THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

School of Professional Education and Executive Development
專業進修學院
SPEED

SEHS4653
Control System Analysis

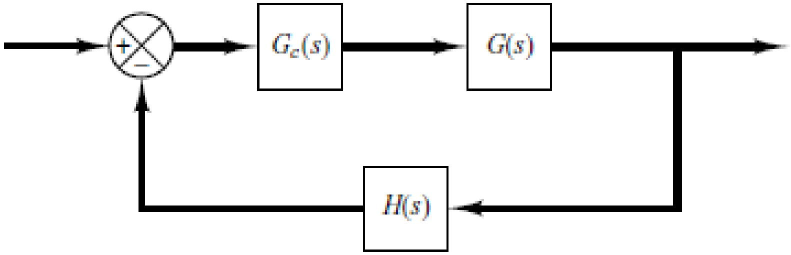
Unit 6
Control System Method
(Reference: [1] chapter 6.6-6.7, 7.10-7.12, 8.2)

Content

- Compensators Design (root-locus and frequency response)
  - Series Lead Compensator
  - Series Lag Compensator
- PID Controllers
  - Ziegler-Nichols Rules for Tuning PID Controllers
    - First and Second Method

Compensators Design

Series Compensation (Unit 5, p.43)



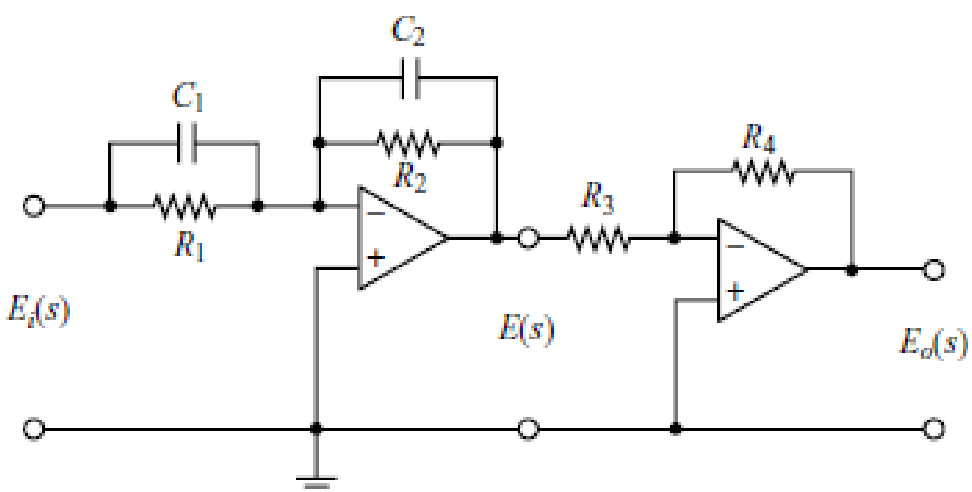
- If a sinusoidal input is applied to the input of a network, and the steady-state output (which is also sinusoidal) has a phase lead, then the network is called a lead network
- If the steady-state output has a phase lag, then the network is called a lag network
- In a lag–lead network, both phase lag and phase lead occur in the output but in different frequency regions
- A compensator having a characteristic of a lead network, lag network, or lag–lead network is called a lead compensator, lag compensator, or lag–lead compensator

Compensators Design

Lead and Lag Compensators

- There are many ways to realize lead compensators and lag compensators, such as electronic networks using operational amplifiers, electrical RC networks, and mechanical spring-dashpot systems

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2R_4}{R_1R_3}\frac{R_1C_1s+1}{R_2C_2s+1} = \frac{R_4C_1}{R_3C_2}\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}} = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$



$$\therefore \alpha = \frac{R_2C_2}{R_1C_1}$$

If $R_1C_1 > R_2C_2$ $(0 < \alpha < 1)$

It is a lead-network

If $R_1C_1 < R_2C_2$

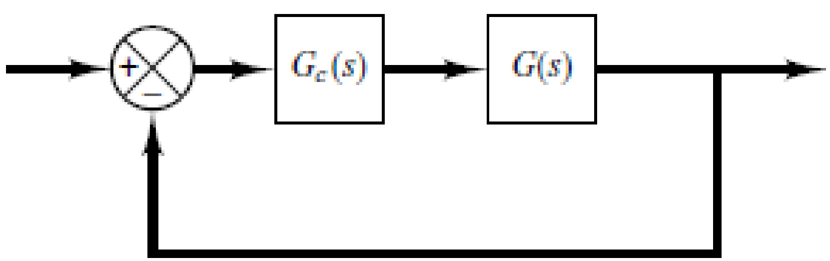
It is a lag-network

Series Lead Compensator Design by Root Locus

- When the specifications are given in terms of time-domain quantities, such as the damping ratio and undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time, and settling time

- Consider a design problem in which the original system either is unstable for all values of gain or is stable but has undesirable transient-response characteristics

- In such a case, the reshaping of the root locus is necessary in the broad neighborhood of the $j\omega$ axis and the origin in order that the dominant closed-loop poles be at desired locations in the complex plane



Series Lead Compensator Design by Root Locus

Procedures for designing a series lead compensator by the root locus approach

1. From the performance specifications, determine the desired location for the dominant closed-loop poles
2. Drawing the root-locus plot of the uncompensated system (original system), calculate the angle deficiency ϕ contributed by the lead compensator
3. Assume the lead compensator $G_c(s)$ to be

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \qquad (0 < \alpha < 1)$$

   where α and T are determined from the angle deficiency. K_c is determined from the requirement of the open-loop grain
4. If static error constants are not specified, determine the location of the pole and zero of the lead compensator so that the lead compensator will contribute the necessary angle ϕ
5. Determine the value of K_c of the lead compensator from the magnitude condition

Example 1

Given the feedforward transfer function

$$G(s) = \frac{10}{s(s+1)}.$$

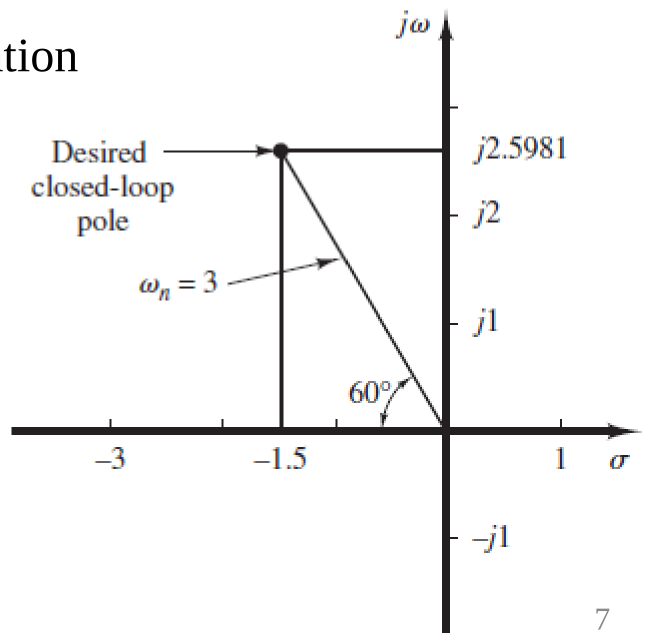
Design a series lead compensator so that the dominant closed-loop poles have the damping ratio, $\zeta = 0.5$, and the undamped natural frequency, $\omega_n = 3$ rad/s.

<u>Answer:</u>

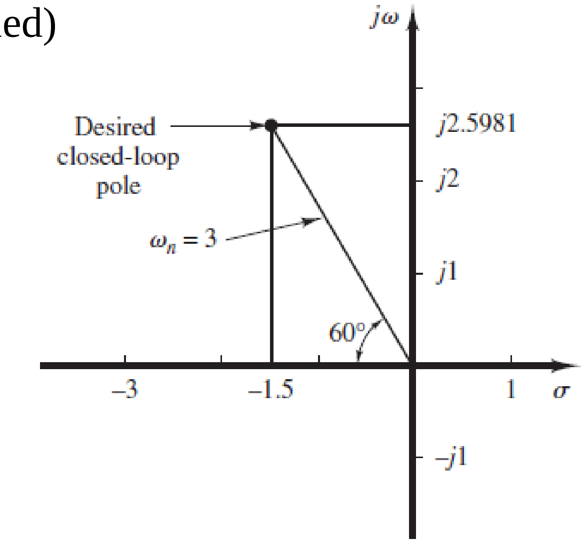
1. Determine the desired closed-loop dominant poles location

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 3s + 9$$

$$s = -1.5 \pm j2.5981$$



Example 1 (continued)



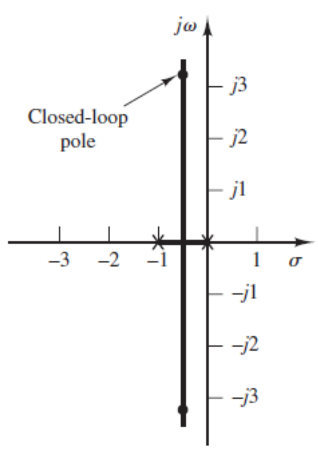
Answer:

2. The root locus of the uncompensated system and find the angle of deficiency

$$\frac{C(s)}{R(s)} = \frac{\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}} = \frac{10}{s^2+s+10}$$

Closed-loop poles: $s = -0.5 \pm j3.1225$

Open-loop poles: $s = 0, -1$



Angle from $s = 0$ to desired pole location:

$$\theta_1 = 180° - \tan^{-1}\frac{2.5981}{1.5} = 120°$$

Angle from $s = -1$ to desired pole location:

$$\theta_2 = 180° - \tan^{-1}\frac{2.5981}{1.5-1} = 100.89°$$

Angel of deficiency (ϕ) = $180° - 120° - 100.89° = -40.894°$

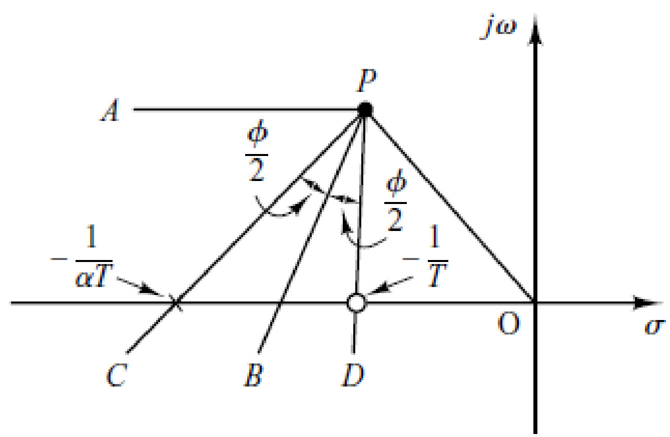
Example 1 (continued)

Answer:

3. Determine the pole and zero of the compensator
   a) Draw a horizontal line (*PA*) passing through point *P*, the desired location for one of the dominant closed-loop poles
   b) Draw a line connecting point *P* and the origin. Bisect the angle $\angle APO$ with a line *PB*
   c) Draw two lines *PC* and *PD* that make angles with the bisector *PB*
   d) The intersections of *PC* and *PD* with the negative real axis give the necessary locations for the pole and zero of the lead network

$$G_c(s) = K_c\alpha \frac{Ts+1}{\alpha Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

$$(0 < \alpha < 1)$$



Example 1 (continued)

Answer:

3. Determine the pole and zero of the compensator

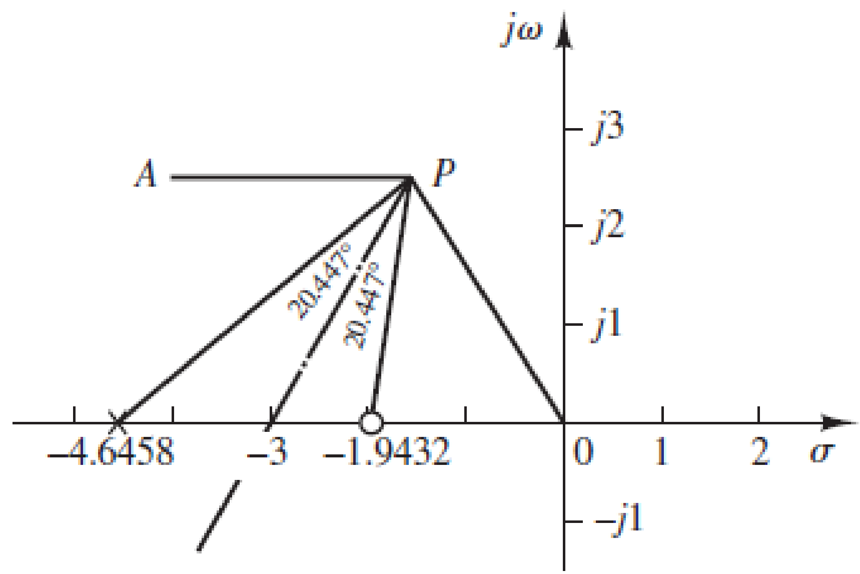
   If we bisect angle $\angle APO$ and take 40.894°/2 each side, then the locations of the zero and pole are found as follows:

   Zero at $s = -1.9432$ and Pole at $s = -4.6458$

   Thus,

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = K_c \frac{s + 1.9432}{s + 4.6458}$$

$$\frac{\frac{1}{T}}{\frac{1}{\alpha T}} = \alpha = \frac{1.9432}{4.6458} = 0.418$$



Example 1 (continued)

Answer:

4. The question did not have static error constants requirement
5. Determine the value of K_c of the lead compensator from the magnitude condition

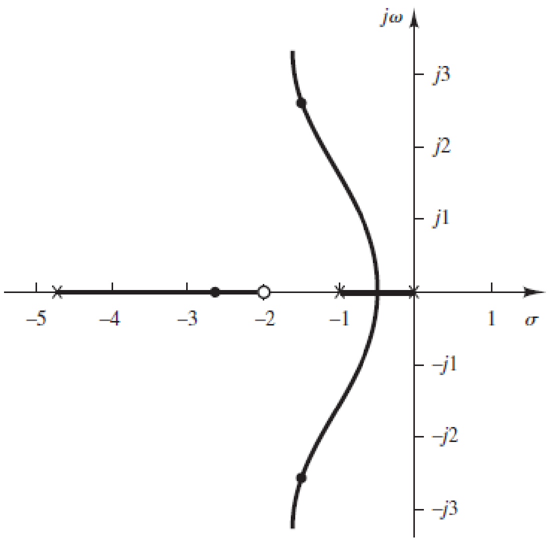
$$|G_c(s)G(s)|_{s=-1.5+j2.891} = 1$$

$$\left| K_c \frac{s+1.9432}{s+4.6458} \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5891} = 1$$

$$K_c = \left| \frac{(s+4.6458)s(s+1)}{10(s+1.9432)} \right|_{s=-1.5+j2.5891} = 1.224$$

Hence, the series lead compensator is given by,

$$G_c(s) = 1.224 \frac{s+1.9432}{s+4.6458}$$

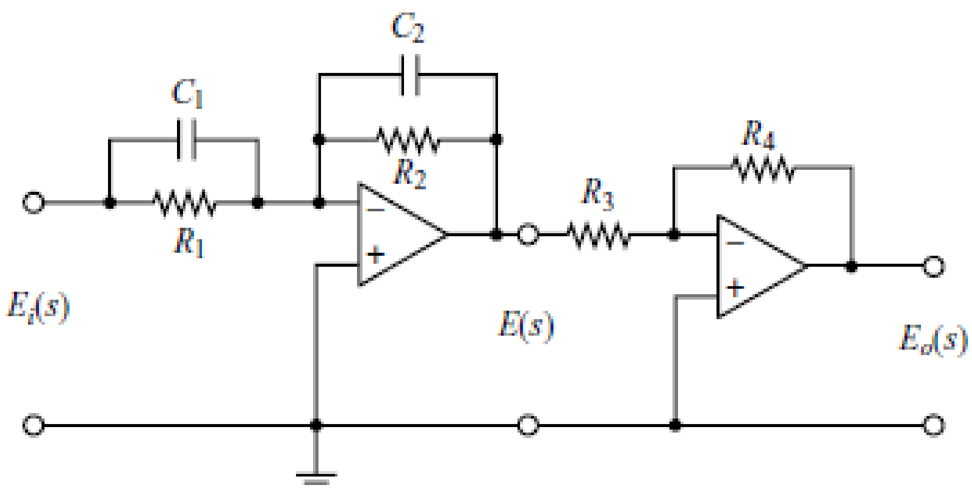


Compensators Design

Lead and Lag Compensators

- The configuration of the electronic lag compensator using operational amplifiers is the same as that for the lead compensator (p. 4)

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2R_4}{R_1R_3}\frac{R_1C_1s+1}{R_2C_2s+1} = \frac{R_4C_1}{R_3C_2}\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}} = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$



If $R_1C_1 < R_2C_2$

It is a lag-network

$$\therefore \beta = \frac{R_2C_2}{R_1C_1} > 1$$

Note that we use β instead of α in the above expressions and we always assume that $0 < \alpha < 1$ and $\beta > 1$.

Series Lag Compensator Design by Root Locus

- The system exhibits satisfactory transient-response characteristics but unsatisfactory steady-state characteristics
- Compensation in this case essentially consists of increasing the open-loop gain without appreciably changing the transient-response characteristics
- This means that the root locus should not be changed appreciably
- This can be accomplished if a lag compensator is put in cascade with the given feedforward transfer function
- To avoid an appreciable change in the root loci, the angle contribution of the lag network should be limited to a small amount, say less than 5°
- Place the pole and zero of the lag network relatively close together and near the origin of the *s*-plane. Hence,

$$|G_c(s)| = \left| K_c \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\beta T}} \right| \approx K_c$$

s_1 is one of the dominant closed-loop poles

Series Lag Compensator Design by Root Locus

Procedures for designing a series lag compensator by the root locus approach

1. From the performance specifications, determine the desired location for the dominant closed-loop poles. Drawing the root-locus plot of the uncompensated system (original system)
2. Assume the lag compensator $G_c(s)$ to be

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

   Then, the open-loop transfer function becomes $G_c(s)G(s)$

3. Evaluate the particular static error constant specified in the problem
4. Determine the amount of increase in the static error constant necessary to satisfy the specifications
5. Determine the pole and zero of the lag compensator that produce the necessary increase in the particular static error constant without appreciably altering the original root loci
6. Draw a new root-locus plot for the compensated system. Locate the desired dominant closed-loop poles on the root locus
7. Adjust gain K_c of the compensator from the magnitude condition so that the dominant closed-loop poles lie at the desired location (K_c will be approximately 1)

Example 2

Given the feedforward transfer function

$$G(s) = \frac{1.06}{s(s+1)(s+2)}.$$

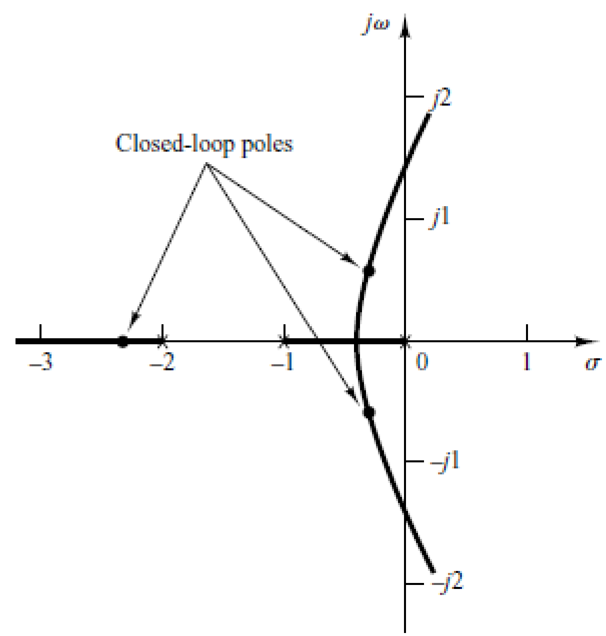
Design a series lag compensator so that the static velocity error constant, K_v, is 5 sec^{-1}.

Answer:

1. Determine the dominant closed-loop poles location
2. Draw the root locus plot

$$\frac{C(s)}{R(s)} = \frac{\frac{1.06}{s(s+1)(s+2)}}{1 + \frac{1.06}{s(s+1)(s+2)}} = \frac{1.06}{s(s+1)(s+2) + 1.06}$$

Dominant closed-loop poles: $s = -0.3307 \pm j0.5864$



Example 2 (continued)

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

Answer:

4. Evaluate the particular static error constant specified in the problem

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{1.06}{s(s+1)(s+2)} = 0.53\ \text{sec}^{-1}$$

5. Determine the amount of increase in the static error constant necessary to satisfy the specifications

$$K_{v,\,new} = 5 = \lim_{s\to 0} sG_cG(s) = \lim_{s\to 0} G_c(s)\lim_{s\to 0} sG(s) = 0.53\lim_{s\to 0} G_c(s) \approx 0.53K_c\beta$$

$$\therefore \beta = 9.4340 \approx 10 \text{ if } K_c \approx 1$$

To increase the static velocity error constant by a factor of about 10, let us choose $\beta = 10$

Example 2 (continued)

$$G_c(s) = K_c\beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

Answer:

6. Determine the pole and zero of the lag compensator

   Select s values for the pole and zero of the lag compensator which results a small amount of phase lag

   Desired closed-loop pole location ($s = -0.3307 + j0.5864$)

   Select *T* = 10

   Zero of the lag compensator is at $s = -0.1$

   Pole of the lag compensator is at $s = -0.01$

   $$\theta_{zero} = 180° - \tan^{-1}\frac{0.5864}{0.3307-0.1} = 111.48°$$

   $$\theta_{pole} = 180° - \tan^{-1}\frac{0.5864}{0.3307-0.01} = 118.67°$$

   Hence, the angle contribution of lag compensator is $\angle G_c(s)|_{s=-0.3307+j0.5864} = -7.19°$

   Select *T* = 20

   Zero of the lag compensator is at $s = -0.05$

   Pole of the lag compensator is at $s = -0.005$

   $$\theta_{zero} = 180° - \tan^{-1}\frac{0.5864}{0.3307-0.05} = 115.58°$$

   $$\theta_{pole} = 180° - \tan^{-1}\frac{0.5864}{0.3307-0.005} = 119.05°$$

   Hence, the angle contribution of lag compensator is $\angle G_c(s)|_{s=-0.3307+j0.5864} = -3.47°$

Example 2 (continued)

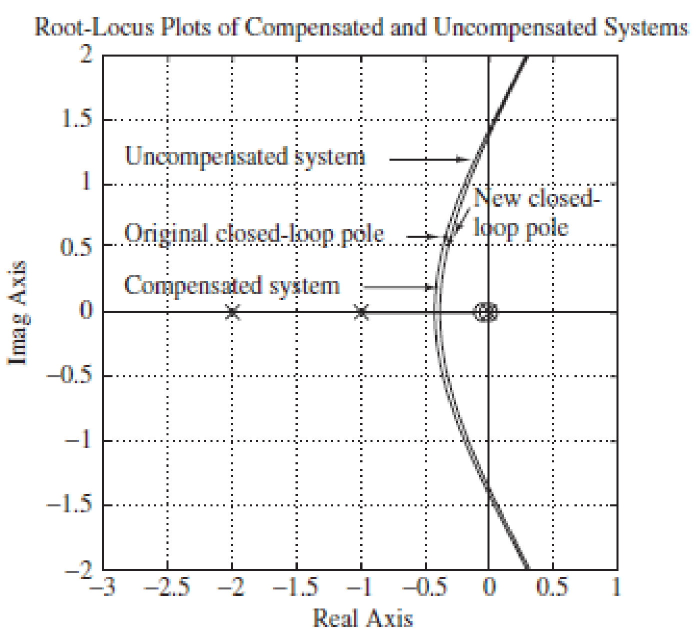
$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

<u>Answer:</u>

Hence, the transfer function of the lag compensator will be given by

$$G_c(s) = \frac{s+0.05}{s+0.005}$$

7. Adjust gain K_c of the compensator from the magnitude condition



If the damping ratio of the new dominant closed-loop poles is kept the same, then these poles are obtained from the new root-locus plot as follows

$$s = -0.31 \pm j0.55$$

$$K_c = \left|\frac{(s+0.005)s(s+1)(s+2)}{1.06(s+0.05)}\right|_{s=-0.31+j0.55} = 0.9656$$

$$G_c(s) = 0.9656\frac{s+0.05}{s+0.005}$$

Compensator Design by Frequency Response

- In the frequency-response approach, the specifications of transient-response performance in an indirect manner, that is, in terms of the phase margin, gain margin, resonant peak magnitude (they give a rough estimate of the system damping); the gain crossover frequency, resonant frequency, bandwidth (they give a rough estimate of the speed of transient response); and static error constants (they give the steady-state accuracy).
- The frequency-response approach can be applied to systems or components whose dynamic characteristics are given in the form of frequency-response data
- Two approaches in the frequency-domain design: Polar Plot approach and Bode Diagram approach

Common Approach

- Adjust the open-loop gain so that the requirement on the steady-state accuracy is met
- Then, the magnitude and phase curves of the uncompensated open loop (with the open-loop gain just adjusted) are plotted
- If the specifications on the phase margin and gain margin are not satisfied, then a suitable compensator that will reshape the open-loop transfer function is determined

Series Lead Compensator Design by Frequency Response

Characteristics of Lead Compensators

- Consider a lead compensator having the following transfer function:

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \qquad (0<\alpha<1)$$

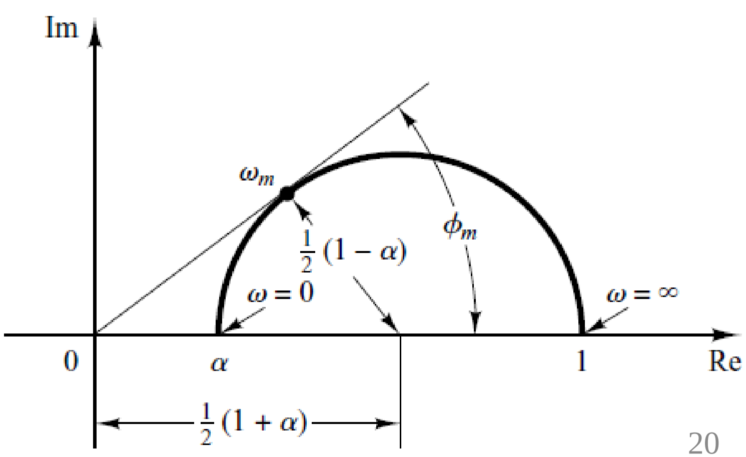
where α is the attenuation factor of the lead compensator

- It has a zero at $s=-1/T$ and a pole at $s=-1/\alpha T$. Since $0<\alpha<1$, we see that the zero is always located to the right of the pole in the complex plane
- Polar plot of the compensator (with K_c =1),

$$G_c(j\omega) = K_c\alpha\frac{j\omega T+1}{j\alpha\omega T+1}$$

Maximum phase-lead angle, ϕ_m

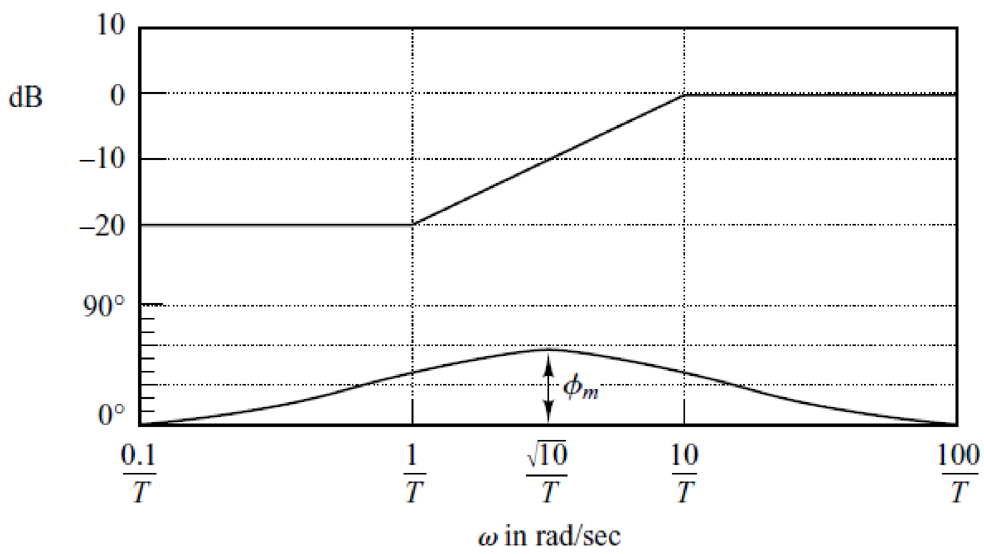
$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$



Series Lead Compensator Design by Frequency Response

Characteristics of Lead Compensators

- Bode diagram of a lead compensator when $K_c = 1$ and $\alpha = 0.1$
- The corner frequencies for the lead compensator are $\omega = 1/T$ and $\omega = 1/\alpha T = 10/T$



$$\log \omega_m = \frac{1}{2}\left(\log\frac{1}{T} + \log\frac{1}{\alpha T}\right)$$

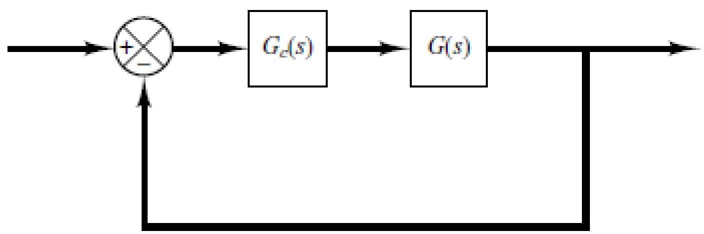
$$\log \omega_m = \log\sqrt{\frac{1}{T}\frac{1}{\alpha T}}$$

$$\therefore \omega_m = \frac{1}{\sqrt{\alpha}T}$$

- Lead compensator is basically a high-pass filter

Series Lead Compensator Design by Frequency Response

Procedure for designing a series lead compensator



1. Assume the following lead compensator,

$$G_c(s) = K_c\alpha \frac{Ts+1}{\alpha Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \qquad (0 < \alpha < 1)$$

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = K\frac{Ts+1}{\alpha Ts+1}G(s) = \frac{Ts+1}{\alpha Ts+1}KG(s) = \frac{Ts+1}{\alpha Ts+1}G_1(s)$$

Determine gain $K(=K_c\alpha)$ to satisfy the requirement of the given static error constant

Series Lead Compensator Design by Frequency Response

Procedure for designing a series lead compensator

2. Using the gain K thus determined, draw a Bode diagram of $G_1(j\omega)$, the gain adjusted but uncompensated system. Evaluate the phase margin.

3. Determine the necessary phase-lead angle to be added to the system. Add an additional 5° to 12° to the phase-lead angle required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin

4. Determine the attenuation factor α by use $\sin\phi_m = \frac{1-\alpha}{1+\alpha}$. Determine the frequency where the magnitude of the uncompensated system $G_1(j\omega)$ is equal to $-20\log\left(\frac{1}{\sqrt{\alpha}}\right)$. Select this frequency as the new gain crossover frequency. This frequency corresponds to $\omega_m = \frac{1}{\sqrt{\alpha}T}$, and the maximum phase shift ϕ_m occurs at this frequency

Series Lead Compensator Design by Frequency Response

Procedure for designing a series lead compensator

5. Determine the corner frequencies of the lead compensator as follows

Zero of lead compensator: $\omega = \dfrac{1}{T}$

Pole of lead compensator: $\omega = \dfrac{1}{\alpha T}$

6. Using the value of K determined in step 1 and that of α determined in step 4, calculate constant K_c $(= K/\alpha)$

7. Check the gain margin to be sure it is satisfactory. If not, repeat the design process by modifying the pole–zero location of the compensator until a satisfactory result is obtained.

Example 3

The open-loop transfer function of a unity feedback system is,

$$G(s) = \frac{4}{s(s+2)}.$$

It is desired to design a series lead compensator for the system so that the static velocity error constant K_v is 20 sec^{-1}, the phase margin is at least 50°, and the gain margin is at least 10 dB.

<u>Answer:</u>

1. The transfer function of a series lead compensator, $G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$

   The compensated system will have the open-loop transfer function $G_c(s)G(s)$,

$$G_c(s)G(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1}\frac{4}{s(s+2)}$$

Define $G_1(s) = KG(s) = \frac{4K}{s(s+2)}$

$$K = K_c\alpha$$

Example 3 (continued)

Answer:

1. Determine gain $K(=K_c\alpha)$ to satisfy the requirement of the given static velocity error constant

$$K_v = \lim_{s\to 0} s\, G_c(s)G(s) = \lim_{s\to 0} s\,\frac{Ts+1}{\alpha Ts+1}\frac{4K}{s(s+2)} = 2K = 20 \Rightarrow K = 10$$

2. Using this gain K, **draw a Bode diagram** of $G_1(j\omega)$, the gain adjusted but uncompensated system.

$$G_1(j\omega) = \frac{4K}{j\omega(j\omega+2)}$$

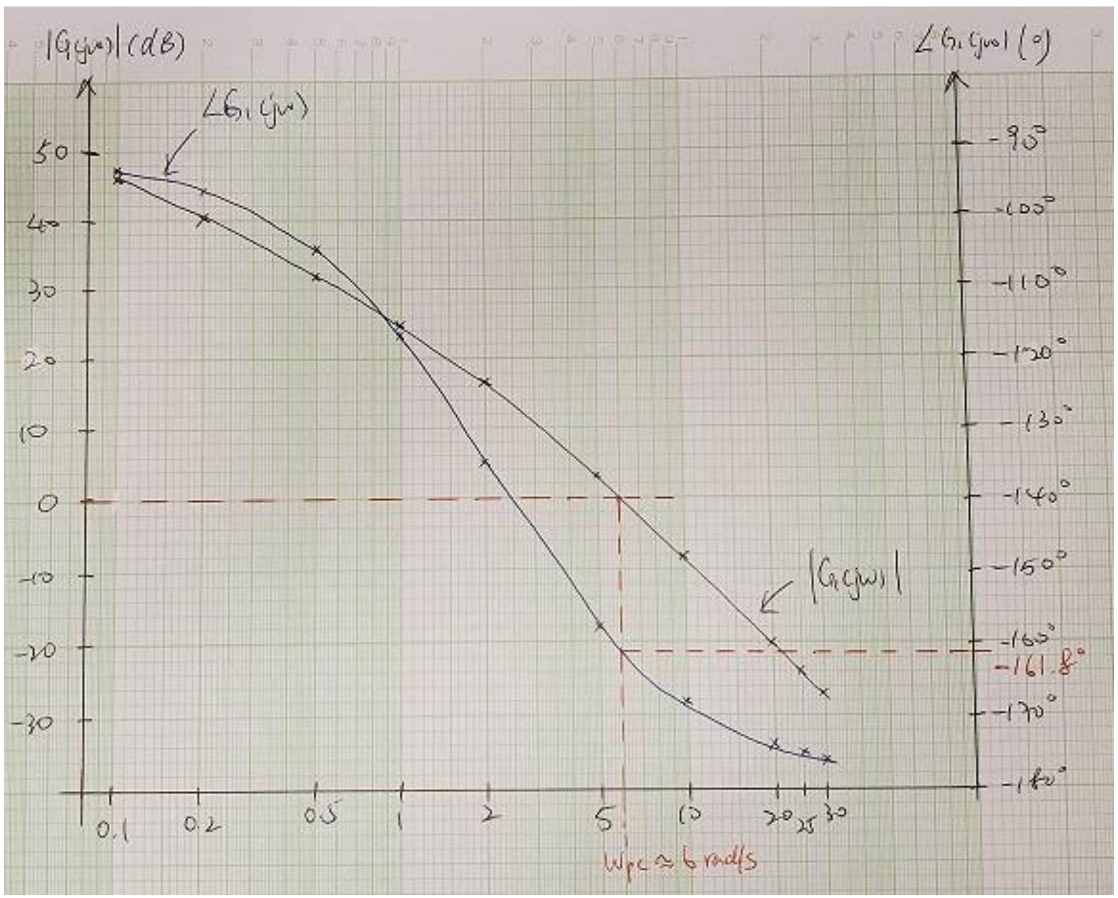
ω (rad/s)	$\lvert G_1\rvert$ (dB)	$\angle G_1$(°)	ω (rad/s)	$\lvert G_1\rvert$ (dB)	$\angle G_1$(°)
0.1	46	-92.86	5	3.44	-158.2
0.2	39.96	-95.71	10	-8.11	-168.69
0.5	31.78	-104.04	20	-20.04	-174.29
1	25.05	-116.57	25	-23.90	-175.43
2	16.99	-135	30	-27.06	-176.19

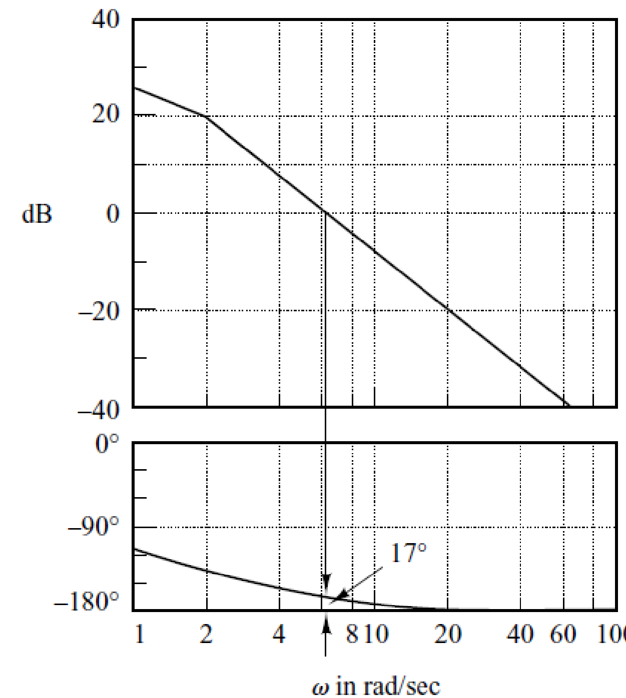
Example 3 (continued)

<u>Answer:</u>

2. Evaluate the phase margin.

From the Bode plot, the phase and gain margins of the system are found to be 18.2° and $+\infty$ dB, respectively





Example 3 (continued)

Answer:

3. Determine the necessary phase-lead angle to be added to the system

   From the question, it requires a phase margin of at least 50°

   Thus, the additional phase lead necessary to satisfy the relative stability requirement is 31.8° (50° – 18.2°) without decreasing the value of *K*, the lead compensator must contribute the required phase angle

4. Determine the attenuation factor α and the new gain crossover frequency

   The maximum phase lead required is then $\phi_{\phi m} = 31.8° + 5° = 36.8°$

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \Rightarrow \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 36.8°}{1+\sin 36.8°} = 0.251$$

Example 3 (continued)

Answer:

4. Determine the attenuation factor α and the new gain crossover frequency

The new gain crossover frequency, $\omega_m = \dfrac{1}{\sqrt{\alpha}T}$

The amount of modification in the magnitude curve at $\omega = \frac{1}{\sqrt{\alpha}T}$ due to the inclusion of $G_c(j\omega)$,

$$\left|\frac{1+j\omega T}{1+j\alpha\omega T}\right|_{\omega=\frac{1}{\sqrt{\alpha}T}} = \left|\frac{1+jT\left(\frac{1}{\sqrt{\alpha}T}\right)}{1+jT\alpha\left(\frac{1}{\sqrt{\alpha}T}\right)}\right| = \frac{1}{\sqrt{\alpha}}$$

$$|G_1(j\omega)| = -20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.251}} = -6.00\text{ dB}$$

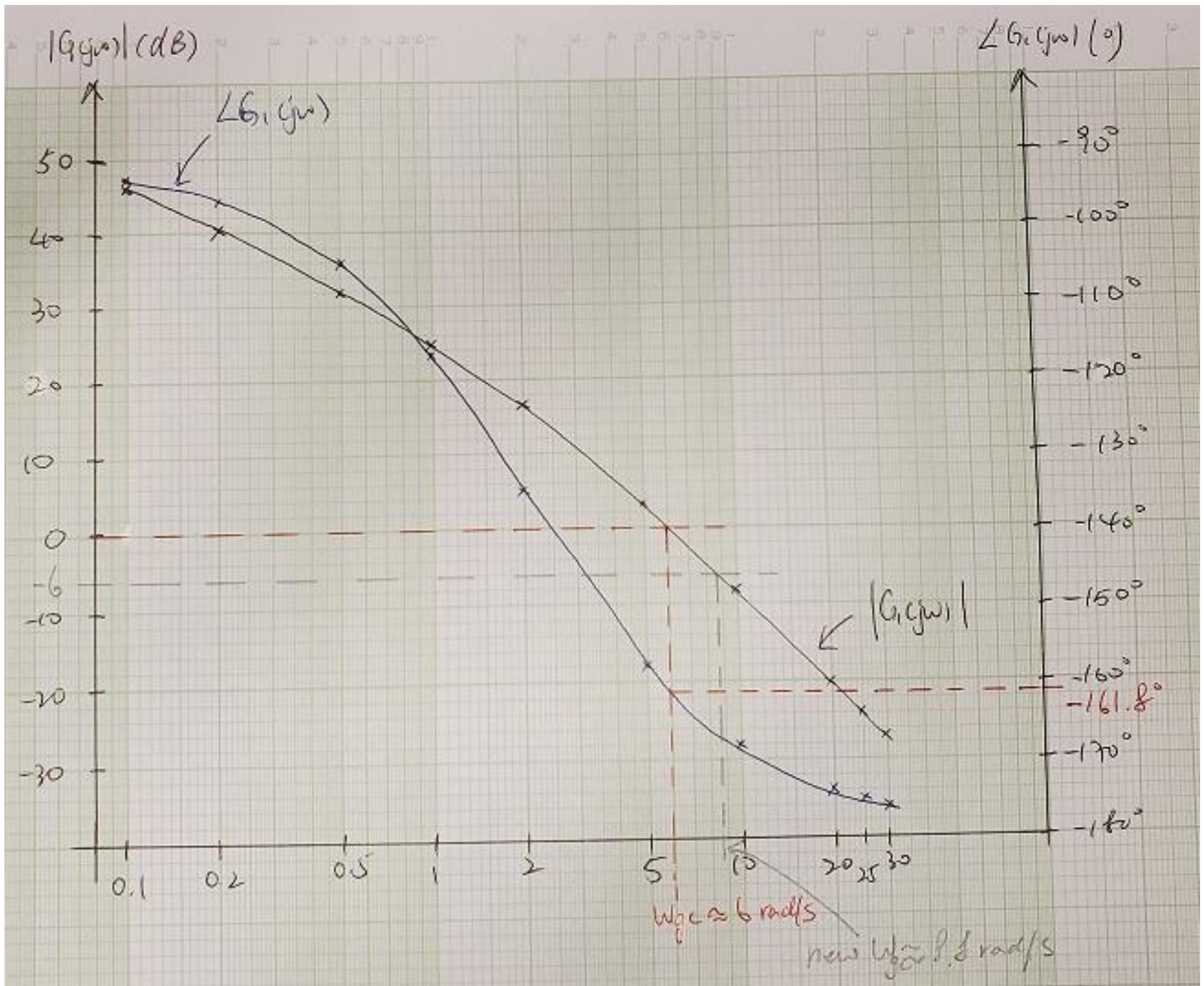
Refer to the Bode diagrams (in Step 2), it corresponds to ω =8.8 rad/s (This is the new gain crossover frequency ω_c)

Example 3 (continued)

Answer:

4. Determine the attenuation factor α and the **new gain crossover frequency**

   Refer to the Bode diagrams (in Step 2), it corresponds to $\omega = 8.8$ rad/s (This is the new gain crossover frequency ω_c)



Example 3 (continued)

Answer:

$\omega_m = 8.8$ rad/s

$\alpha = 0.251$

5. Determine the corner frequencies of the lead compensator

Zero of lead compensator: $\omega_C = \frac{1}{T}$ $\quad \omega_m = \frac{1}{\sqrt{\alpha}T} \Rightarrow \frac{1}{T} = \omega_m\sqrt{\alpha} = 8.8\sqrt{0.251} = 4.41$

Pole of lead compensator: $\omega_C = \frac{1}{\alpha T}$ $\quad \omega_m = \frac{1}{\sqrt{\alpha}T} \Rightarrow \frac{1}{\alpha T} = \frac{\omega_m\sqrt{\alpha}}{\alpha} = \frac{8.8\sqrt{0.251}}{0.251} = 17.56$

The series lead compensator is, $G_c(s) = K_c\frac{s+4.41}{s+17.56}$

With $K = K_c\alpha$, $K_c = \frac{10}{0.251} = 39.84$. Thus, the transfer function of the compensator is,

$$G_c(s) = 39.84\frac{s+4.41}{s+17.56}$$

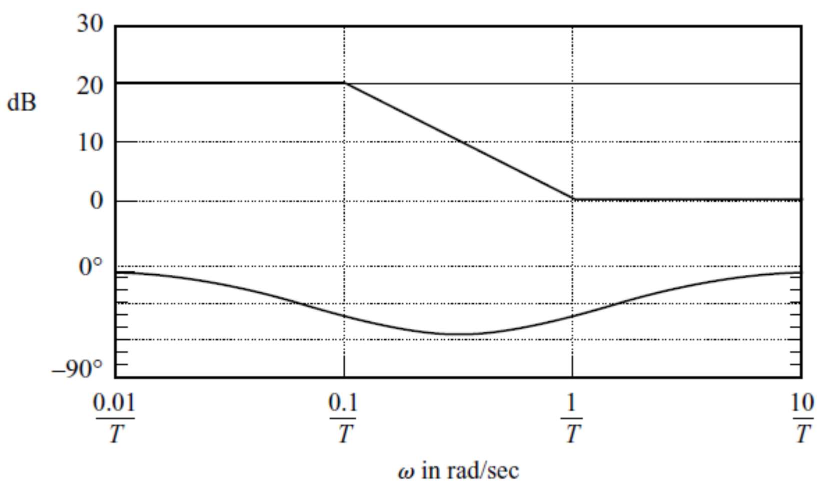
Series Lag Compensator Design by Frequency Response

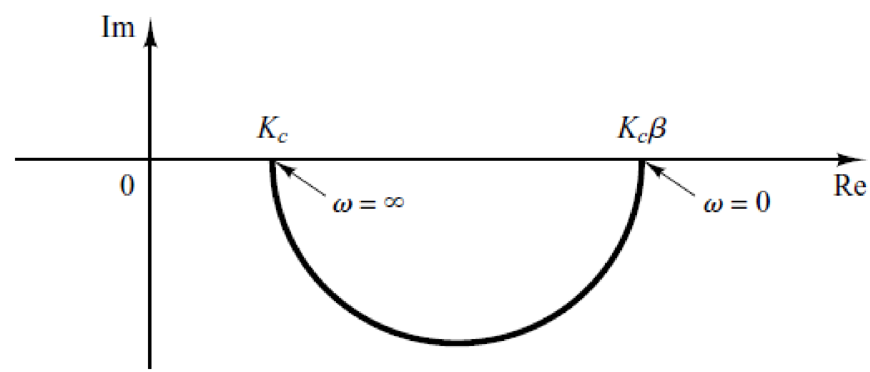
Characteristics of Lag Compensators

- Consider a lag compensator having the following transfer function:

$$G_c(s) = K_c\beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad (\beta > 1)$$

- It has a zero at $s = -1/T$ and a pole at $s = -1/\beta T$.
- Polar plot and Bode diagrams of the compensator (with K_c =1, $\beta = 10$),





- The lag compensator is essentially a low-pass filter

Series Lag Compensator Design by Frequency Response

Procedure for designing a series lag compensator

1. Assume the following lead compensator,

$$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad (\beta > 1)$$

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = K\frac{Ts+1}{\beta Ts+1}G(s) = \frac{Ts+1}{\beta Ts+1}KG(s) = \frac{Ts+1}{\beta Ts+1}G_1(s)$$

Determine gain $K(=K_c\beta)$ to satisfy the requirement of the given static error constant

Series Lag Compensator Design by Frequency Response

Procedure for designing a series lag compensator

2. Using the gain K thus determined, draw a Bode diagram of $G_1(j\omega)$, the gain adjusted but uncompensated system. Evaluate the phase margin.

3. Determine the necessary phase-lag angle to be added to the system. Add an additional 5° to 12° to compensate for the phase lag of the lag compensator. Choose this frequency as the new gain crossover frequency

$$\phi = -180° + \text{phase margin in specs} + (5° \sim 12°)$$

4. To prevent detrimental effects of phase lag due to the lag compensator, the pole and zero of the lag compensator must be located substantially lower than the new gain crossover frequency. Choose the corner frequency $\omega = 1/T$ (corresponding to the zero of the lag compensator) 1 octave to 1 decade below the new gain crossover frequency.

Series Lag Compensator Design by Frequency Response

Procedure for designing a series lead compensator

5. Determine the attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency. Hence $20 \log \beta$, determine the value of β. Then the other corner frequency (corresponding to the pole of the lag compensator) is determined from $1/(\beta T)$.

6. Using the value of K determined in step 1 and that of β determined in step 5, calculate constant K_c $(= K/\beta)$

Example 4

The open-loop transfer function of a unity feedback system is,

$$G(s) = \frac{1}{s(s+1)(0.5s+1)}.$$

It is desired to design a series lag compensator for the system so that the static velocity error constant K_v is 5 sec^{-1}, the phase margin is at least 40°, and the gain margin is at least 10 dB.

<u>Answer:</u>

1. The transfer function of a series lag compensator, $G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$

   The compensated system will have the open-loop transfer function $G_c(s)G(s)$,

$$G_c(s)G(s) = K_c\beta\frac{Ts+1}{\beta Ts+1}\frac{1}{s(s+1)(0.5s+1)} \qquad K = K_c\beta$$

Define $G_1(s) = KG(s) = \dfrac{K}{s(s+1)(0.5s+1)}$

Example 4 (continued)

Answer:

1. Determine gain $K(= K_c\beta)$ to satisfy the requirement of the given static velocity error constant

$$K_v = \lim_{s\to 0} s\, G_c(s)G(s) = \lim_{s\to 0} s\, \frac{Ts+1}{\beta Ts+1}\, \frac{K}{s(s+1)(0.5s+1)} = K = 5$$

2. Using $K = 5$, draw a **Bode diagram** of $G_1(j\omega)$, the gain adjusted but uncompensated system.

$$G_1(j\omega) = \frac{5}{j\omega(j\omega+1)(0.5j\omega+1)}$$

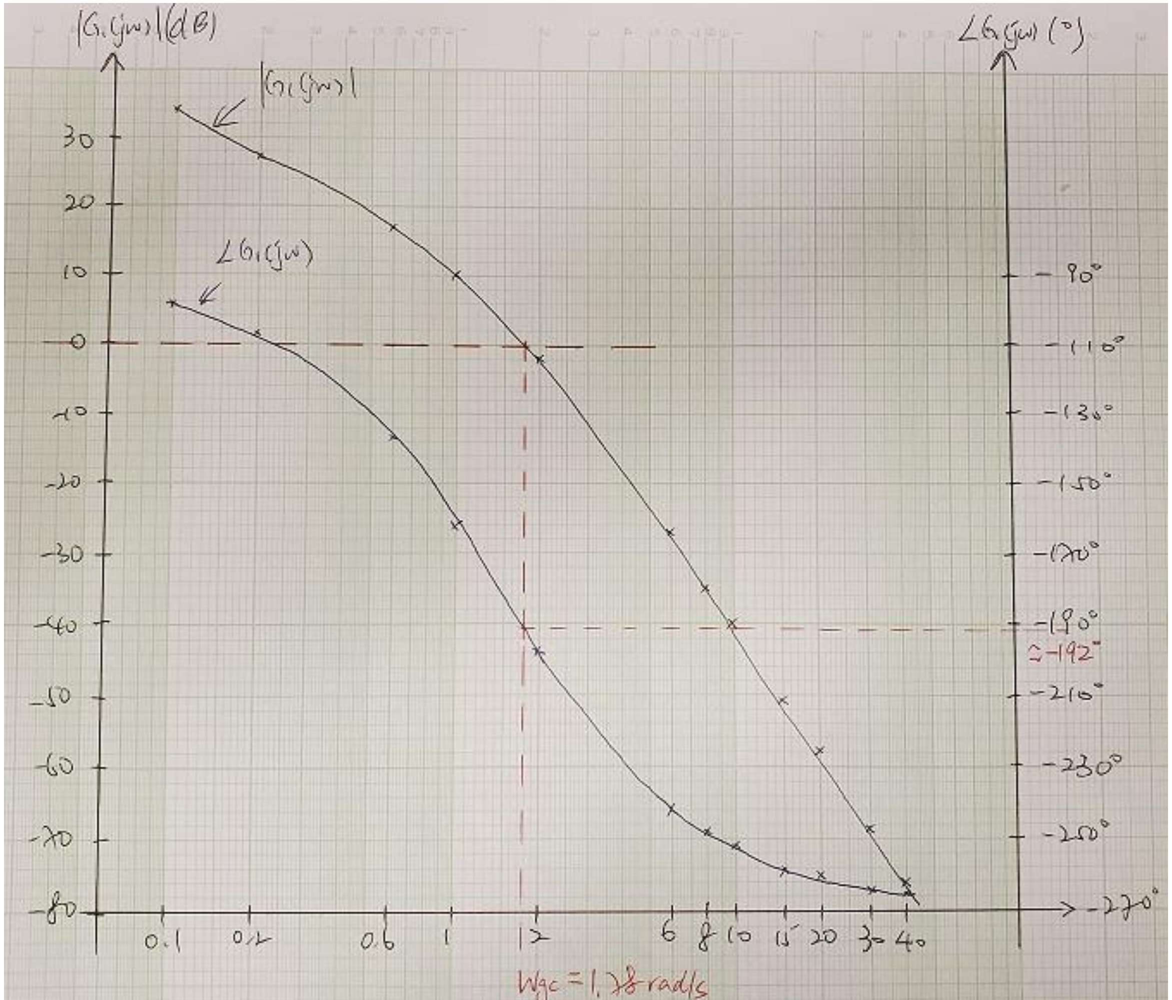
ω (rad/s)	$\lvert G_1 \rvert$ (dB)	$\angle G_1$(°)	ω (rad/s)	$\lvert G_1 \rvert$ (dB)	$\angle G_1$(°)
0.1	33.93	-98.57	8	-34.52	-248.84
0.2	27.75	-107.02	10	-40.21	-252.98
0.6	16.71	-137.06	15	-50.66	-258.59
1	10	-161.57	20	-58.12	-261.43
2	-2.04	-198.43	30	-68.65	-264.28
6	-27.27	-242.10	40	-76.14	-265.71

Example 4 (continued)

Answer:

2. Evaluate the phase margin.

From the Bode plot, the phase margin = –12° which means that the gain-adjusted but uncompensated system is unstable.



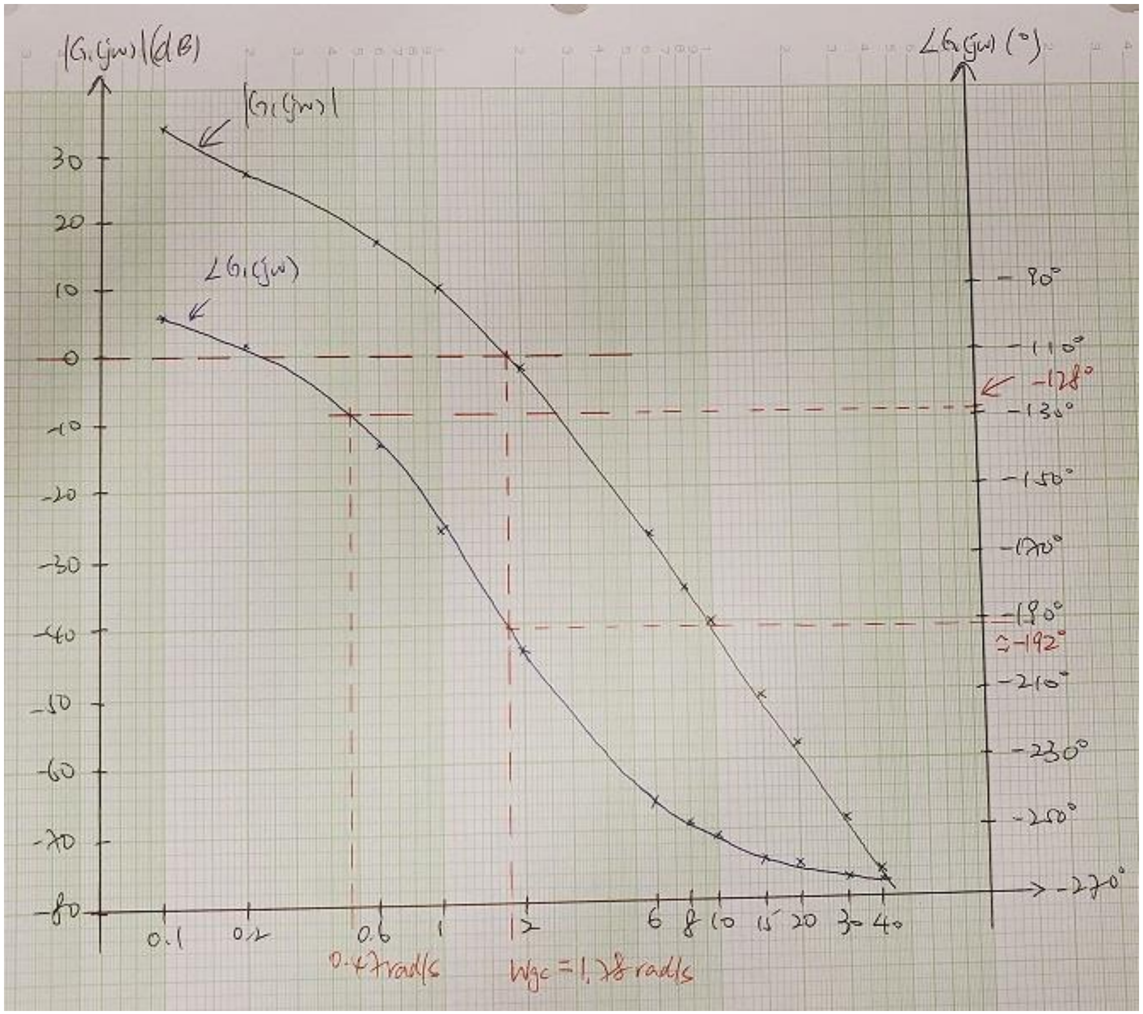
Example 4 (continued)

Answer:

3. Determine the necessary phase-lag angle to be added to the system

$$\phi = -180° + \text{phase margin in specs} + (5° \sim 12°)$$

$$\phi = -180° + 40° + 12° = -128°$$



The new gain crossover frequency (of the compensated system) = 0.47 rad/s

Example 4 (continued)

Answer:

4. Locate the pole and zero of the lag compensator

   Generally chosen the frequency that is one-tenth of the new gain crossover frequency as the location of zero, $\omega = 1/T$, of the lag compensator

   Hence, we have the zero of the lag compensator = 0.047 rad/s.

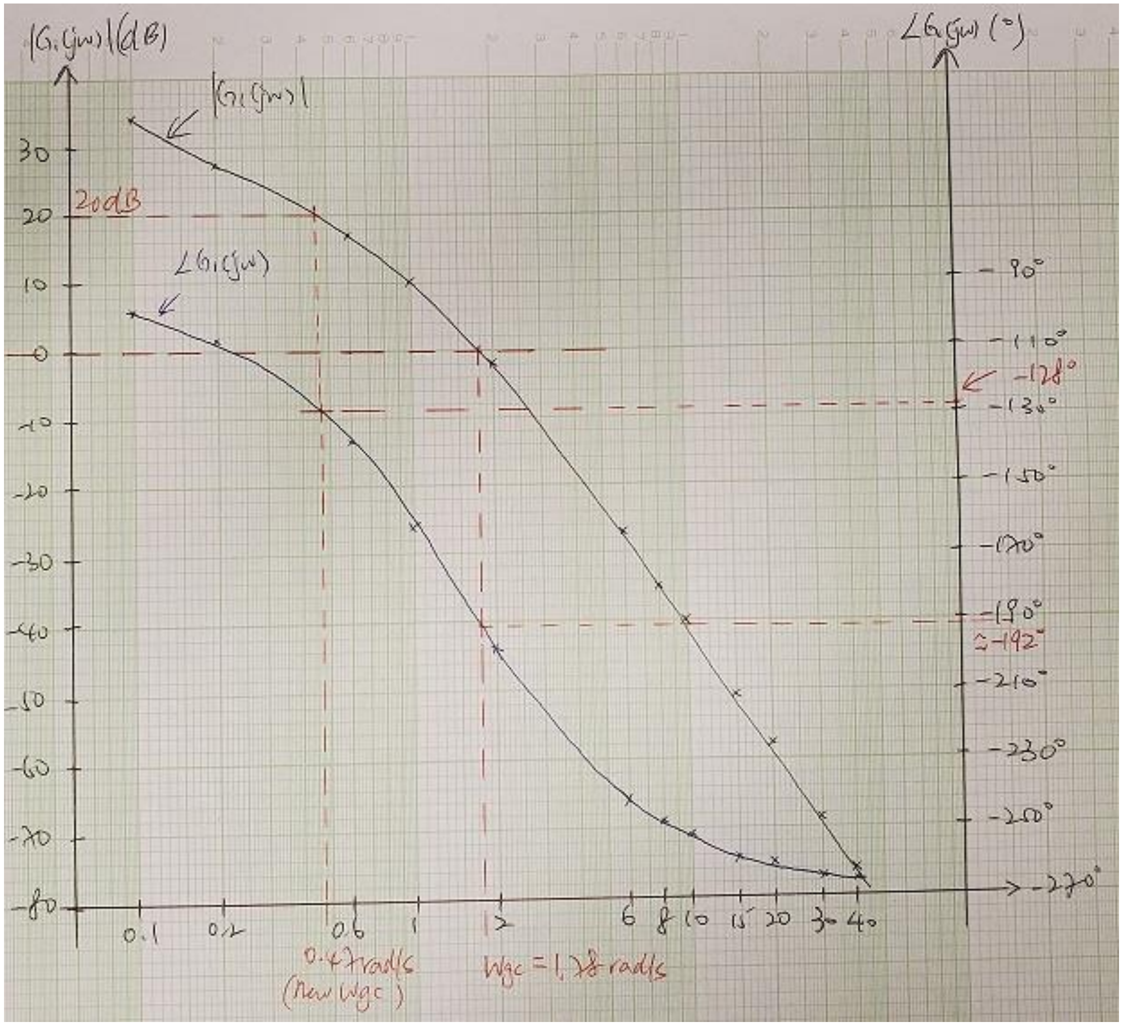
5. Determine the attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency (from the Bode diagram)

$$|G_c(j\omega)| = 20\log\beta = \mathbf{20} \Rightarrow \beta = 10$$

Example 4 (continued)

<u>Answer:</u>

5. Determine the attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency (from the Bode diagram)



$$|G_c(j\omega)| = 20\log\beta = \mathbf{20}$$
$$\Rightarrow \beta = 10$$

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Example 4 (continued)

Answer:

6. The other corner frequency is, $\omega = \frac{1}{\beta T} = \frac{0.047}{10} = 0.0047 \text{ rad/s}$

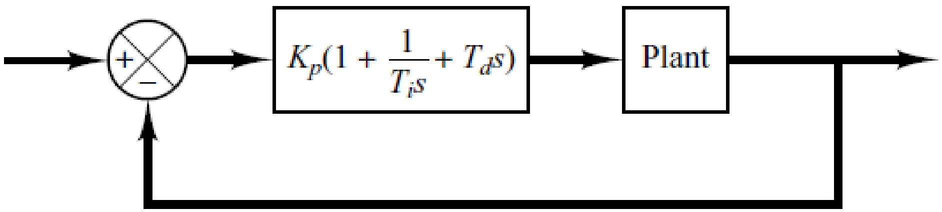
The series lag compensator is,

With $K = K_c\beta$, $K_c = \frac{5}{10} = 0.5$. Thus, the transfer function of the compensator is

$$G_c(s) = 0.5\frac{s + 0.047}{s + 0.0047}$$

PID Controllers

- PID control of a plant



- If a mathematical model of the plant can be derived, we can apply various design techniques for determining parameters of the controller that will meet the transient and steady-state specifications of the closed-loop system
- If a mathematical model of a plant is not obtained, then an analytical or computational approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to the tuning of PID controllers
- The process of selecting the controller parameters to meet given performance specifications is known as controller tuning
- Ziegler and Nichols suggested rules for tuning PID controllers (based on experimental step responses or based on the value of K_p that results in marginal stability when only proportional control action is used

PID Controllers

- However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable.
- Hence, we need series of fine tunings until an acceptable result is obtained
- In fact, the Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning, rather than giving the final settings for and in a single shot

Ziegler–Nichols Rules for Tuning PID Controllers

- Based on the transient response characteristics of a given plant
- Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant
- Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal

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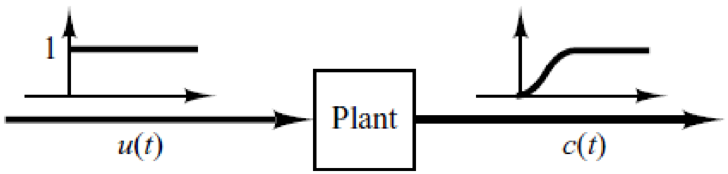
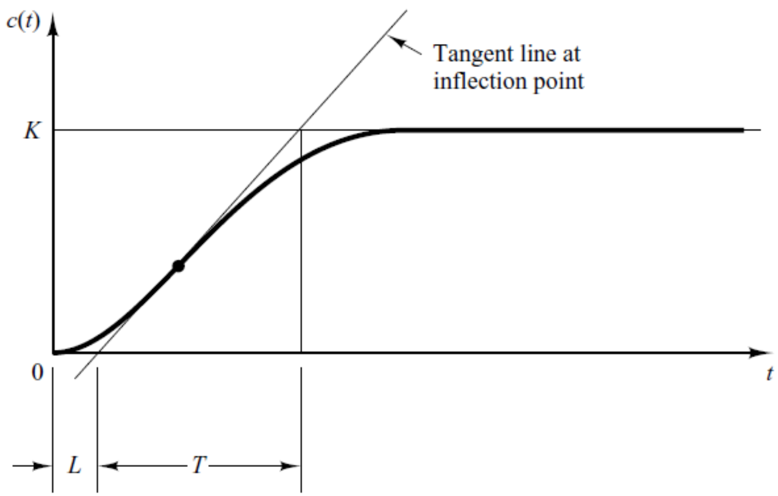
Tuning Rules for PID Controllers

First Method

- Obtain experimentally the response of the plant to a unit-step input
- The delay time (L) and time constant (T) are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$





- The transfer function $C(s)$ / $U(s)$ may then be approximated by a first-order system with time lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1}$$

Tuning Rules for PID Controllers

First Method

- Ziegler–Nichols Tuning Rule Based on Step Response of Plant

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

- Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

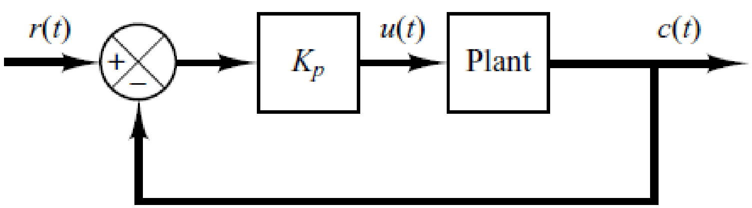
$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) = 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

Tuning Rules for PID Controllers

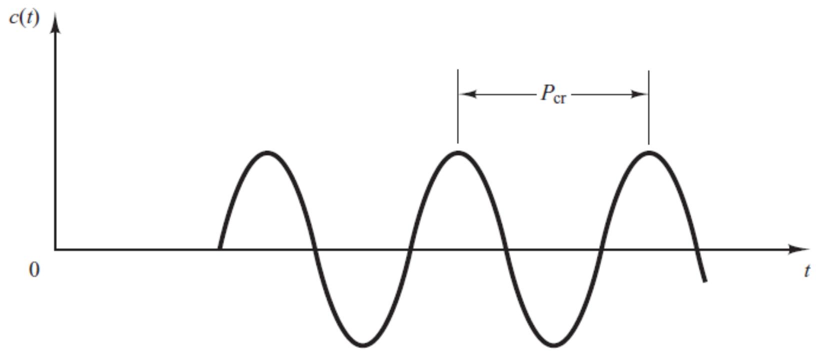
Second Method

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

- First set $T_i = \infty$ and $T_d = 0$
- Using the proportional control action only, increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations



- Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined



Tuning Rules for PID Controllers

Second Method

- Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr}

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

- Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$

$$= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

Tuning Rules for PID Controllers

Second Method

- If the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillations P_{cr}, where $2\pi/\omega_{cr} = P_{cr}$
- How to obtain such information from root-locus plot?

Comments

- Ziegler–Nichols tuning rules have been widely used to tune PID controllers in process control systems where the plant dynamics are not precisely known
- Over many years, such tuning rules proved to be very useful
- Ziegler–Nichols tuning rules can be applied to plants whose dynamics are known as well
- If the plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler–Nichols tuning rules