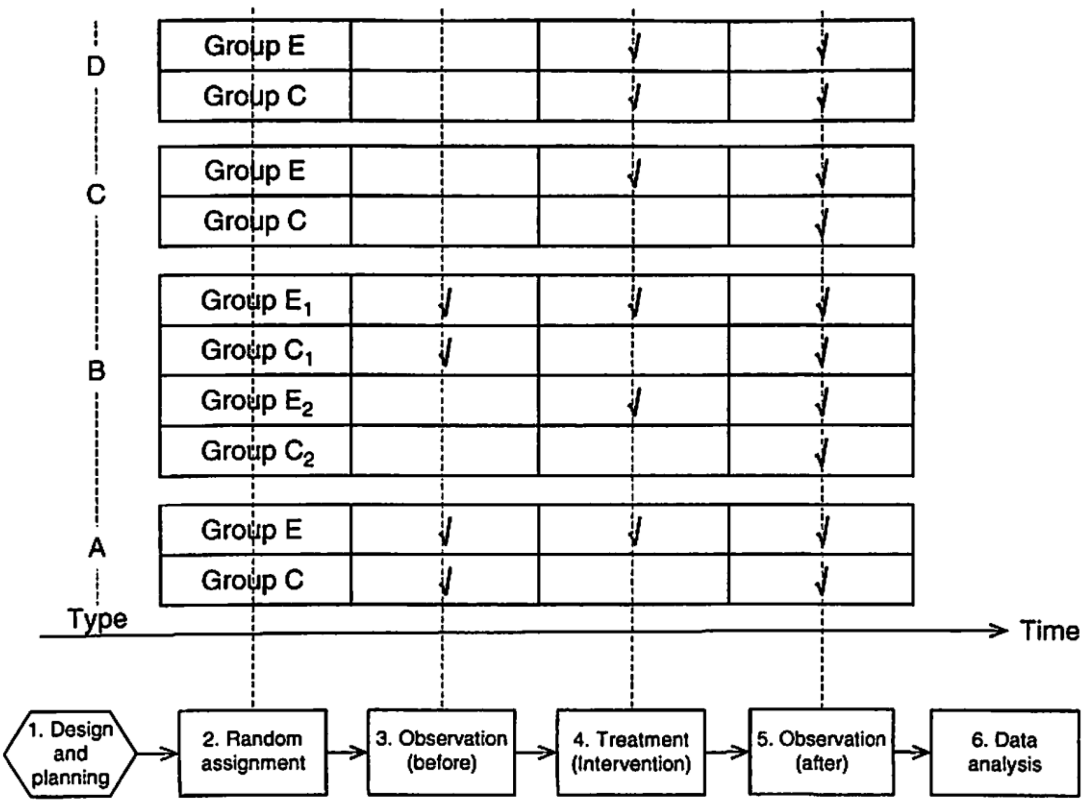


Figure 5.13 Four types of true experimental studies and overall process.

Figure 5.13 shows the common types of true experimental designs, where Group E has experimental treatments, but Group C is a control group without treatment. The process of true experimental studies has six stages. Each of the observations often has multiple measurements.

#2 Pre-Experimental Study

Pre-experimental Study. This type of study is the simplest form. In such a study, either a single group or multiple groups are observed presuming a treatment causes the change. The experimental and control groups in this type of study are not composed of equivalent or randomly selected members or there may be no control group. Pre-experimental study should be considered as a merely test and should be followed by more controlled experiments.

#3 Quasi-Experimental Study

Quasi-experimental Design. This type of experiment is similar to regular true experiment designs, except its randomness is not practical or ensured. In such experiments, two groups (a control group and an experimental treatment group) are still matched. We apply some criterion other than random assignments in the experiments. e.g. planning your experimental runs based on one factor that is hard to change over by running all the experiments where that factor is at one setting, then the remaining experiments with that factor at another setting. In this way, it becomes more feasible to set up than true experimental designs.

Due to the lack of randomness, the statistical analysis, results, and conclusions from a quasiexperiment do not have as strong internal validity. Quasi experiments are often used for causal relationships with two or more variables. Quasi-experimental studies are not often used for engineering and technology research.

#4 Ex-Post Facto Design

Ex Post Facto Design. This is a type of quasi-experimental study but is based on the existing (secondary) data. In such studies, there is no direct control or manipulation of the independent variables. Using this kind of study, we may reveal the presumed cause that has already occurred for certain effects. Therefore, we are to collect existing data to investigate a possible relationship between the factors and subsequent characteristics. There is a chance of searching in the wrong areas and getting incorrect conclusions.

4. Factorial Design of Experiment (DOE)

Design of experiments (DOEs) is a methodology that has various methods, design techniques, and processes for engineering study, research, and optimization. DOE is powerful and practical for the cases with multiple factors. The applications of DOE involve experimental planning, conducting experiments, fitting models to the outputs, and so on.

We often use DOE to find the relationship between the controlled variation of input factors and the variation of a system outcome under certain conditions.

Conventional comparative experiments are sometimes called OFAT (One-Factor-at-a-Time). If multiple input factors are considered, a conventional experiment design has to hold all factors constant except one factor to vary at a time during experiments. Obviously, OFAT is inefficient and ineffective to study multiple factors.

One strength of DOE is about handling multifactors in an experimental design. Figure 5.14 shows a diagram that DOE can be used for studying the relationship between three input factors and one response of a process. The input factors are called "treatments" in DOE, and they are assignable at different levels.

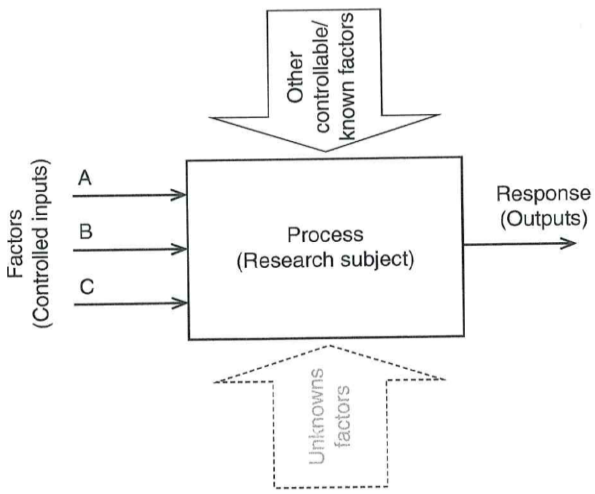


Figure 5.14 A process with inputs and output for DOE.

Two Factor Design

5.4.1.3 Two-level Factorial Design

A two-level factorial design means that each input factor has two values, i.e. low and high. A design of full two-level factors is the foundation of DOE and is widely used in practice because it is simple and efficient to design and execute. If there are k factors, the full two-level factorial design consists of 2^k treatment combinations. The simplest design is 2^2, meaning two levels for each of the two input factors.

The 2^2 design of an experiment needs to run at least four times. In addition, under each combination of levels and factors, we may have multiple runs (replicates). Replicates provide an estimate of the pure trial-to-trial "noise" in an experiment and allow for more precise estimates of the effects. In other words, with two replicates, a 2^2 design needs to run eight times (see Table 5.8). We normally generate a DOE design matrix is using software, such as Minitab.

Similarly, three-level factors have three levels of values: low, normal, high presented by 3^k. Three-level factorial design can yield more information, particularly possible nonlinear relationship. However, four-level factors are uncommon due to the complexity of experiments.

Table 5.8 An example of 2^2 DOE with two replicates.

Trial			Factors or treatments		Response
Standard order	Run order	Blocks	*A*	*B*	*Y*
1	5	1	−1	−1	y_1
2	8	1	1	−1	y_2
3	1	1	−1	1	y_3
4	6	1	1	1	y_4
5	4	2	−1	−1	y_5
6	2	2	1	−1	y_6
7	7	2	−1	1	y_7
8	3	2	1	1	y_8

Steps of DOE

(1) Design and planning

Define input factors and output response, determine reasonable range, assign specific levels.

For most engineering and technical projects, input factors and the output in an experiment are quantitative (numerical) variables, rather than attributes (e.g. pass/fail). In addition, even if the exact output may be unknown before running a DOE, the experimenter should estimate the range of the output and appropriately prepare its measurements.

(2) Execution

Following the experimental design, we need to run the experiments based on the design (or at the levels of each factor) and record the output responses. There is no physical difference between conducting DOE tests and doing any other types of experiments. For example, we must record the inputs, experiment conditions, and output. We should follow the running order based on randomized combinations of factors and levels, which is included in the design matrix.

(3) Analysis

Many textbooks have relevant mathematic equations and calculation examples. However, manual analysis of DOE results are tedious. Using computer software, the analysis is quick and easy, as we just need to input the experiment data and select the correct functions of software. The output of software analysis is often in

Other Considerations of DOE

Replication, as mentioned, means a test under each factor combination runs multiple times. Running DOE with replications is an effective way to reduce and estimate the errors associated with an experiment. Generally, the more an experiment is replicated, the more reliable the results can be. When testing replication is technically and financially feasible, it is recommended that we run the test under each condition three times.

Experiments have some factors that likely affect the output but are not of primary concern to the research objectives. These factors are called nuisance factors if they are controllable. To control these factors, we may use a blocking approach, or assigning some tests into groups, in an experiment design. Using a blocking approach, we can increase the probability of revealing the true differences in identification of the main effects influencing the output from a DOE test. DOE software packages have blocking functions to use in experiment designs.

5. Summary

Statistical Analyses

1. Two major types of statistical analysis are descriptive statistics (to reveal and summarize the information about a data set) and inferential statistics (to analyze sample data to make inferences about the population).
2. A data set is often analyzed and described by its central tendency (mode, median, and/or mean) and variability (range, standard deviation, and/or average deviation).
3. The association of two data sets is often analyzed for their correlation. The correlation coefficient r indicates mathematical correlation, but does not necessarily present a causational relationship between the two data sets.
4. ANOVA and regression analysis are often used for relationship between data sets.
5. A meta-analysis is a statistical analysis of previous studies.
6. A hypothesis testing is a process following certain format and steps and often concludes based on the calculated p-value. The analysis has types I (α) and II (β) errors.
7. There are several methods to detect outliers in a data set.

Engineering Quantitative Research

8. There are various mathematical approaches to model real-world situations or cases.
9. Optimization has wide applications in applied research and R&D, good research potentials, and significant benefits.
10. CI in real world is different from optimization.
11. Computer simulation is a research method and process to produce the functions, behaviors, and outcomes of a physical system. Simulation has growing applications for complex real-world situations.
12. Research can be the development of new technologies, methods, and processes.

Experimental Studies

13. Experimental study is a common research approach, including comparative studies and factorial DOEs.
14. Comparative studies, simple and effective, include true experimental, pre-experimental, quasiexperimental, and ex post facto design. However, they are not suitable for the studies with interactive and confounding variables.
15. When randomness is not designed into a comparative study, it is called quasi-experimental design. If using existing data, a comparative study is called an ex post facto design.

Factorial Design of Experiment (DOE)

16. DOE is an approach and a process that has various methods and design techniques for research.
17. DOE is more complex than the comparative studies but can handle multiple variables and reveal possible interactions between variables.
18. DOE has certain processes and considerations and often use software for the designs, experimental plan, and consequent data analysis.

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