Q4

Given the following characteristic equations, determine the system stability

(a) $\Delta(s) = s^3 + 4s^2 + 8s + 12 = 0$

(b) $\Delta(s) = 2s^3 + 4s^2 + 4s + 12 = 0$

Ans: (a) Stable; (b) Unstable

Solution

(a) The Routh's array,

$$
\begin{array}{c|cc}
s^3 & 1 & 8 \\
s^2 & 4 & 12 \\
s^1 & \dfrac{(4)(8)-(1)(12)}{4} = 5 & \\
s^0 & \dfrac{(5)(12)-(4)(0)}{5} = 12 &
\end{array}
$$

Since there is no sign change on the 1st column of the Routh's array, the system is **Stable.**

(b) The Routh's array,

$$
\begin{array}{c|cc}
s^3 & 2 & 4 \\
s^2 & 4 & 12 \\
s^1 & \dfrac{(4)(4)-(2)(12)}{4} = -2 & \\
s^0 & \dfrac{(-2)(12)-(4)(0)}{-2} = 12 &
\end{array}
$$

Since there are 2 sign changes on the 1st column of the Routh's array, the system is **Unstable.**

Q5

Determine the range of K such that the system with the characteristic equation, $\Delta(s) = s^4 + 6s^3 + 11s^2 + 6s + K = 0$, is stable.

Ans: $0 < K < 264$

Solution

From the Routh's array,

$$
\begin{array}{c|ccc}
s^4 & 1 & 11 & K \\
s^3 & 6 & 6 & \\
s^2 & \dfrac{(6)(11)-(1)(6)}{6} = 10 & \dfrac{(6)(K)-(1)(0)}{6} = K & \\
s^1 & \dfrac{(10)(6)-(6)(K)}{10} = 6-0.6K & & \\
s^0 & \dfrac{(6-0.6K)(K)-(10)(0)}{6-0.6K} = K & &
\end{array}
$$

The system is stable if there is no sign change on the 1st column of the Routh's array. Hence, we have $6-0.6K > 0, \quad K < 10$ and $K > 0 \Rightarrow 0 < K < 10$.