Question 1

Bode Plot

1. Plot the asymptotic and exact Bode diagrams of the open-loop transfer function,

$$G(s) = \frac{10(s+1)}{(s+2)(s+5)}.$$

Q1 solution

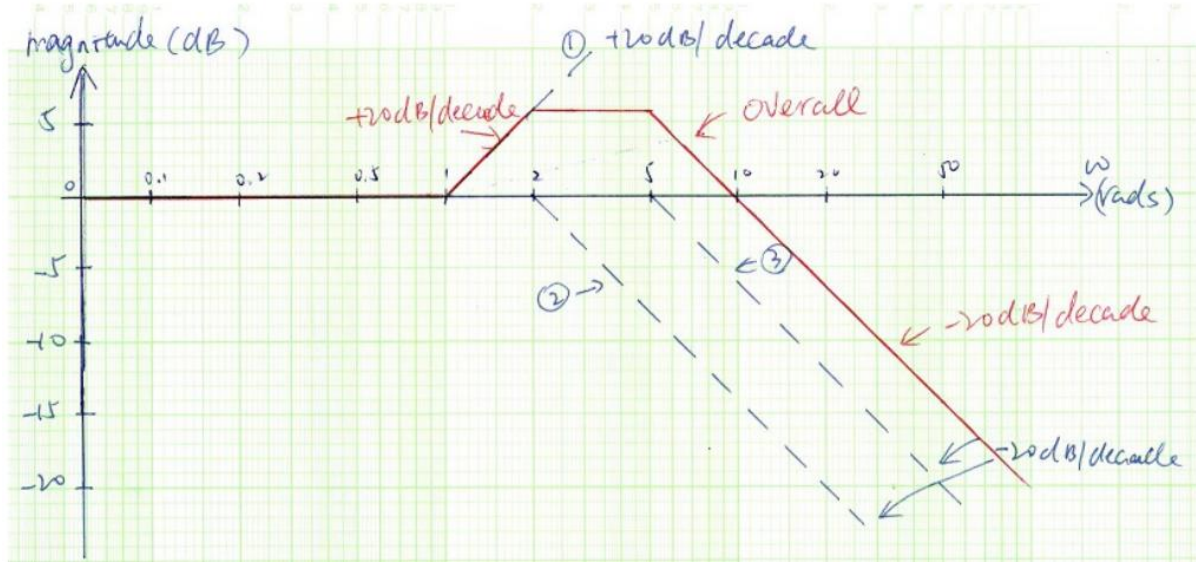
1. (a) $G(s) = \dfrac{10(s+1)}{(s+2)(s+5)}$

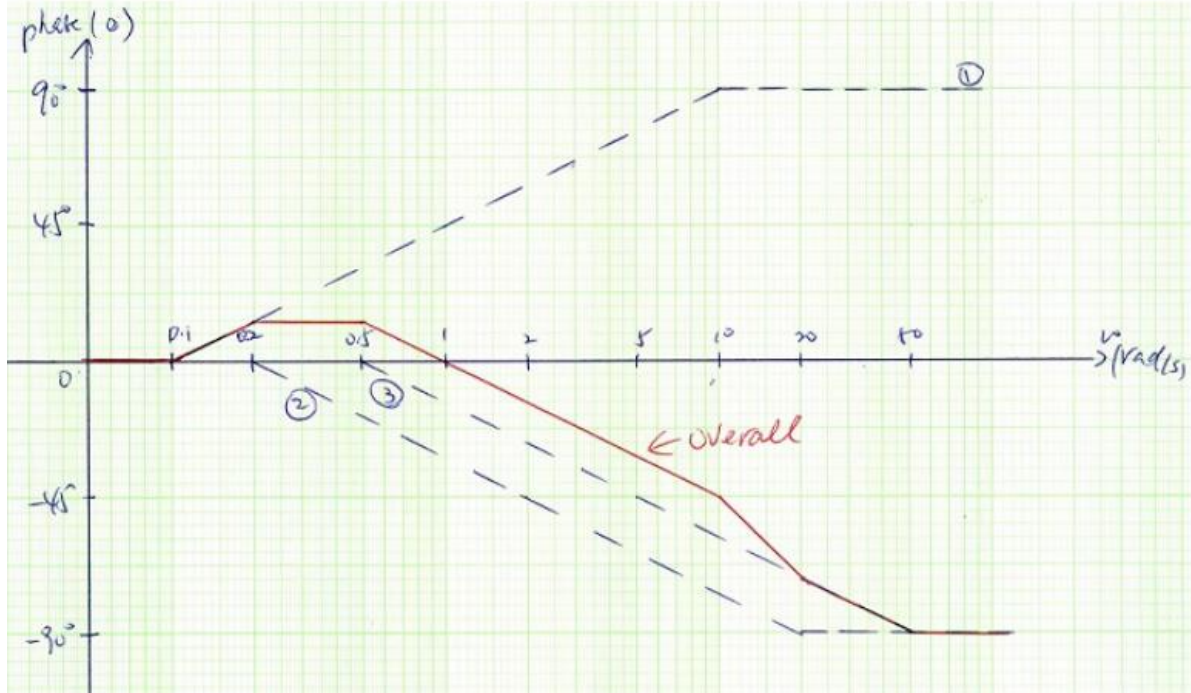
Asymptotic Bode Diagram:

$$G(j\omega) = \frac{10(j\omega+1)}{(j\omega+2)(j\omega+5)} = \frac{10(j\omega+1)}{\left(1+\frac{j\omega}{2}\right)(2)\left(1+\frac{j\omega}{5}\right)(5)} = \frac{j\omega+1}{\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{5}\right)}$$

From the standard form, we have identified 3 basic factors:

1. Zero: $j\omega + 1$; Corner frequency: $\omega = 1$ rad/s
2. Pole: $1 + \frac{j\omega}{2}$; Corner frequency: $\omega = 2$ rad/s
3. Pole: $1 + \frac{j\omega}{5}$; Corner frequency: $\omega = 5$ rad/s


magnitude (dB)
① +20dB/decade
+20dB/decade
overall
5
0
-5
-10
-15
-20
0.1
0.2
0.5
1
2
5
10
20
50
ω (rad/s)
②
③
-20dB/decade
-20dB/decade


phase (°)
90°
45°
0
-45°
-90°
0.1
0.2
0.5
1
2
5
10
20
50
ω (rad/s)
①
②
③
overall

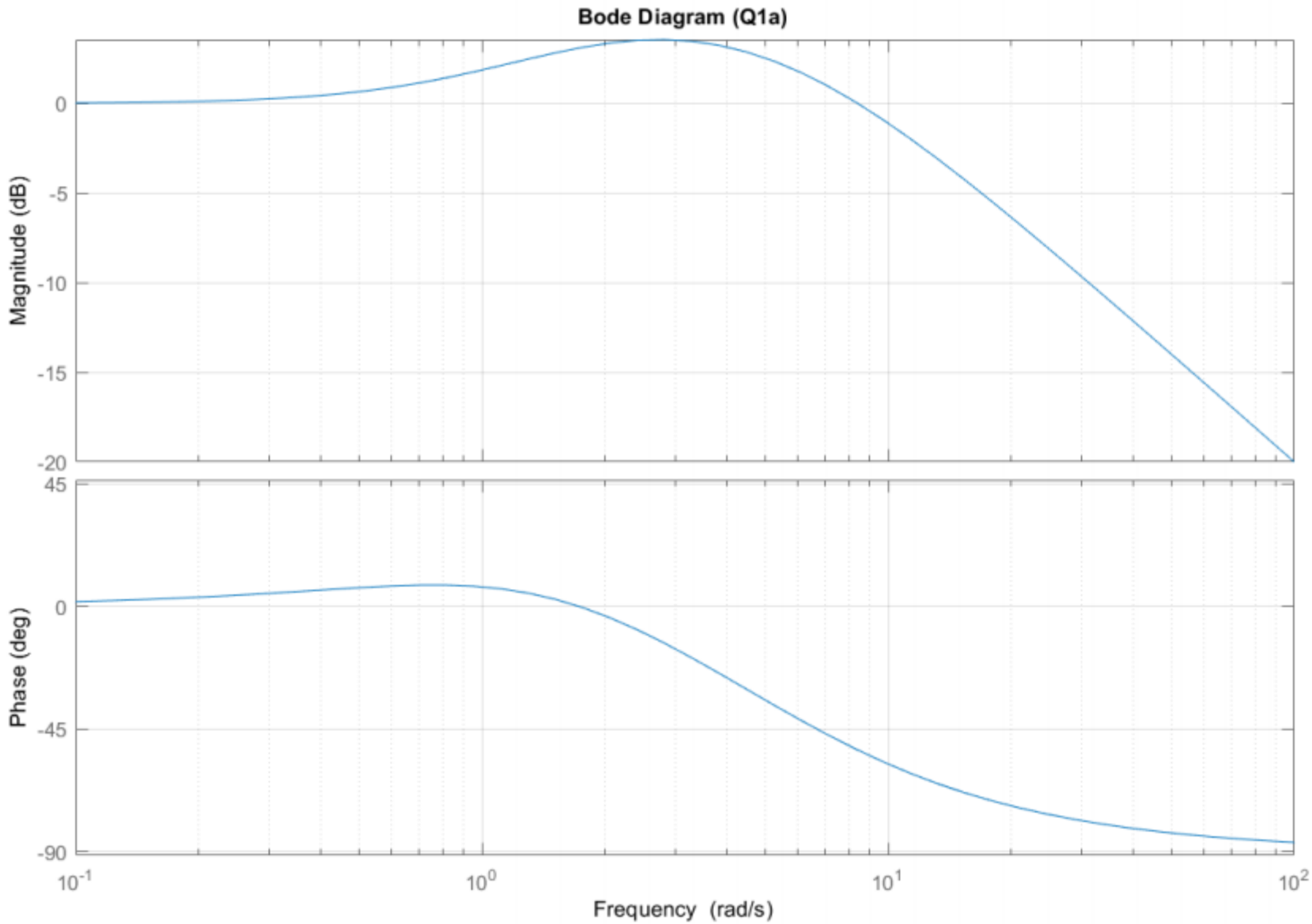
Exact Bode Diagram:

$$G(j\omega) = \frac{10(j\omega + 1)}{(j\omega + 2)(j\omega + 5)}$$

$$|G(j\omega)| = 20 \log \frac{10\sqrt{1^2 + \omega^2}}{\sqrt{2^2 + \omega^2}\sqrt{5^2 + \omega^2}} \quad \text{(dB)}$$

$$\angle G(j\omega) = \tan^{-1} \omega - \tan^{-1} \frac{\omega}{2} - \tan^{-1} \frac{\omega}{5} \quad (°)$$

ω (rad/s)	Magnitude (dB)	Phase (°)
0.1	0.0306	1.702
0.5	0.663	6.818
1	1.871	7.125
2	3.335	–3.366
5	2.536	–34.508
8	0.310	–51.083
10	–1.096	–57.836
15	–4.037	–67.784
20	–6.316	–73.116
30	–9.676	–78.633
50	–14.028	–83.145
80	–20.012	–85.708


Bode Diagram (Q1a)
Magnitude (dB)
0
-5
-10
-15
-20
Phase (deg)
45
0
-45
-90
10^-1
10^0
10^1
10^2
Frequency (rad/s)

Question 2

2. Draw the exact Bode diagram for the following open-loop transfer function using a frequency range from 0.05 to 0.25 rad/s, in step of 0.05 rad/s.

$$G(s) = \frac{2}{(1+2s)(1+3s)(1+11s)^2}$$

Solution

Exact Bode Diagram:

$$G(j\omega) = \frac{10(j\omega + 1)}{(j\omega + 2)(j\omega + 5)}$$

$$|G(j\omega)| = 20\log\frac{10\sqrt{1^2 + \omega^2}}{\sqrt{2^2 + \omega^2}\sqrt{5^2 + \omega^2}} \quad \text{(dB)}$$

$$\angle G(j\omega) = \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{5} \quad (°)$$

ω (rad/s)	Magnitude (dB)	Phase (°)
0.1	0.0306	1.702
0.5	0.663	6.818
1	1.871	7.125
2	3.335	–3.366
5	2.536	–34.508
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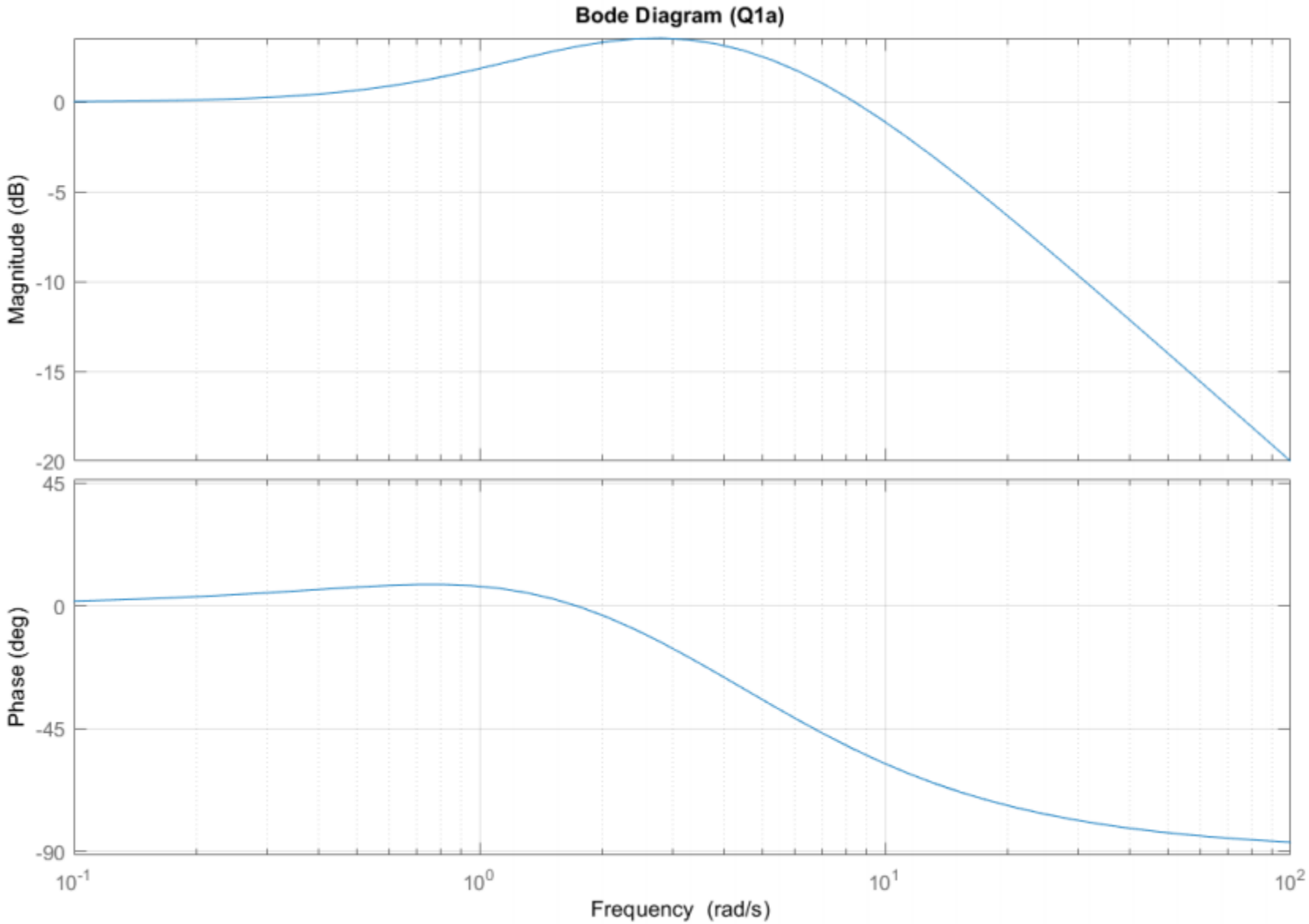
50	–14.028	–83.145
80	–20.012	–85.708


Bode Diagram (Q1a)
Magnitude (dB)
0
-5
-10
-15
-20
Phase (deg)
45
0
-45
-90
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)