THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

School of Professional Education and Executive Development
專業進修學院
SPEED

SEHS4653 Control System Analysis
Tutorial Questions (Part 1)

Laplace Transform

1. Solve the following differential equations using Laplace transform method.

(a) $\dot{y}(t) + y(t) = 2e^{t}$, $y(0) = 4$ Ans: $y(t) = e^{t} + 3e^{-t}$

(b) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 1$, $y(0) = -1$, $\dot{y}(0) = 0$ Ans: $y(t) = \frac{1}{2} - 3e^{-t} + \frac{3}{2}e^{-2t}$

2. Find the unit-step and unit-impulse response of the following systems in time domain.

(a) $\ddot{y}(t) + 2\dot{y}(t) + y(t) = r(t)$ Ans: $u(t) = 1 - te^{-t} - e^{-t}$, $h(t) = te^{-t}$

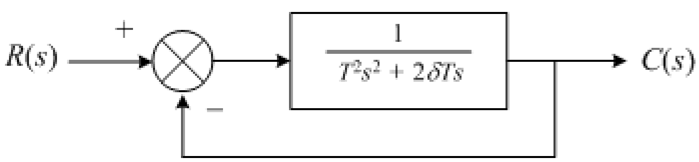
(b) $\ddot{y}(t) + \dot{y}(t) = r(t)$ Ans: $u(t) = t - 1 + e^{-t}$, $h(t) = 1 - e^{-t}$

(c) $\ddot{y}(t) + 2\dot{y}(t) + 2y(t) = r(t)$ Ans: $u(t) = \frac{1}{2} - \frac{1}{2}e^{-t}\cos t - \frac{1}{2}e^{-t}\sin t$, $h(t) = e^{-t}\sin t$

Block Diagrams

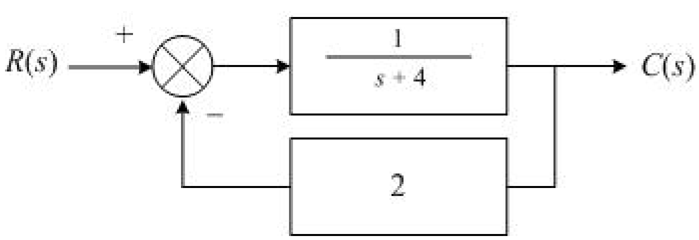
3. Find the closed-loop transfer function for the systems shown below.

(a)



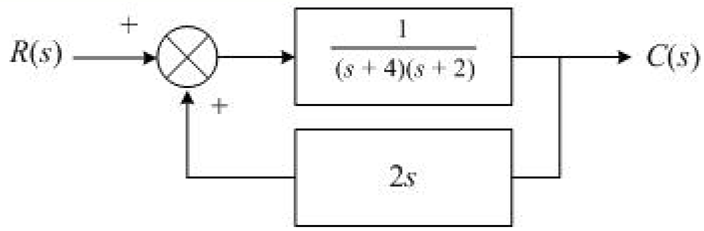
Ans: $\frac{C(s)}{R(s)} = \frac{1}{T^2s^2 + 2\delta Ts + 1}$

(b)



Ans: $\frac{C(s)}{R(s)} = \frac{1}{s + 6}$

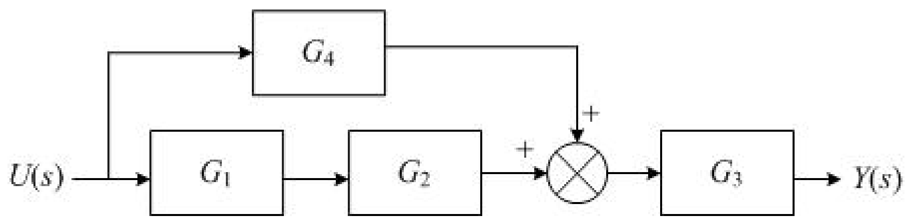
(c)



Ans: $\dfrac{C(s)}{R(s)} = \dfrac{1}{s^2 + 4s + 8}$

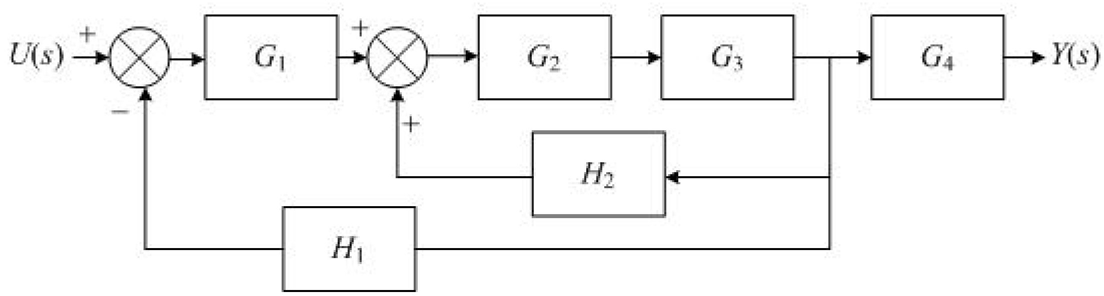
4. Reduce the block diagrams shown below to obtain the transfer function using block combination rules.

(a)



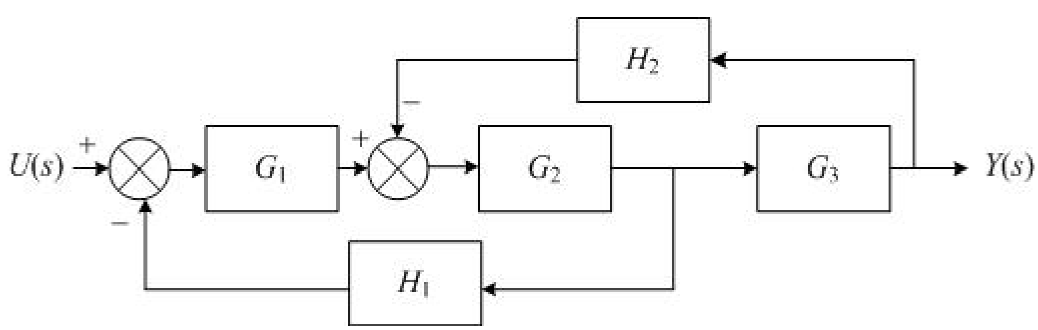
Ans: $\dfrac{Y(s)}{U(s)} = (G_1 G_2 + G_4) G_3$

(b)



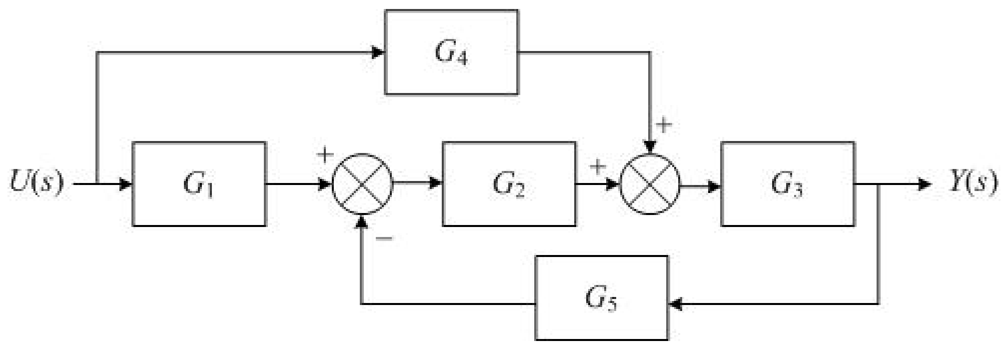
Ans: $\dfrac{Y(s)}{U(s)} = \dfrac{G_1 G_2 G_3 G_4}{1 - G_2 G_3 H_2 + G_1 G_2 G_3 H_1}$

(c)



Ans: $\dfrac{Y(s)}{U(s)} = \dfrac{G_1 G_2 G_3}{1 + G_2 G_3 H_2 + G_1 G_2 H_1}$

(d)



Ans: $\frac{Y(s)}{U(s)} = \frac{G_3(G_1G_2 + G_4)}{1 + G_2G_3G_5}$

Signal Flow Graphs

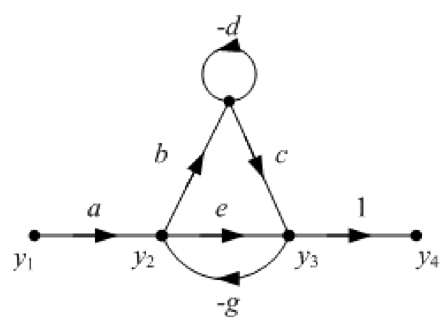
5. Construct the signal flow graph for the following set of simultaneous equations.

$$x_2 = A_{12}x_1 + A_{32}x_3$$
$$x_3 = A_{13}x_1 + A_{23}x_2 + A_{33}x_3$$
$$x_4 = A_{24}x_2 + A_{34}x_3$$

Hence determine the transfer function (x_4/x_1) using the Mason's rule.

Ans: $\frac{x_4}{x_1} = \frac{A_{21}A_{42}(1 - A_{33}) + A_{23}A_{31}A_{42} + A_{31}A_{43} + A_{21}A_{32}A_{43}}{1 - A_{23}A_{32} - A_{33}}$

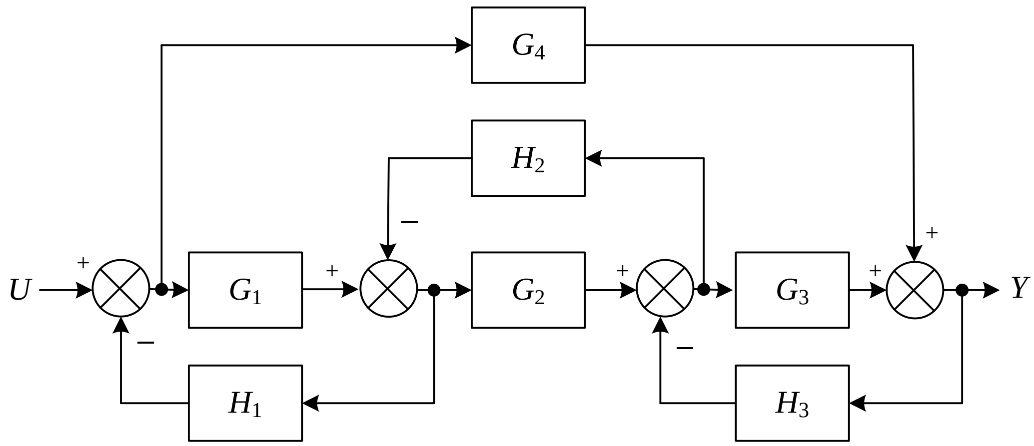
6. Consider the signal flow graph below:



Hence determine the transfer function (y_4/y_1) using the Mason's rule.

Ans: $\frac{y_4}{y_1} = \frac{ae(1 + d) + abc}{1 + d + eg + bcg + edg}$

7. Construct a signal flow graph for the below block diagram and hence determine its transfer function by using Mason's rule.



Ans: $\frac{Y(s)}{U(s)} = \frac{G_1G_2G_3 + G_4 + G_2G_4H_2}{1 + G_1H_1 + G_2H_2 + G_3H_3 + G_4H_1H_2H_3 + G_1G_3H_1H_3}$

System Modelling

8. A very simplified version of the suspension system is shown in Figure 1 below. Assuming that the motion x_i at point P is the input to the system and the vertical motion x_o of the body is the output, obtain the transfer function $X_o(s)/X_i(s)$. (Consider the motion of the body only in the vertical direction.) Displacement x_o is measured from the equilibrium position in the absence of input x_i.

Ans: $\frac{X_o(s)}{X_i(s)} = \frac{bs + k}{ms^2 + bs + k}$

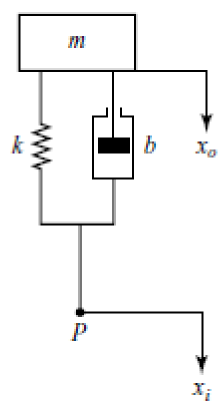


Figure 1

9. Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical systems shown in Figures 3 and 4 below **using impedance method.**

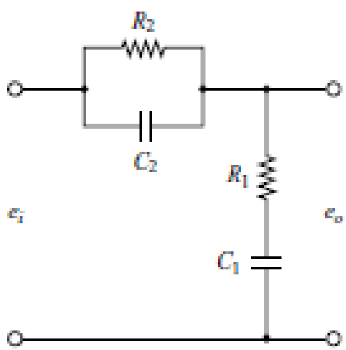


Figure 3

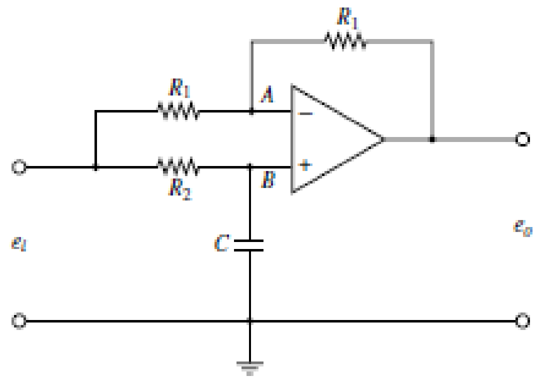


Figure 4

Ans: $\dfrac{E_o(s)}{E_i(s)} = \dfrac{(sR_1C_1+1)(sR_2C_2+1)}{sR_2C_1+(sR_1C_1+1)(sR_2C_2+1)}$

Ans: $\dfrac{E_o(s)}{E_i(s)} = \dfrac{1-sR_2C}{sR_2C+1}$

End of Tutorial Questions (Part 1)