Q8 question

A very simplified version of the suspension system is shown in Figure 1 below. Assuming that the motion x_i at point P is the input to the system and the vertical motion x_o of the body is the output, obtain the transfer function $X_o(s)/X_i(s)$. (Consider the motion of the body only in the vertical direction.) Displacement x_o is measured from the equilibrium position in the absence of input x_i.

Ans: $\dfrac{X_o(s)}{X_i(s)} = \dfrac{bs + k}{ms^2 + bs + k}$

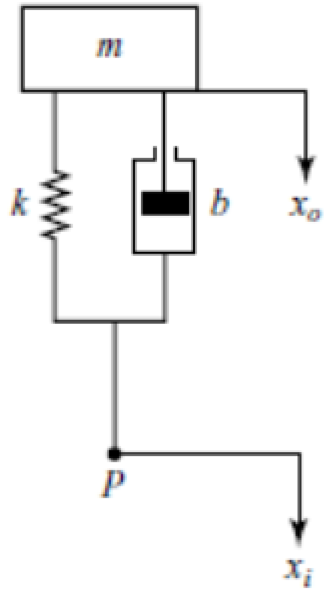
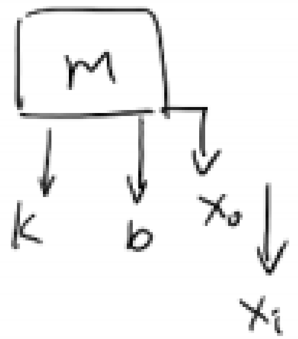


Figure 1

Q8 solution



$$m\ddot{x}_o = k(x_i - x_o) + b(\dot{x}_i - \dot{x}_o)$$

$$m\ddot{x}_o + b\dot{x}_o + kx_o = b\dot{x}_i + kx_i$$

Taking Laplace Transform, we have

$$ms^2X_o(s) + bsX_o(s) + kX_o(s) = bsX_i(s) + kX_i(s)$$

$$\therefore \frac{X_o(s)}{X_i(s)} = \frac{bs + k}{ms^2 + bs + k}$$

Q9 question

Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical systems shown in Figures 3 and 4 below **using impedance method.**

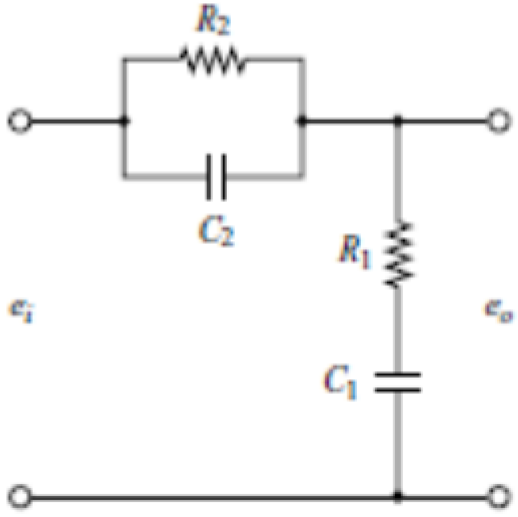


Figure 3

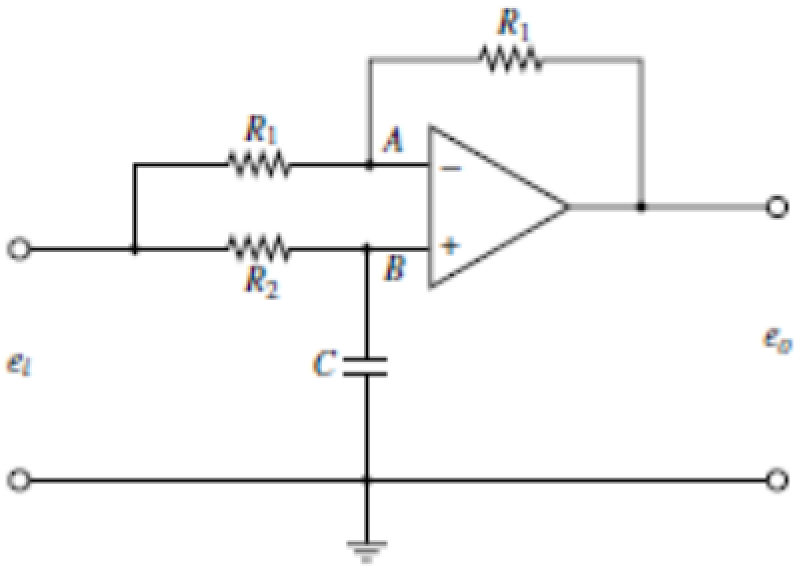


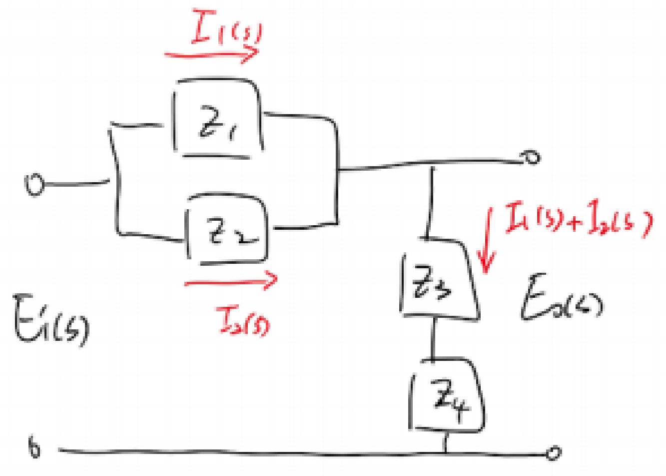
Figure 4

Ans: $$\frac{E_o(s)}{E_i(s)} = \frac{(sR_1C_1 + 1)(sR_2C_2 + 1)}{sR_2C_1 + (sR_1C_1 + 1)(sR_2C_2 + 1)}$$

Ans: $$\frac{E_o(s)}{E_i(s)} = \frac{1 - sR_2C}{sR_2C + 1}$$

End of Tutorial Questions (Part 1)

Q9a solution



The impedance after taking Laplace transform, we have

$$Z_1 = R_2 \quad Z_2 = \frac{1}{sC_2} \quad Z_3 = R_1 \quad Z_4 = \frac{1}{sC_1}$$

$$E_o(s) = \big(I_1(s) + I_2(s)\big)(Z_3 + Z_4)$$

$$I_1(s) = \frac{E_i(s) - E_o(s)}{Z_1}, \quad I_2(s) = \frac{E_i(s) - E_o(s)}{Z_2}$$

$$\therefore E_o(s) = \left(\frac{E_i(s) - E_o(s)}{Z_1} + \frac{E_i(s) - E_o(s)}{Z_2}\right)(Z_3 + Z_4)$$

$$Z_1 Z_2 E_o(s) = [(Z_1 + Z_2)E_i(s) - (Z_1 + Z_2)E_o(s)](Z_3 + Z_4)$$

$$Z_1 Z_2 E_o(s) + (Z_1 + Z_2)(Z_3 + Z_4)E_o(s) = (Z_1 + Z_2)(Z_3 + Z_4)E_i(s)$$

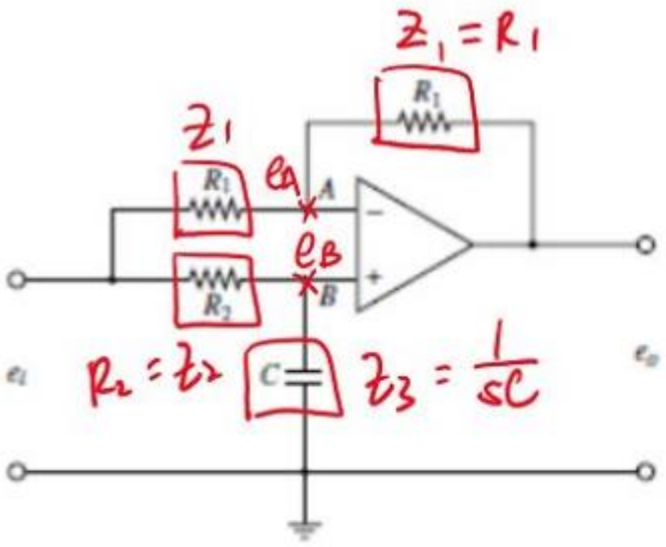
$$\frac{E_o(s)}{E_i(s)} = \frac{(Z_1 + Z_2)(Z_3 + Z_4)}{(Z_1 + Z_2)(Z_3 + Z_4) + Z_1 Z_2}$$

Substituting the corresponding impedances into the equation, we have

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(R_2 + \frac{1}{sC_2}\right)\left(R_1 + \frac{1}{sC_1}\right)}{\left(R_2 + \frac{1}{sC_2}\right)\left(R_1 + \frac{1}{sC_1}\right) + R_2\left(\frac{1}{sC_2}\right)}$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{(sR_1C_1 + 1)(sR_2C_2 + 1)}{sR_2C_1 + (sR_1C_1 + 1)(sR_2C_2 + 1)}$$

Q9b solution



From the above circuit, we have the following equations,

$$e_B = \frac{Z_3}{Z_2 + Z_3} E_i(s) = \frac{\frac{1}{sC}}{R_2 + \frac{1}{sC}} E_i(s) = \frac{1}{R_2Cs + 1} E_i(s)$$

$$\frac{E_i(s) - e_A}{R_1} = \frac{e_A - E_o(s)}{R_1} \Rightarrow e_A = \frac{1}{2}[E_i(s) + E_o(s]$$

Since $e_A \approx e_B$, we have

$$\frac{1}{R_2Cs + 1} E_i(s) = \frac{1}{2}[E_i(s) + E_o(s]$$

$$\frac{1}{R_2Cs + 1} E_i(s) - \frac{1}{2} E_i(s) = \frac{1}{2} E_o(s)$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{1 - sR_2C}{sR_2C + 1}$$

End of Tutorial Questions (Part 1) Solution