Question

Polar (Nyquist) Plot

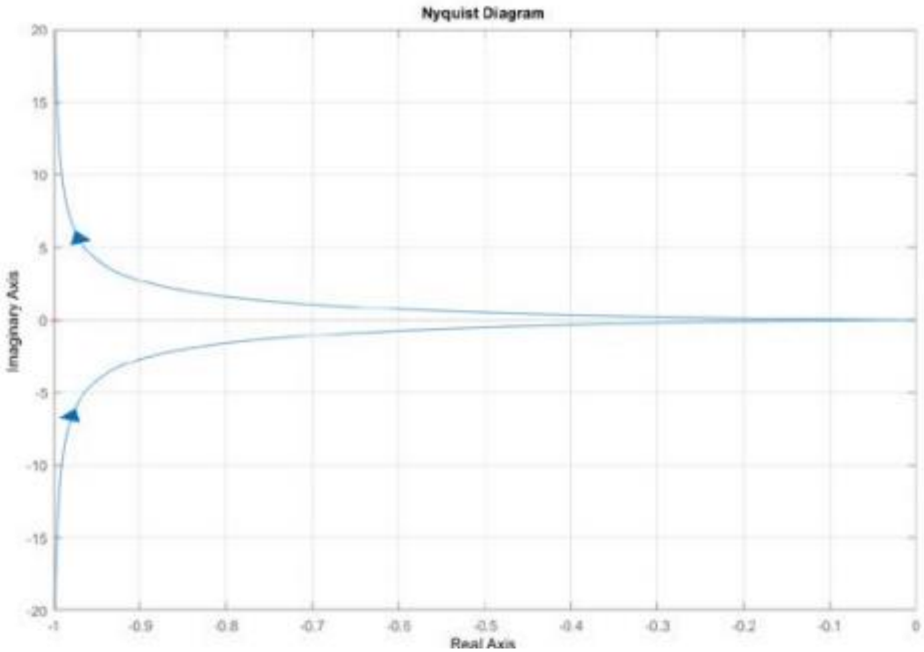
1. Sketch the Nyquist plot for the system,

$$G(s) = \frac{1}{s(s+1)}.$$

Solution

1. $|G(j\omega)| = \dfrac{1}{\omega\sqrt{\omega^2+1}}$ and $\angle G(j\omega) = -90° - \tan^{-1}\omega$

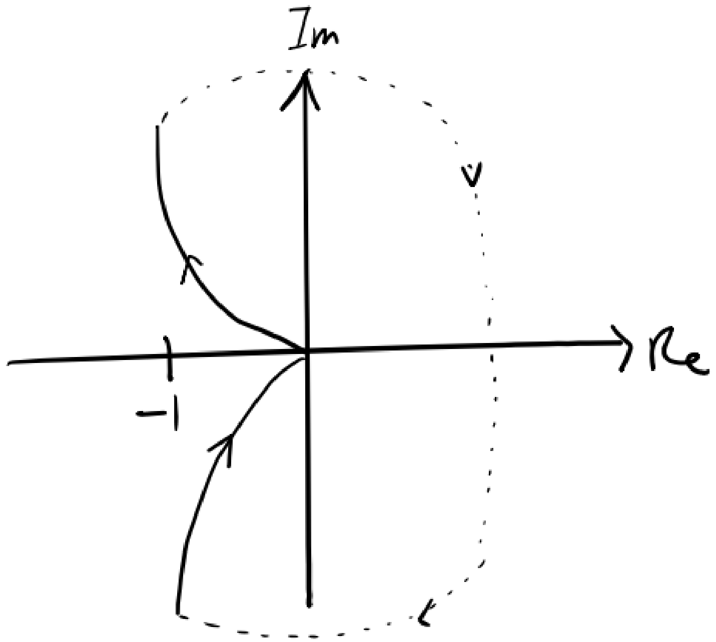
From Matlab:



Sketch:

$\omega = 0^+; |G(j\omega)| = \infty$ and $\angle G(j\omega) = -90°$

$\omega = +\infty; |G(j\omega)| = 0$ and $\angle G(j\omega) = -180°$



Question

2. Given the open-loop transfer function of a system,

$$G(s)H(s) = \frac{K}{s(s+1)(2s+1)}.$$

(a) Sketch the Nyquist plot of the above system with $K = 2$.

(b) Use Nyquist stability criterion to determine the absolute stability of the closed-loop system.

(c) Find the critical value of gain K for stability.

(Ans: (b) unstable; (c) $0 < K < 3/2$)

Solution

2. (a) $$|G(j\omega)H(j\omega)| = \frac{2}{\omega\sqrt{\omega^2+1}\sqrt{(2\omega)^2+1}}$$

$$\angle G(j\omega)H(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$-90° - \tan^{-1}\omega - \tan^{-1}2\omega = -180°, \quad -\tan^{-1}\omega - \tan^{-1}2\omega = -90°$$

$$\because \tan^{-1}X + \tan^{-1}Y = \tan^{-1}\left(\frac{X+Y}{1-XY}\right)$$

$$\therefore \tan^{-1}\left(\frac{\omega+2\omega}{1-(\omega)(2\omega)}\right) = 90°$$

$$\frac{3\omega}{1-2\omega^2} = \infty$$

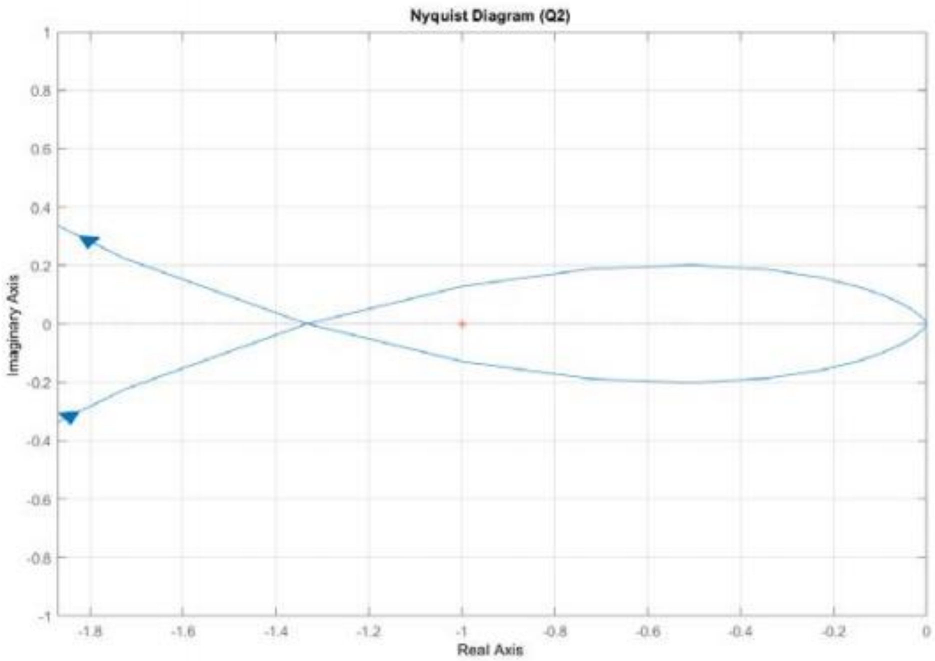
$$\therefore 1-2\omega^2 = 0, \quad \omega = \sqrt{\frac{1}{2}} = 0.707 \text{ rad/s}$$

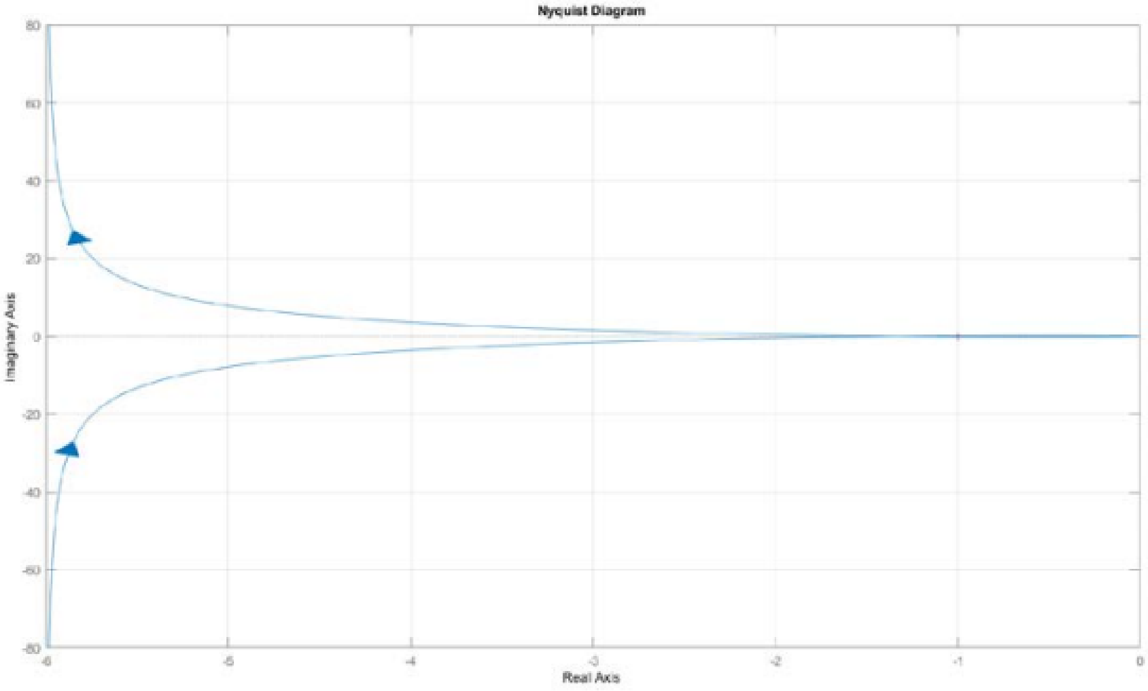
At $\omega = 0.707$ rad/s, the system magnitude will be

$$|G(j\omega)H(j\omega)| = \frac{2}{\sqrt{1/2}\sqrt{\left(\sqrt{1/2}\right)^2+1}\sqrt{\left(2\sqrt{1/2}\right)^2+1}} = 1.333$$

Hence, the Nyquist plot will cross in the x-axis at -1.333.

From Matlab:



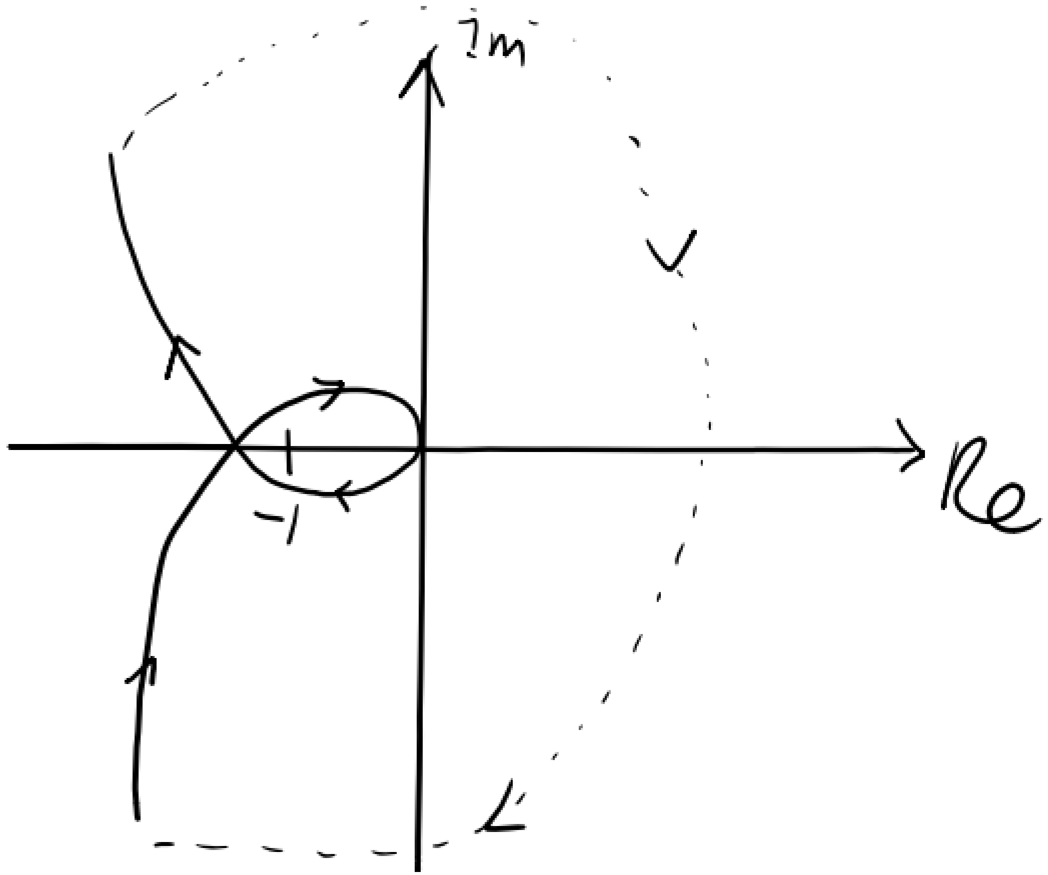


Solution

Sketch:

$\omega = 0^+; |G(j\omega)| = \infty$ and $\angle G(j\omega) = -90°$

$\omega = +\infty; |G(j\omega)| = 0$ and $\angle G(j\omega) = -270°$



Solution

2. (b) Nyquist stability criterion:

$P = 0$

$N = 2$

$\therefore Z = 2$

Hence, the system is unstable since there are 2 closed-loop poles in the right-half s plane.

Solution

2. (c) Let find the point where the Nyquist plot crosses the negative real axis $\Rightarrow$ the imaginary part of $G(j\omega)H(j\omega) = 0$, which

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega+1)(2j\omega+1)} = \frac{K}{-3\omega^2 + j(\omega - 2\omega^3)}$$

$$\omega - 2\omega^3 = 0 \Rightarrow \omega(1 - 2\omega^2) = 0$$

$$\therefore \omega = 0, \quad 1 - 2\omega^2 = 0 \Rightarrow \omega = \pm\frac{1}{\sqrt{2}}$$

Substituting $\omega = 1/\sqrt{2}$,

$$\frac{K}{-3\left(\frac{1}{\sqrt{2}}\right)^2 + j(0)} = -\frac{2K}{3}$$

The critical value of the gain K is obtained by equating $-\frac{2K}{3} = -1 \Rightarrow K = \frac{3}{2}$.

The system is stable if

$$0 < K < \frac{3}{2}$$

OR using Routh Array

The characteristic equation, $\Delta(s) = s(s+1)(2s+1) + K = 2s^3 + 3s^2 + s + K = 0$

s^3	2	1
s^2	3	K
s^1	$\frac{3-2K}{2}$	
s^0	K	

$$\frac{3-2K}{2} > 0, \quad K < \frac{3}{2}$$

Hence the system is stable if

$$0 < K < \frac{3}{2}$$