Compensators / Controllers Design

Question

5. An open-loop frequency response test on a unit feedback control system produced the data below.

Frequency ω (rad/s)	Gain (dB)	Phase (degree)
0.3	20	–19
1	18	–51
3	10.5	–91
6	5	–116
10	–1	–135
20	–12	–163
30	–19	–177
60	–31.5	–201
100	–40	–218

(a) A phase-lead series compensating network, having the transfer function, $G_c(s) = \frac{0.4(1+0.08s)}{1+0.032s}$, is then incorporated to improve the system performance. Plot the gain and phase characteristics on a Nichols' chart.

(b) If the system gain is then increased by 16 dB, determine the characteristics of the compensated system the resulting

(i) gain margin;

(ii) and phase margin.

(Ans: (b)(i) 14.6 dB, (b)(ii) 43°)

Solution

5. (a) The transfer function of the phase-lead compensator is,

$$G_c(j\omega) = \frac{0.4(1 + 0.08j\omega)}{1 + 0.032j\omega}$$

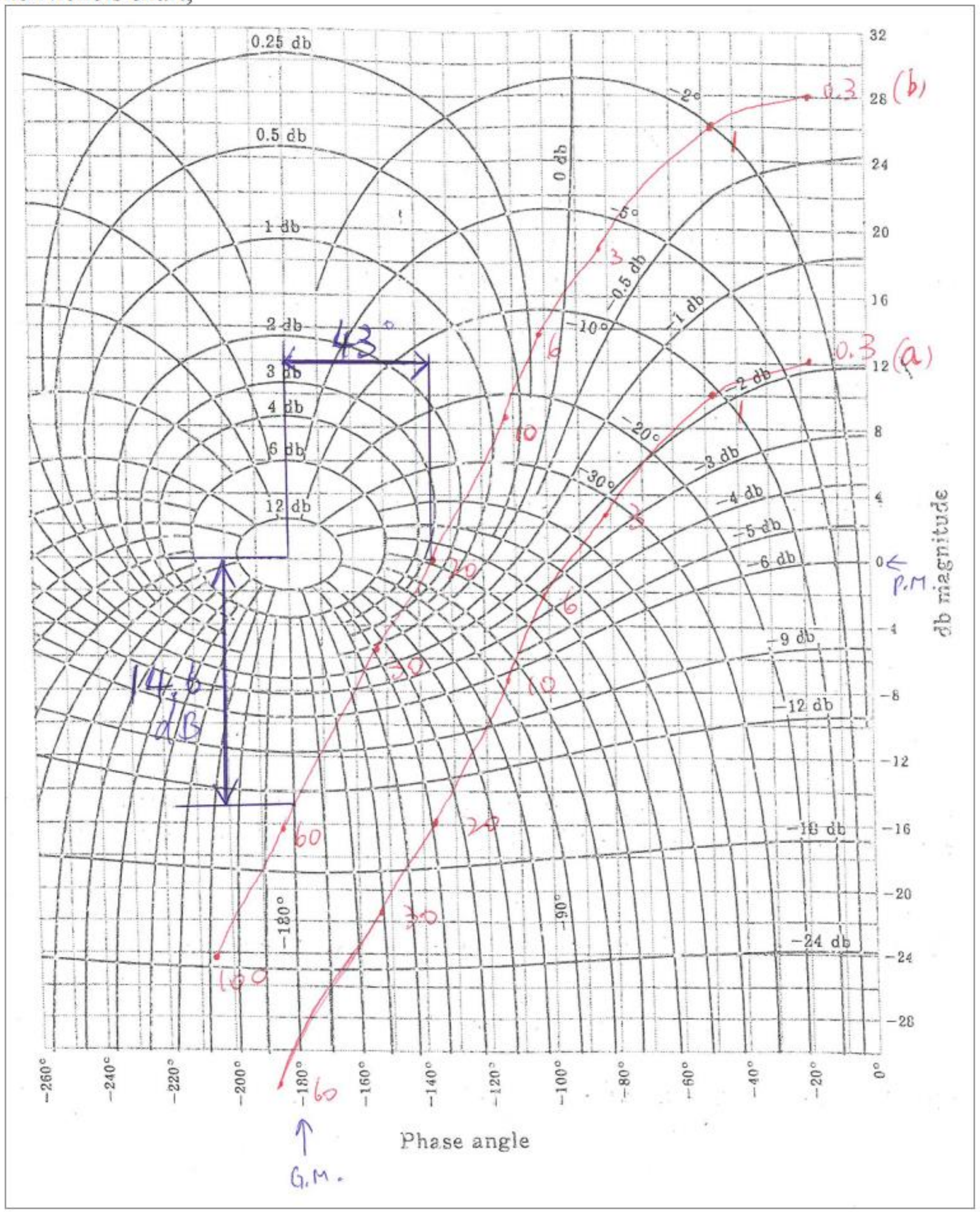
$$|G_c(j\omega)| = 20\log\frac{0.4\sqrt{(0.08\omega)^2 + 1}}{\sqrt{(0.032\omega)^2 + 1}} \quad \text{and} \quad \angle G_c(j\omega) = \tan^{-1}0.08\omega - \tan^{-1}0.032\omega$$

ω	$G_p(j\omega)$ Gain (dB)	$G_p(j\omega)$ Phase (°)	$G_c(j\omega)$ Gain (dB)	$G_c(j\omega)$ Phase (°)	$G_c(j\omega)G_p(j\omega)$ Gain (dB)	$G_c(j\omega)G_p(j\omega)$ Phase (°)
0.3	20	−19	−7.96	0.82	12.04	−18.19
1	18	−51	−7.94	2.74	10.06	−48.26
3	10.5	−91	−7.76	8.01	2.74	−82.99
6	5	−116	−7.22	14.77	−2.22	−101.23
10	−1	−135	−6.23	20.92	−7.23	−114.08
20	−12	−163	−3.94	25.38	−15.94	−137.62
30	−19	−177	−2.50	23.55	−21.50	−153.45
60	−31.5	−201	−0.86	15.74	−32.36	−185.26
100	−40	−218	−0.34	10.23	−40.34	−207.77

5. (b) (i) & (ii) If the system gain is increased by 16 dB, the curve in the Nichols chart in part (a) will shift up by 16 dB which is shown in the Nichols chart in part (a) as well.

From the new curve, Gain margin = 14.6 dB and Phase margin = 43°

The Nichols chart,



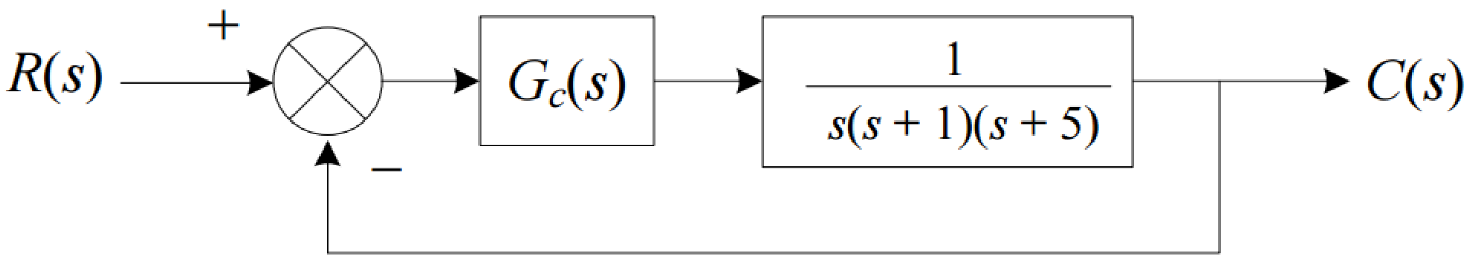
Question

6. Consider the control system shown below in which a PID controller is used to control the system. The PID controller has the transfer function,

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right).$$

Although many analytical methods are available for the design of a PID controller for the present system, lets us apply Ziegler-Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d.

(Ans: K_p =18, T_i =1.405, T_d =0.351)



Solution

6. By setting $T_i = \infty$ and $T_d = 0$ (i.e. only K_p) , we obtain the closed-loop transfer function as follow,

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5)+K_p}$$

The value of K_p that makes the system marginally stable (so that sustained oscillation occurs) by using the Routh's stability criterion, $\Delta(s) = s^3 + 6s^2 + 5s + K_p = 0$

s^3	1	5
s^2	6	K_p
s^1	$\frac{(6)(5)-(1)(K_p)}{6}$	
s^0	K_p	

$$\therefore \frac{30-K_p}{6} = 0 \Rightarrow K_p = 30$$

With gain K_p sets equal to K_c (= 30), the characteristic equation becomes,

$$\Delta(s) = s^3 + 6s^2 + 5s + 30 = 0$$

We substitute $s = j\omega$ into the $\Delta(s)$, we have

$$(j\omega)^3 + 6(j\omega)^2 + 5j\omega + 30 = 0$$
$$-j\omega^3 - 6\omega^2 + 5j\omega + 30 = 0$$

Rearranging the terms, we have

$$j\omega(5-\omega^2) + 6(5-\omega^2) = 0$$

Hence, we have

$$5 - \omega^2 = 0 \Rightarrow \omega = \pm\sqrt{5} \quad \text{and} \quad \omega = 0$$

The period of sustained oscillation is,

$$T_c = \frac{2\pi}{\omega} = 2.8099 \text{ sec}$$

Referring to Ziegler and Nichols tuning rule,

$K_p = 0.6K_c = (0.6)(30) = 1.8$

$T_i = 0.5T_c = (0.5)(2.8099) = 1.405$

$T_d = 0.125T_c = (0.125)(2.8099) = 0.351$