Question

Nichols Chart

3. Open-loop frequency response tests on a control system with unity gain negative feedback yield the following data.

Frequency ω (rad/s)	Gain (Output / Input)	Phase (degree)
0.4	2.452	–101.31
0.8	1.161	–111.80
1.2	0.715	–120.96
1.4	0.585	–124.99
1.8	0.413	–131.99
2.0	0.354	–135.00
4.0	0.112	–153.43
8.0	0.030	–165.96

A first-order lag with a time constant of 1 sec, $G_1(s) = \frac{1}{s+1}$, is now inserted in the forward path of the control loop. Use a Nichols chart to determine for the modified system

(a) the gain margin;
(b) the phase margin.

(Ans: (a) 9.4 dB; (b) 33°)

Solution

3. (a) & (b) First-order lag system

$$G_1(j\omega) = \frac{1}{j\omega + 1}$$

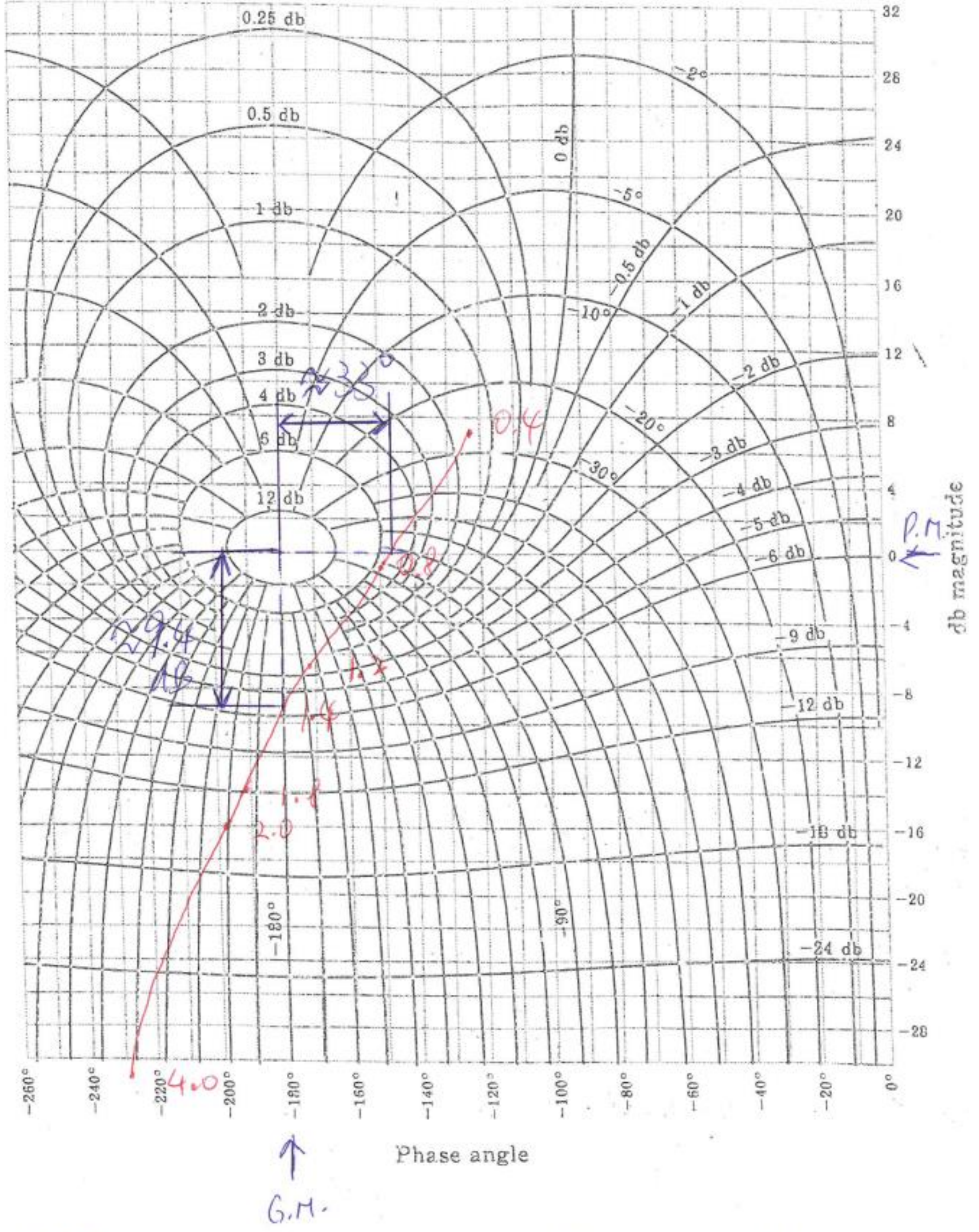
$$|G(j\omega)| = 20\log\left(\frac{1}{\sqrt{\omega^2+1}}\right) = -10\log(\omega^2+1)$$

$$\angle G(j\omega) = -\tan^{-1}\omega$$

Change the gain value to dB for the given open-loop frequency response.

ω (rad/s)	Open-loop system: $G_p(j\omega)$		$G_1(j\omega)$		Combined system: $G_1(j\omega)G_p(j\omega)$	
	Gain (dB)	Phase (°)	Gain (dB)	Phase (°)	Gain (dB)	Phase (°)
0.4	7.79	−101.31	−0.64	−21.80	7.14	−123.11
0.8	1.29	−111.80	−2.15	−38.66	−0.85	−150.46
1.2	−2.92	−120.96	−3.87	−50.19	−6.79	−171.16
1.4	−4.65	−124.99	−4.71	−54.46	−9.37	−179.45
1.8	−7.68	−131.99	−6.27	−60.95	−13.96	−192.94
2.0	−9.03	−135	−6.99	−63.43	−16.02	−198.43
4.0	−19.03	−153.43	−12.30	−75.96	−31.33	−229.40
8.0	−30.37	−165.96	−18.13	−82.88	−41.50	−248.84

Use the data of the combined system for drawing on the Nichlos chart.



From the Nichlos chart, gain margin = 9.4 dB and phase margin = 33°

Question

4. A closed-loop control system consists of three elements A, B and C in series in the forward path and a unity gain feedback loop.

Component A is an amplifier with gain G.

Component B has the transfer function $\frac{1}{1+0.2s}$.

The transfer function of component C is not known, but frequency response tests on this component give the following results.

Frequency ω (rad/s)	Gain (Output / Input)	Phase (degree)
5.0	0.894	–63.26
7.0	0.714	–89.12
10.0	0.447	–116.34
10.8	0.394	–121.42
20.0	0.124	–152.39

(a) Show that if $G > 5$, the system will be unstable.

(b) If the gain G is fixed at 6, the system can be stabilized by putting an additional component with transfer function, $G_p(s) = 1 + Ks$, in series with A, B and C. Show that neutral stability now occurs when $K = 0.0111$.

Solution

4. (a) Similar to Q3

$$G_B(j\omega) = \frac{1}{1 + 0.2j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{(0.2\omega)^2 + 1}} \quad \text{and} \quad \angle G(j\omega) = -\tan^{-1} 0.2\omega$$

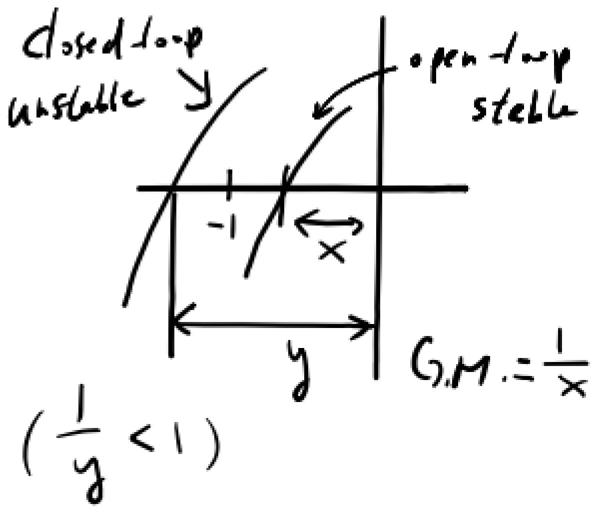
ω	$G_A(j\omega)$		$G_B(j\omega)$		$G_C(j\omega)$		$G_A(j\omega)G_B(j\omega)G_C(j\omega)$	
	Gain	Phase (°)	Gain	Phase (°)	Gain	Phase (°)	Gain	Phase (°)
5	G	0	0.707	–45	0.894	–63.26	0.632G	–108.26
7	G	0	0.581	–54.46	0.714	–89.12	0.415G	–143.58
10	G	0	0.447	–63.43	0.447	–116.34	0.2G	–179.77
10.8	G	0	0.4201	–65.16	0.394	–121.42	0.1655G	–186.58
20	G	0	0.243	–75.96	0.124	–152.39	0.03G	–228.35

Gain margin: G.M = 1 / 0.2G

For stability,

$$\frac{1}{0.2G} > 1 \quad or\ G < 5$$

Hence, the system is unstable if $G > 5$.



Solution

4. (b) The transfer function of the compensation element: $G_c(j\omega) = 1 + Kj\omega$, then $|G(j\omega)| = \sqrt{(\omega K)^2 + 1}$ and $\angle G(j\omega) = \tan^{-1} K\omega$

ω	$G_A(j\omega)G_B(j\omega)G_C(j\omega)$		$G_c(j\omega)$	
	Gain	Phase (°)	Gain	Phase (°)
5	3.792	−108.26	$\sqrt{25K^2+1}$	$\tan^{-1} 5K$
7	2.49	−143.58	$\sqrt{49K^2+1}$	$\tan^{-1} 7K$
10	1.2	−179.77	$\sqrt{100K^2+1}$	$\tan^{-1} 10K$
10.8	0.993	−186.58	$\sqrt{116.64K^2+1}$	$\tan^{-1} 10.8K$
20	0.18	−228.35	$\sqrt{400K^2+1}$	$\tan^{-1} 20K$

Consider the design point at $\omega = 10.8$ rad/s, for neutral stability,

$$\left(\sqrt{116.64K^2 + 1}\right)(0.993) = 1 \Rightarrow K = 0.0110$$

(It will be more accurate if you take more decimal places in calculating the Gain of various elements.)