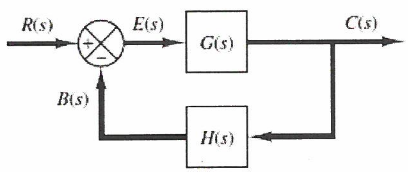
Block Diagrams

Open-Loop Transfer Function & Feedforward Transfer Function

$B(s)$: Feedback Signal; $E(s)$: Actuating Error Signal

$$\text{Open} - \text{loop transfer function} = \frac{B(s)}{E(s)} = G(s)H(s)$$

$$\text{Feedforward transfer function} = \frac{C(s)}{E(s)} = G(s)$$



If the feedback transfer function $H(s)$ is unity, then the open-loop transfer function and the feedforward transfer function are the same

Block Diagrams

Closed-Loop Transfer Function

$$\text{Closed} - \text{loop transfer function} = \frac{\mathcal{L}[\text{Output}]}{\mathcal{L}[\text{Input}]} = \frac{C(s)}{R(s)}$$

$$\left.\begin{aligned} C(s) &= G(s)E(s) \\ E(s) &= R(s) - B(s) = R(s) - H(s)C(s) \\ C(s) &= G(s)[R(s) - H(s)C(s)] \end{aligned}\right\} \quad \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}}$$

From the above closed-loop transfer function,

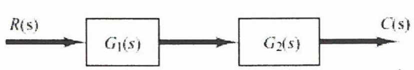
$$C(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$

Example 1

Determine the transfer function $C(s) / R(s)$ of the below systems

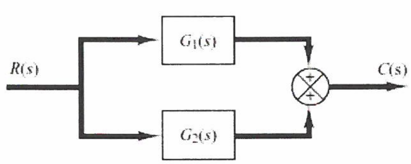
Cascaded

(a)



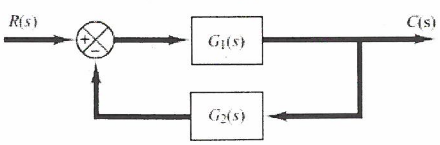
Parallel

(b)



Feedback

(c)



Answer:

$$\frac{C(s)}{R(s)} = G_1(s)G_2(s)$$

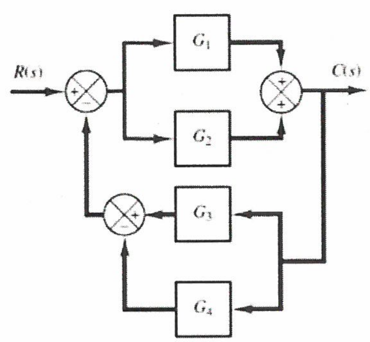
$$C(s) = R(s)G_1(s) + R(s)G_2(s)$$

$$\frac{C(s)}{R(s)} = G_1(s) + G_2(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$

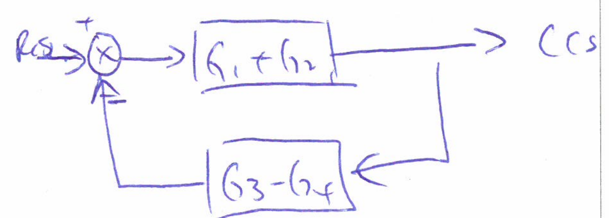
Example 2

Obtain the closed-loop transfer function $C(s) / R(s)$.



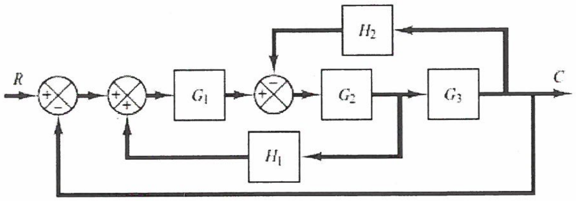
Answer:

$$\frac{C(s)}{R(s)} = \frac{G_1 + G_2}{1 + (G_1 + G_2)(G_3 - G_4)}$$



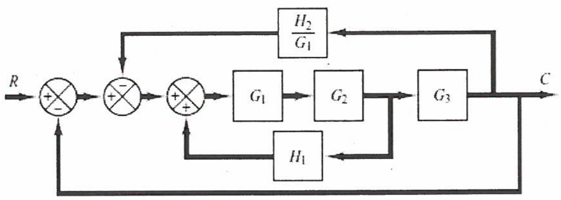
Example 3

Simplify the below block diagram.



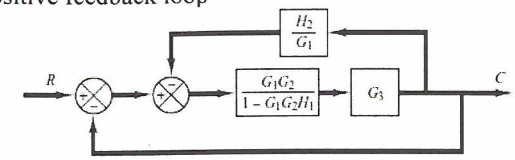
Answer:

Moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1

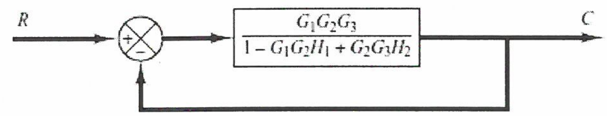


Example 3

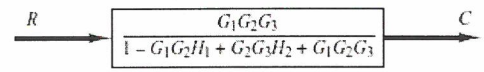
Eliminating the positive feedback loop



Eliminating the loop containing H_2/G_1 gives,



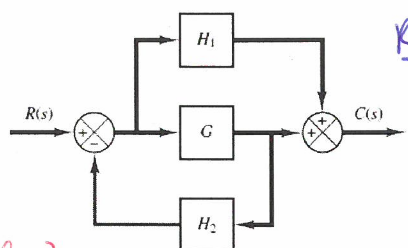
Finally, eliminating the feedback loop results,

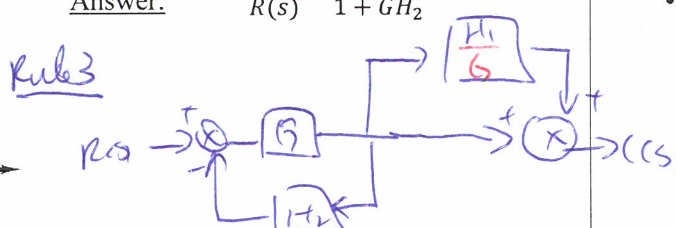


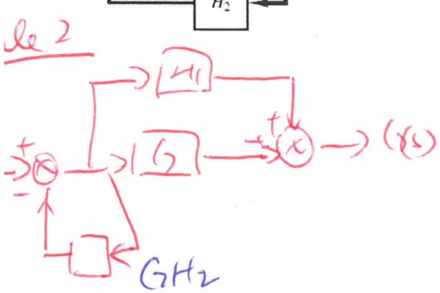
Example 4

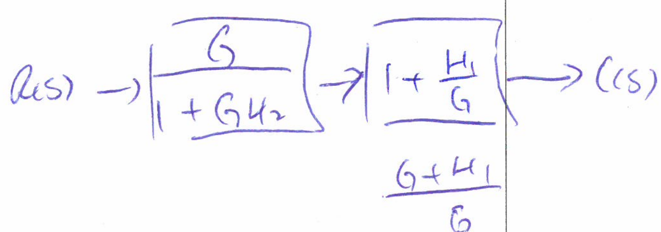
Simply the below block diagram.

Answer: $\frac{C(s)}{R(s)} = \frac{G+H_1}{1+GH_2}$



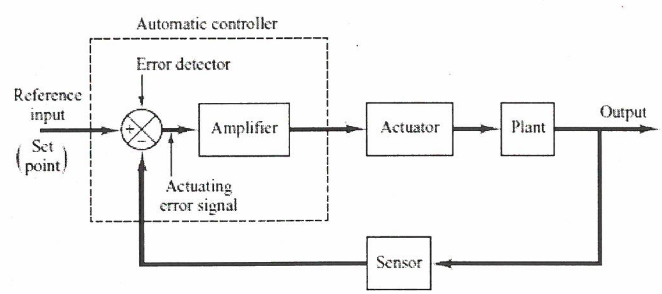






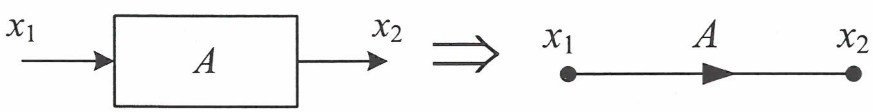
Modelling of Automatic Controllers

- An automatic controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value



Signal Flow Graphs

- SFG is another pictorial representation of a system

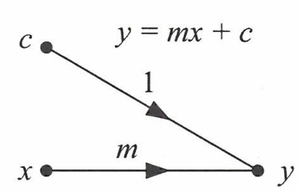


- Every variable becomes a node and every transmission function A is designated by a branch
- Thus, A represents the system transfer function

Signal Flow Graphs

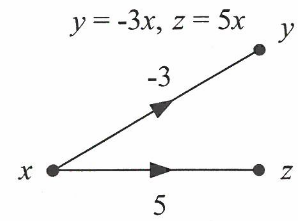
- Signal flow graph algebra

Addition



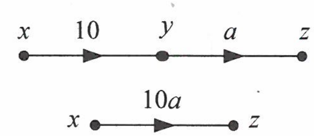
The variable at a node is equal to the sum of all signal entering the node

Transmission



The variable designated by a node is transmitted on every branch leaving the node

Multiplication



Cascades are reduced as in block diagrams

Signal Flow Graphs

Properties

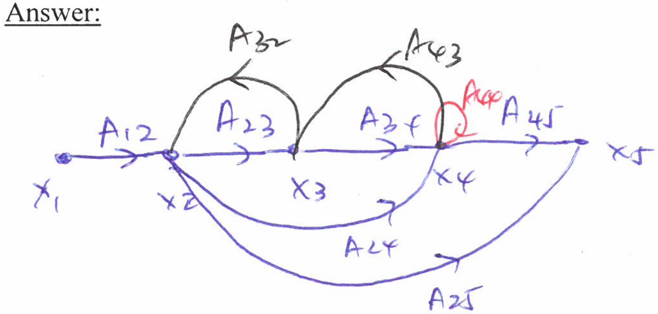
1. SFG applies only to linear systems
2. The equations for which an SFG is drawn must be algebraic equations in the form of cause-and-effect
3. Nodes are used to represent variables. Normally, the nodes are arranged from *left to right*, from the input to the output, following a succession of cause-and-effect relations through the system
4. Signals travel along branches only in the direction described by the arrows of the branches.
5. The branch directing from node x_k to x_j represents the dependence of x_j upon x_k , but not the reverse
6. A signal x_k traveling along a branch between x_k and x_j is multiplied by the gain (A_{kj}) of the branch, so a signal $A_{kj}x_k$ is delivered at x_j

Example 9

Construct the signal flow graph of a system described by the following set of algebraic equations:

$$\left.\begin{aligned} x_2 &= A_{12}x_1 + A_{32}x_3 \\ x_3 &= A_{23}x_2 + A_{43}x_4 \\ x_4 &= A_{24}x_2 + A_{34}x_3 + A_{44}x_4 \\ x_5 &= A_{25}x_2 + A_{45}x_4 \end{aligned}\right\} \text{Same as example 10}$$

Answer:



Signal Flow Graphs

Definitions

- Input Node (Source): An input node is a node that has only outgoing branches
- Output Node (Sink): An output node is a node that has only incoming branches. However, this condition is not always readily met by an output node
- Path: A path is any collection of a continuous succession of branches traversed in the same direction
- Forward Path: A path of an input node to an output node, no node is traversed more than once
- Feedback Path or Loop: Originates and ends at the same node, no node is traversed more than once
- Self Loop: A feedback loop consisting of one branch
- Path Gain: Product of the branch gains encountered in traversing a path
- Loop Gain: Path gain of a loop
- Non-touching Loops: Two parts of an SFG are non-touching if they do not share a common node

Signal Flow Graphs

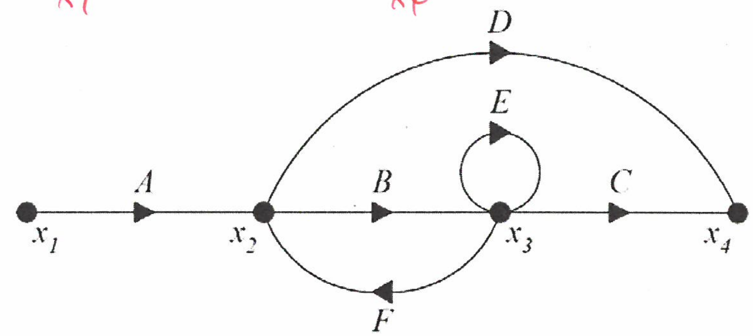
ABC / AD BF E

Forward path? Feedback path? Self loop?

Gain? A→F Path gain? ABC AD Loop gain? BF, E

Input node? Output node?

x1 x4



Signal Flow Graphs

Mason's rule

$$\boxed{M = \frac{Y}{U} = \frac{1}{\Delta}\sum_{k=1}^{N}(P_k\Delta_k)}$$

Y = Output-node variable

U = Input-node variable

N = Total number of forward paths between Y and U

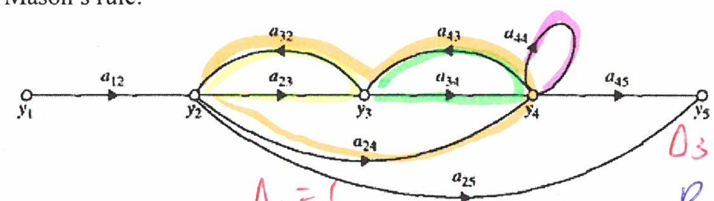
P_k = Gain of the kth forward paths between Y and U

$\Delta = 1 -$ (sum of all individual loop gains) + (sum of gain products of 2 non-touching loops) – (sum of gain products of 3 non-touching loops) + …

$\Delta_k =$ Δ evaluated with all loops touching P_k eliminated (i.e. set equal to zero)

Example 10

Consider the signal flow graph constructed in Example 9. Determine the gain by using the Mason's rule.



$\Delta_3 = 1-(L_2+L_4)$

$P_3 = a_{12}a_{25}$

$\Delta_1 = 1$

Answer:

Forward Path (2) 3: $P_1 = a_{12}a_{23}a_{34}a_{45}$

$P_2 = a_{12}a_{24}a_{45}$ $\Delta_2 = 1$

Loop (4): $L_1 = a_{23}a_{32}$, $L_2 = a_{34}a_{43}$, $L_3 = a_{24}a_{43}a_{32}$

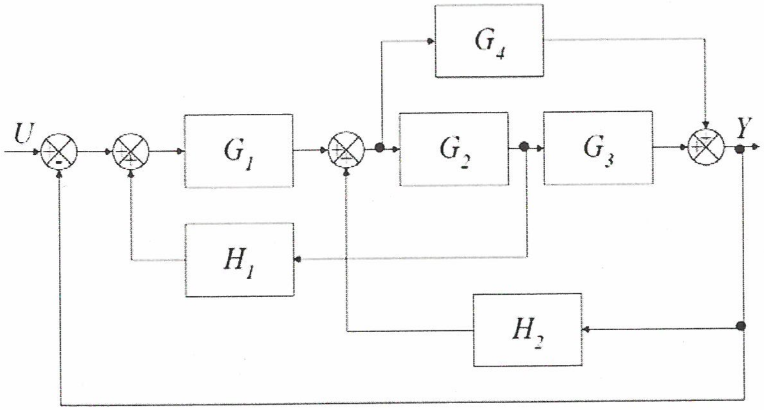
$L_4 = a_{44}$

Non-touching Loop (1):

$L_1L_4 = a_{23}a_{32}a_{44}$ $\Delta = 1-(L_1+L_2+L_3+L_4) + (L_1L_4)$

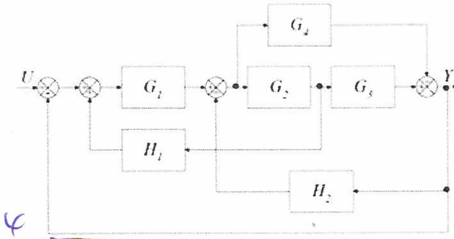
Example 12

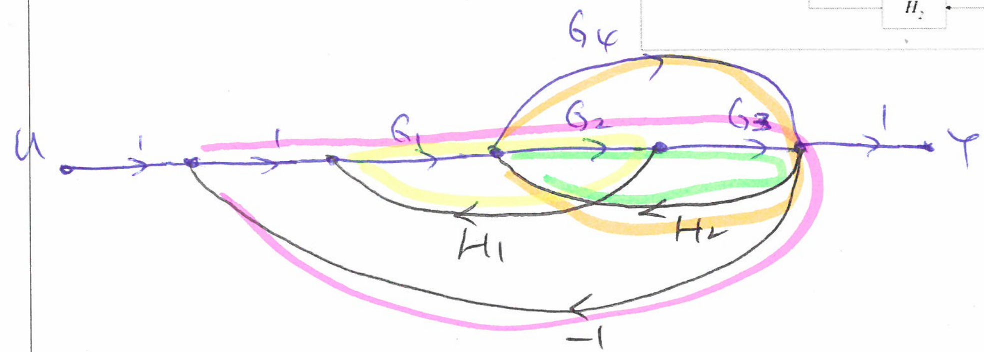
Construct a signal flow graph for the following block diagram and hence determine the transfer function (Y / U).



Example 12

Answer:





$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_4$

$L_1 = G_1 G_2 H_1$

$L_2 = G_2 G_3 H_2$

$L_3 = G_4 H_2$

$L_4 = -G_1 G_2 G_3$

$L_5 = -G_4 G_1$

$\Delta = 1-(L_1 + L_2 + L_3 + L_4 + L_5)$

$\Delta_1 = 1$

$\Delta_2 = 1$

Example 12

Answer:
Try it yourself !

$$\therefore \frac{Y}{U} = \frac{G_1G_2G_3 + G_1G_4}{1-(G_1G_2H_1 + G_2G_3H_2 - G_1G_2G_3 - G_1G_4 + G_4H_2)}$$