SEHS4653 Control System Analysis
Tutorial Questions (Part 4)

Polar (Nyquist) Plot

1. Sketch the Nyquist plot for the system,

$$G(s) = \frac{1}{s(s+1)}.$$

2. Given the open-loop transfer function of a system,

$$G(s)H(s) = \frac{K}{s(s+1)(2s+1)}.$$

(a) Sketch the Nyquist plot of the above system with $K = 2$.
(b) Use Nyquist stability criterion to determine the absolute stability of the closed-loop system.
(c) Find the critical value of gain K for stability.
(Ans: (b) unstable; (c) $0 < K < 3/2$)

Nichols Chart

3. Open-loop frequency response tests on a control system with unity gain negative feedback yield the following data.

Frequency ω (rad/s)	Gain (Output / Input)	Phase (degree)
0.4	2.452	–101.31
0.8	1.161	–111.80
1.2	0.715	–120.96
1.4	0.585	–124.99
1.8	0.413	–131.99
2.0	0.354	–135.00
4.0	0.112	–153.43
8.0	0.030	–165.96

A first-order lag with a time constant of 1 sec, $G_1(s) = \frac{1}{s+1}$, is now inserted in the forward path of the control loop. Use a Nichols chart to determine for the modified system

(a) the gain margin;
(b) the phase margin.
(Ans: (a) 9.4 dB; (b) 33°)

4. A closed-loop control system consists of three elements A, B and C in series in the forward path and a unity gain feedback loop.

   Component A is an amplifier with gain *G*.

   Component B has the transfer function $\frac{1}{1+0.2s}$.

   The transfer function of component C is not known, but frequency response tests on this component give the following results.

Frequency ω (rad/s)	Gain (Output / Input)	Phase (degree)
5.0	0.894	–63.26
7.0	0.714	–89.12
10.0	0.447	–116.34
10.8	0.394	–121.42
20.0	0.124	–152.39

   (a) Show that if $G > 5$, the system will be unstable.
   (b) If the gain G is fixed at 6, the system can be stabilized by putting an additional component with transfer function, $G_p(s) = 1 + Ks$, in series with A, B and C. Show that neutral stability now occurs when $K = 0.0111$.

<u>Compensators / Controllers Design</u>

5. An open-loop frequency response test on a unit feedback control system produced the data below.

Frequency ω (rad/s)	Gain (dB)	Phase (degree)
0.3	20	–19
1	18	–51
3	10.5	–91
6	5	–116
10	–1	–135
20	–12	–163
30	–19	–177
60	–31.5	–201
100	–40	–218

   (a) A phase-lead series compensating network, having the transfer function, $G_c(s) = \frac{0.4(1+0.08s)}{1+0.032s}$, is then incorporated to improve the system performance. Plot the gain and phase characteristics on a Nichols' chart.

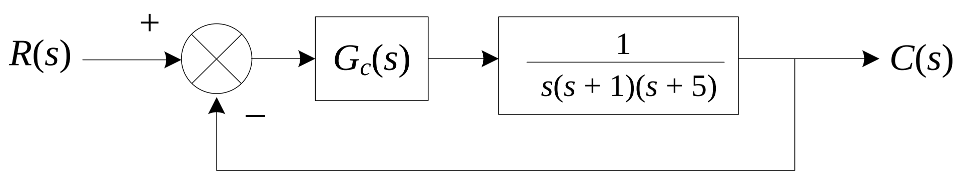
   (b) If the system gain is then increased by 16 dB, determine the characteristics of the compensated

system the resulting
(i) gain margin;
(ii) and phase margin.
(Ans: (b)(i) 14.6 dB, (b)(ii) 43°)

6. Consider the control system shown below in which a PID controller is used to control the system. The PID controller has the transfer function,

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right).$$

Although many analytical methods are available for the design of a PID controller for the present system, lets us apply Ziegler-Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d.
(Ans: K_p =18, T_i =1.405, T_d =0.351)



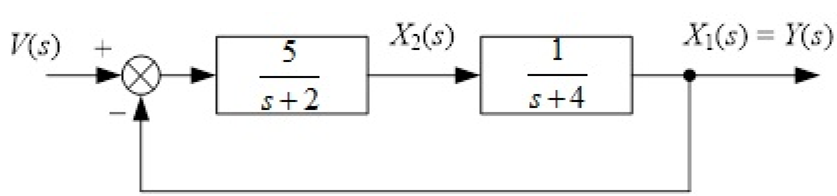
State Space Analysis

7. Consider a system defined by the following state-space equations:

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-5 & -1\\3 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}2\\5\end{bmatrix}u \text{ and } y = \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

Obtain the transfer function $G(s)$ of the system.
(Ans: $G(s) = \frac{12s+59}{s^2+6s+8}$)

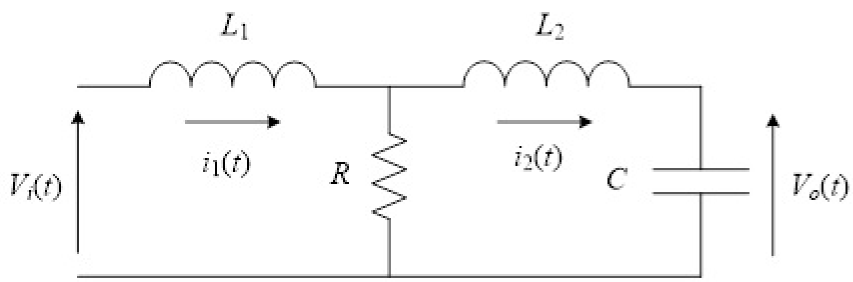
8. For the system shown below, find the state-space equations and calculate the state transition matrix in time domain.



(Ans: $\begin{bmatrix}\dot{x}_1(t)\\\dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}-4 & 1\\-5 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix} + \begin{bmatrix}0\\5\end{bmatrix}v(t),\ y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix},$

$\phi(t) = \begin{bmatrix}e^{-3t}\cos(2t) - \frac{1}{2}e^{-3t}\sin(2t) & \frac{1}{2}e^{-3t}\sin(2t)\\ -\frac{5}{2}e^{-3t}\sin(2t) & e^{-3t}\cos(2t) + \frac{1}{2}e^{-3t}\sin(2t)\end{bmatrix}$)

9. Obtaint the state space equations for the following circuit.



(Ans: $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{C} \\ 0 & -\frac{R}{L_1} & \frac{R}{L_1} \\ -\frac{1}{L_2} & \frac{R}{L_2} & -\frac{R}{L_2} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ 0 \end{bmatrix} u(t),\ y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$)

End of Tutorial Questions (Part 4)