SEHS4653 Useful Formulae

	Time Function $f(t)$	Laplace Transform $F(s)$
1	Unit-impulse function $\delta(t)$	1
2	Unit-step function $u_s(t)$	$\frac{1}{s}$
3	Unit-ramp function t	$\frac{1}{s^2}$
4	t^n $(n =$ positive integer$)$	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{s+a}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}$ $(n =$ positive integer$)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
10	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
11	$\frac{1}{b-a}(e^{-at}-e^{-bt})$ $(a \neq b)$	$\frac{1}{(s+a)(s+b)}$
12	$\frac{1}{b-a}(be^{-bt}-ae^{-at})$ $(a \neq b)$	$\frac{s}{(s+a)(s+b)}$
13	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
14	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
15	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
16	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
17	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

18	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ where $\phi=\cos^{-1}\zeta$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
19	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ where $\phi=\cos^{-1}\zeta$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
20	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
21	$\frac{d}{dt}f(t)$	$sF(s)-f(0)$
22	$\frac{d^2}{dt^2}f(t)$	$s^2F(s)-sf(0)-f'(0)$
23	$\frac{d^n}{dt^n}f(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots$ $-sf^{(n-2)}(0)$ $-f^{(n-1)}(0)$
24	$\int f(t)dt$	$\frac{F(s)}{s}+\frac{1}{s}\left[\int f(t)dt\right]$
25	$f(t-T)$	$e^{-Ts}F(s)$
26	$f(\infty)=\lim_{t\to\infty}f(t)$	$=\lim_{s\to 0}sF(s)$
27	$f(0^+)=\lim_{t\to 0^+}f(t)$	$=\lim_{s\to\infty}sF(s)$

Rise Time: $t_r = \frac{\pi - \beta}{\omega_d}$

Peak Time: $t_p = \frac{\pi}{\omega_d}$

Maximum Overshoot: $M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

Settling Time (2%): $t_s = \frac{4}{\zeta\omega_n}$

Settling Time (5%): $t_s = \frac{3}{\zeta\omega_n}$

Damped Natural Frequency: $\omega_d = \omega_n\sqrt{1-\zeta^2}$

<u>Series-lead Compensator:</u>

Transfer Function:

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Maximum Phase Lead: $\sin\phi_m = \frac{1-\alpha}{1+\alpha}$

New Gain Crossover Frequency and its Corresponding Gain: $\omega_m = \frac{1}{\sqrt{\alpha}T}$ and $|G_1(j\omega)| = -20\log\frac{1}{\sqrt{\alpha}}$

<u>Series-lag Compensator:</u>

Transfer Function:

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

At New Gain Crossover Frequency: $|G_c(j\omega)| = 20\log\beta$