THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

COLLEGE OF PROFESSIONAL AND CONTINUING EDUCATION
專業及持續教育學院

School of Professional Education and Executive Development
專業進修學院
SPEED

SEHS
Division of Science, Engineering and Health Studies
科技、工程及健康學部

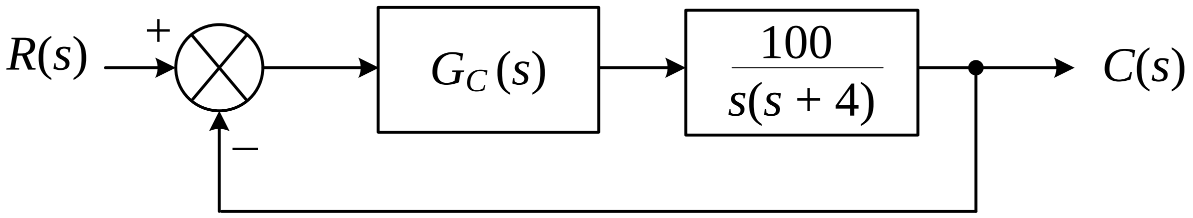
SEHS4653 Control System Analysis

Past Paper Revision (Part 4)

Dr Kenneth LO

Question 5 Sem 2, 2023/24

A series phase-lead compensator, $G_c(s)$, is used to control the position of a servo motor as shown in Figure 2 below.



(a) Find the value of the gain, $K_C\alpha$, of the compensator such that the system has statics velocity error constant of 10 sec^{-1}. (5 marks)

From equation list, we have

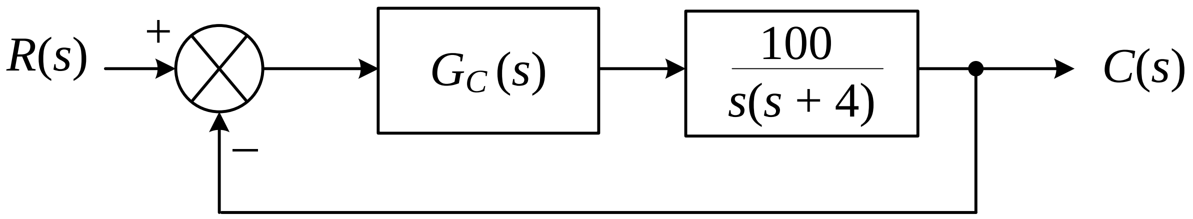
$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

$$K_V = \lim_{s\to 0} s\left(\frac{100}{s(s+4)}\right)\left(K_C\alpha\frac{Ts+1}{\alpha Ts+1}\right) = \frac{100K_C\alpha}{4} = 25K_C\alpha = 10$$

$$\therefore K_C\alpha = 0.4$$

Question 5 Sem 2, 2023/24

A series phase-lead compensator, $G_c(s)$, is used to control the position of a servo motor as shown in Figure 2 below.



(a) $K_C\alpha = 0.4$

(b) With the result obtained in part (a), plot the exact Bode diagrams at angular frequencies of 1 rad/s, 2 rad/s, 5 rad/s, 8 rad/s, and 10 rad/s for $G_1(s) = K_c\alpha\frac{100}{s(s+4)}$. (15 marks)

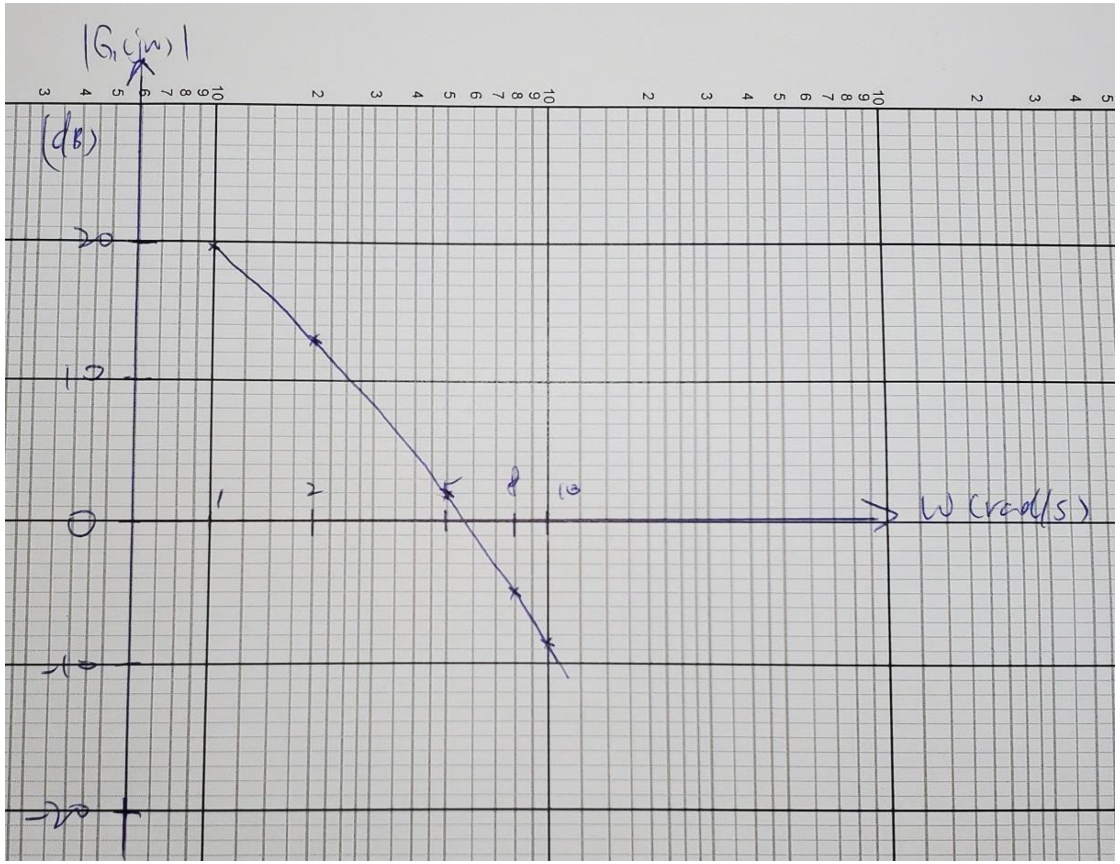
$$G_1(s) = (0.4)\left(\frac{100}{s(s+4)}\right) = \frac{40}{s(s+4)}$$

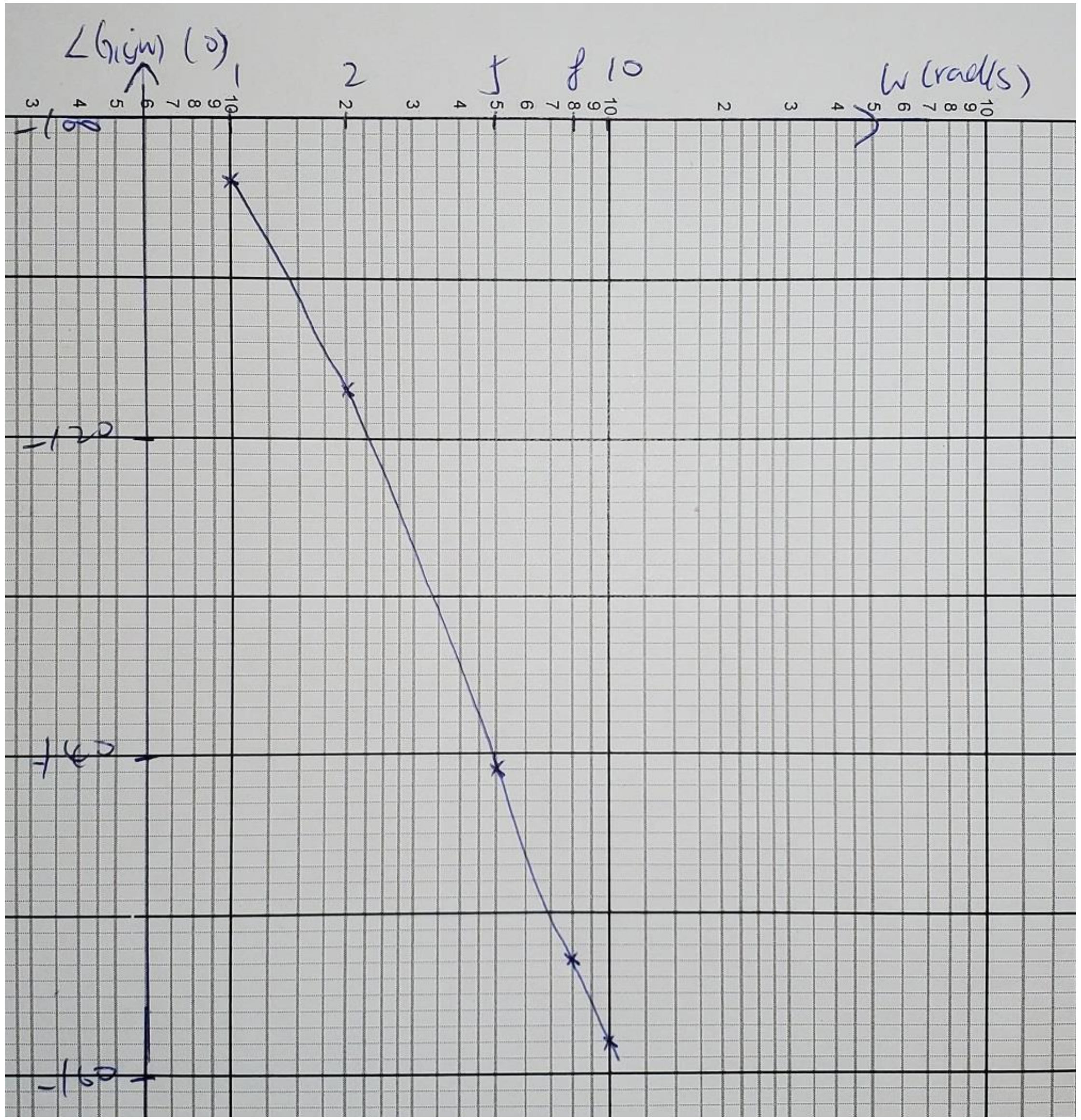
Replace s with $j\omega$, we have $$G_1(j\omega) = \frac{40}{j\omega(j\omega+4)}$$

$$|G_1(j\omega)| = 20\log\frac{40}{\omega\sqrt{\omega^2+4^2}} \qquad \angle G_1(j\omega) = -90° - \tan^{-1}\frac{\omega}{4}$$

ω (rad/ s)	$\lvert G_1(j\omega)\rvert$ (dB)	$\angle G_1(j\omega)$ (°)
1	19.7	–104
2	13	–116.6
5	1.9	–141.3
8	–5.1	–153.4
10	–8.6	–158.2

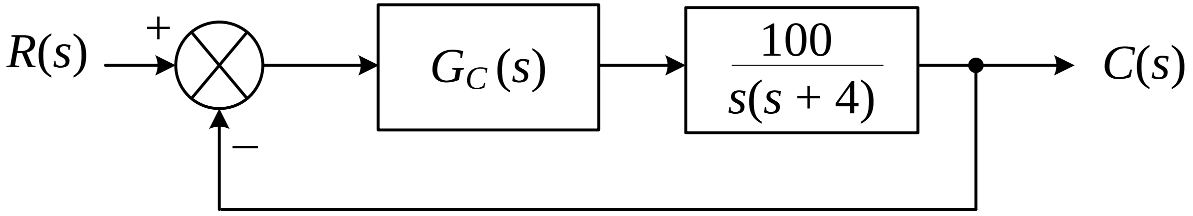
Question 5 Sem 2, 2023/24





Question 5 Sem 2, 2023/24

A series phase-lead compensator, $G_c(s)$, is used to control the position of a servo motor as shown in Figure 2 below.



(a) $K_C \alpha = 0.4$

(c) Hence, find the transfer function of the phase-lead compensator such that the phase margin is at least 60°. (10 marks)

(Total: 30 marks)

From the Bode diagrams, find the phase margin (P.M.) first:

At 0 dB, $\omega = 5.6$ rad/ s, the system phase is –143°.
The phase margin = 180° + (–143°) = 37°

Hence, the maximum phase-lead required is $\phi_M = 60° - 37° + 5° = 28°$

$$\sin 28° = \frac{1-\alpha}{1+\alpha} \rightarrow \boldsymbol{\alpha = 0.361}$$

The new system gain is now,

$$|G_1(j\omega)| = -20\log\frac{1}{\sqrt{0.361}} = -4.42 \text{ dB}$$

At this new system gain, from the Bode Diagram, the new crossover frequency is, $\boldsymbol{\omega_m = 7.8}$ **rad/s**.

Question 5 Sem 2, 2023/24

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

(a) $K_C\alpha = 0.4$

$\omega_m = 7.8$ rad/s

$$\omega_m = \frac{1}{\sqrt{\alpha}T} \rightarrow \frac{1}{T} = (7.8)\left(\sqrt{0.361}\right) = 4.686$$

$$\frac{1}{\alpha T} = \frac{4.686}{0.361} = 12.98$$

$$K_C\alpha = 0.4 \rightarrow K_C = \frac{0.4}{0.361} = 1.11$$

So, the transfer function of the phase-lead compensator is,

$$G_C(s) = 1.11\frac{s+4.686}{s+12.98}$$

Question 5 Sem 1, 2023/24

A series phase-lag compensator is added in the forward path of the position control system, as shown in Figure Q5 below, to enhance the stability of the system. The transfer function of the compensated system is listed as,

$$G_c(s)G(s) = \left(\frac{s+0.4}{s+4}\right)\left(\frac{100}{s(s+3)(s+5)}\right).$$

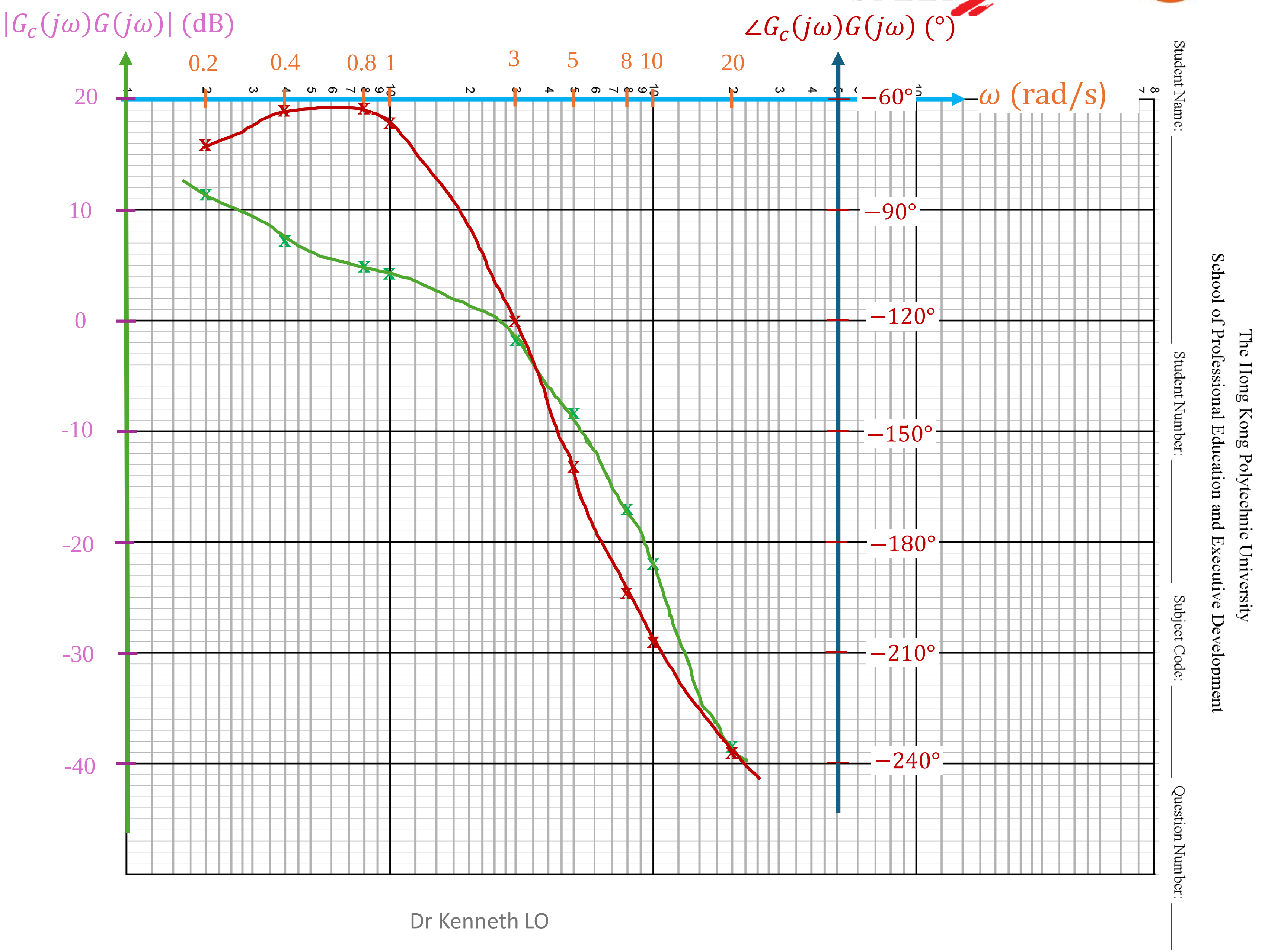
Given the following open-loop frequency response of the compensated system ($G_c(s)G(s)$).

ω (rad/ s)	Gain (dB)	Phase (°)
0.2	11.4	–72.4
0.4	7.3	–62.9
0.8	4.8	–61.9
1	4.2	–65.6
3	–1.8	–120.4
5	–8.4	–160
8	–17.1	–193.7
10	–22	–207.2
20	–38.6	–237.3

(a) Plot the Bode diagrams of the compensated system in the semi-log graph paper provided. (8 marks)

Question 5

Sem 1, 2023/24

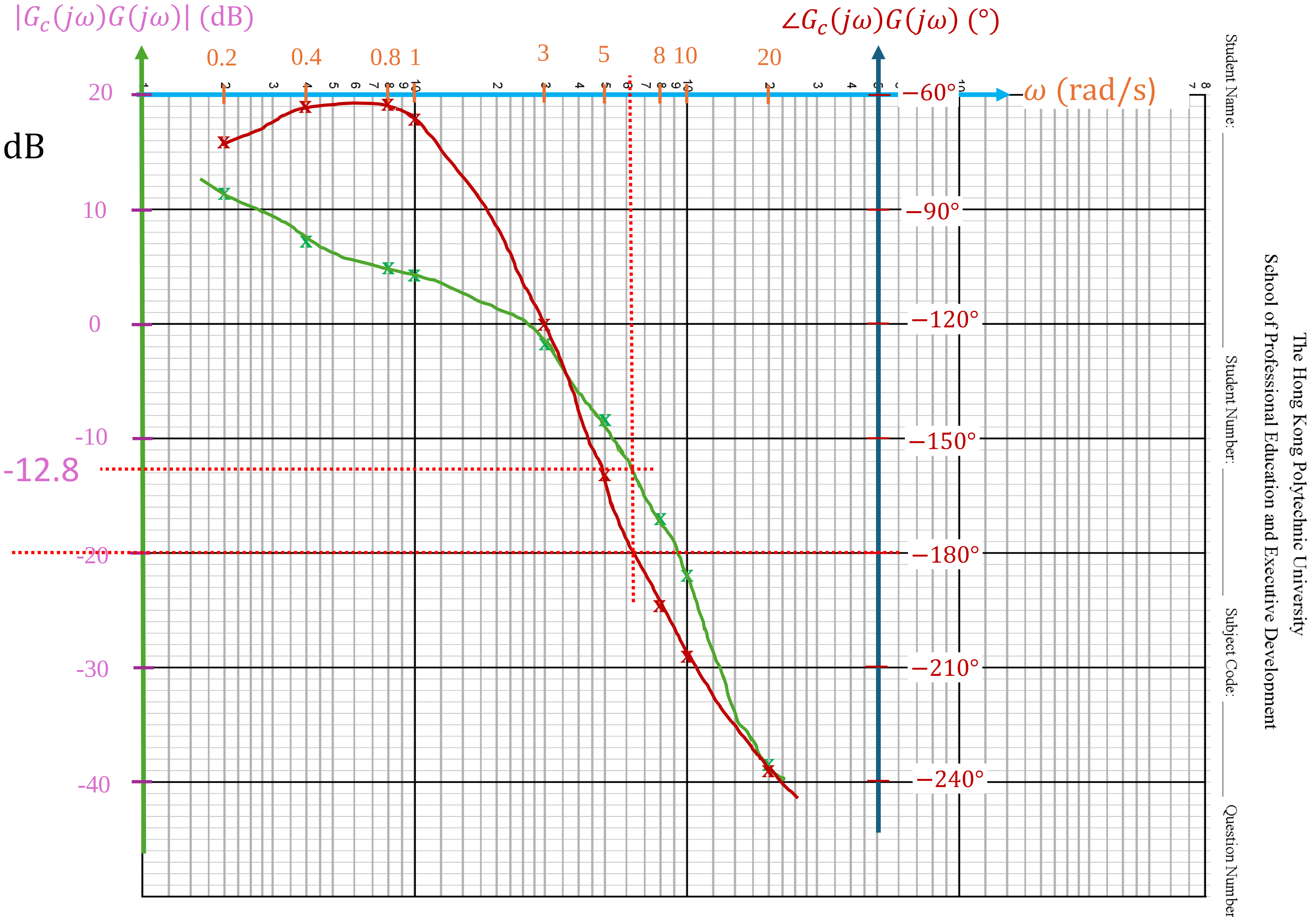


Question 5 (b) Hence, find the gain margin and phase margin of the compensated system. (4 marks)

Sem 1, 2023/24

$$\text{G. M.} = 0 - (-12.8) = 12.8 \text{ dB}$$

$$\omega_{pc} = 6.1 \text{ rad/s}$$



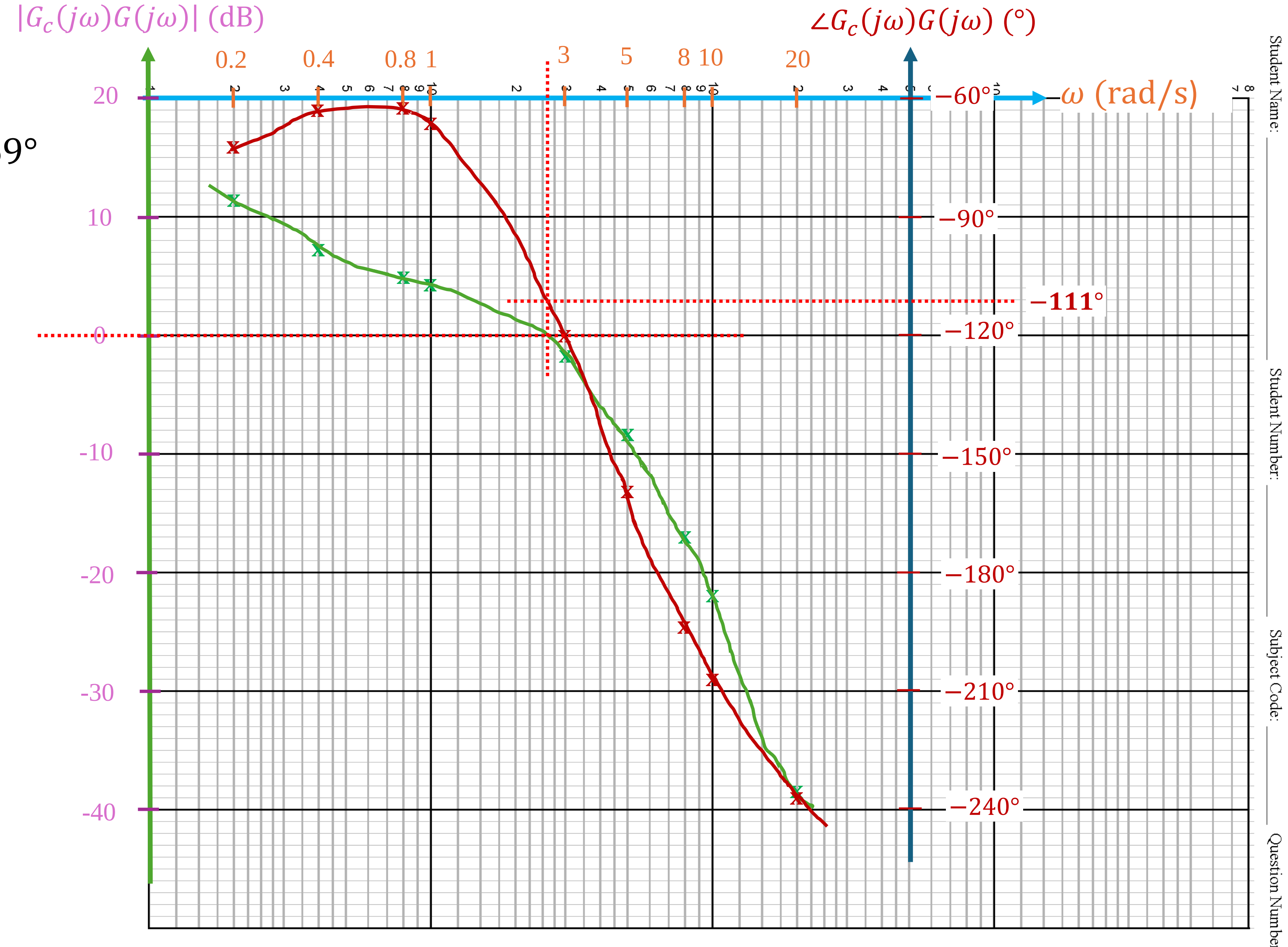
<u>Question 5</u>

<u>Sem 1, 2023/24</u>

(b) Hence, find the gain margin and phase margin of the compensated system. (4 marks)

$$\text{P.M.} = 180° + (-111°) = 69°$$

$$\omega_{gc} = 2.6 \text{ rad/s}$$



Question 5 Sem 1, 2023/24

(c) Identify the new gain margin and phase margin if a gain of magnitude 2 is added before $G_c(s)$. How will be the system stability affected?
(8 marks)
(Total: 20 marks)

Only the magnitude bode plot will be shifted up by $\mathbf{20\log 2 = 6\ dB}$.

ω (rad/ s)	Gain (dB)	New Gain (dB)
0.2	11.4	17.4
0.4	7.3	13.3
0.8	4.8	10.8
1	4.2	10.2
3	–1.8	4.2
5	–8.4	–2.4
8	–17.1	–11.1
10	–22	–16
20	–38.6	–32.6

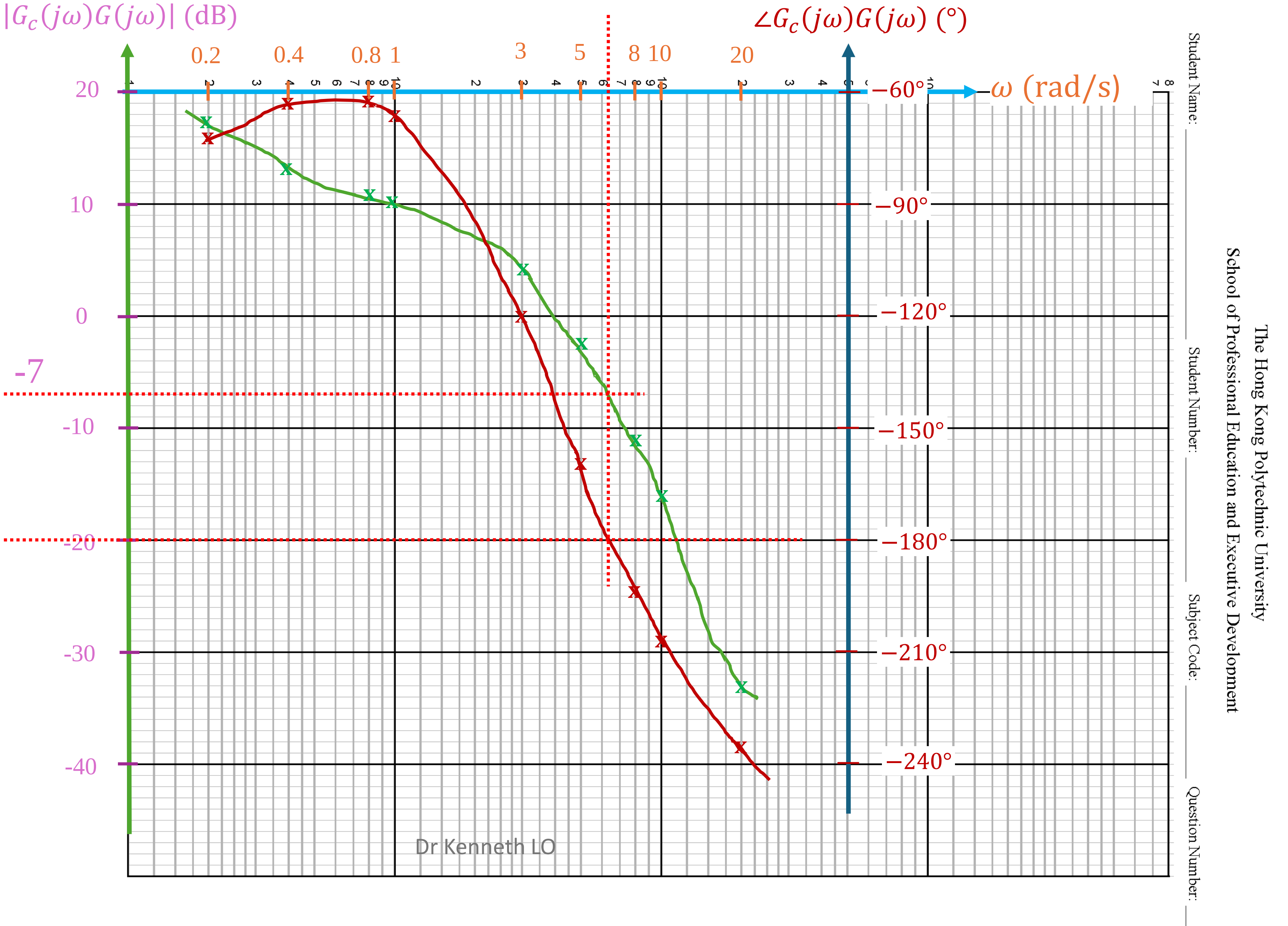
$$G_c(s)G(s) = \left(\frac{s+0.4}{s+4}\right)\left(\frac{100}{s(s+3)(s+5)}\right)$$

Question 5

Sem 1, 2023/24

$$\text{G.M.} = 0 - (-7) = 7 \text{ dB}$$

$$\omega_{pc} = 6.1 \text{ rad/s}$$

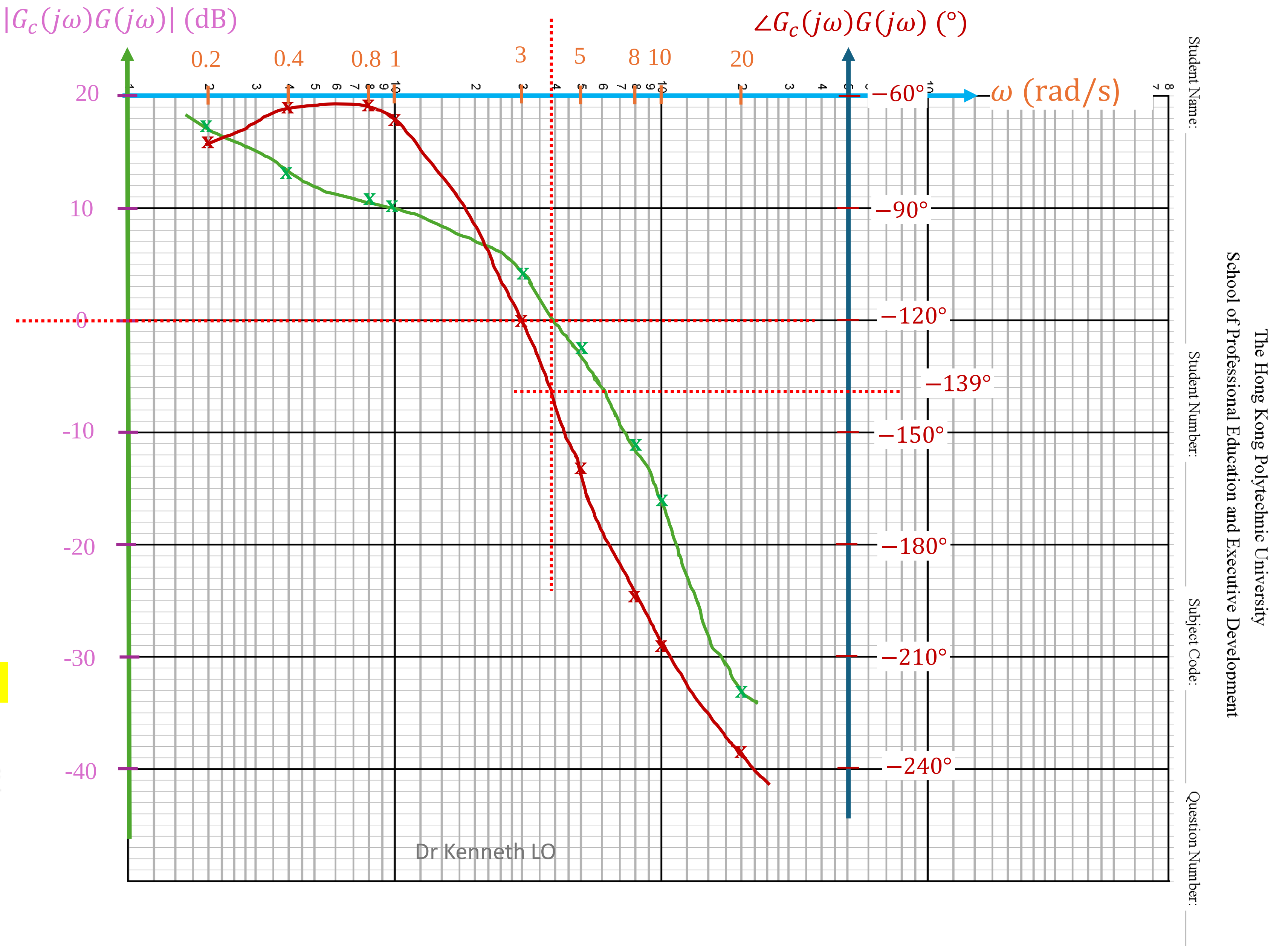


Question 5

Sem 1, 2023/24

$$\text{P.M.} = 180° + (-139°) = 41°$$

$$\omega_{gc} = 3.9 \text{ rad/s}$$



When the system gain increased by 6 dB, the magnitude Bode plot will be shift up by 6 dB, which results a **gain margin decreased** from 12.8 dB to 7 dB, and the **phase margin is also decreased** from 69° to 41°. Hence, the system stability will be lowered.

Question 6 Sem 2, 2020/21

The open-loop transfer function of a pressure process control system is given by $G(s) = \dfrac{100}{(s+1)(s+5)(s+10)}$.

a) Evaluate the magnitude (in dB) and phase (in degrees) of the system at angular frequencies of 0.1, 0.5, 1, 5, 10, 12, 15 and 18 rad/s. (4 marks)

Replace s with $j\omega$, we have

$$G(j\omega) = \frac{100}{(j\omega+1)(j\omega+5)(j\omega+10)}$$

$$|G(j\omega)| = 20\log\left(\frac{100}{\sqrt{\omega^2+1^2}\sqrt{\omega^2+5^2}\sqrt{\omega^2+10^2}}\right)$$

$$\angle G(j\omega) = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{5} - \tan^{-1}\frac{\omega}{10}$$

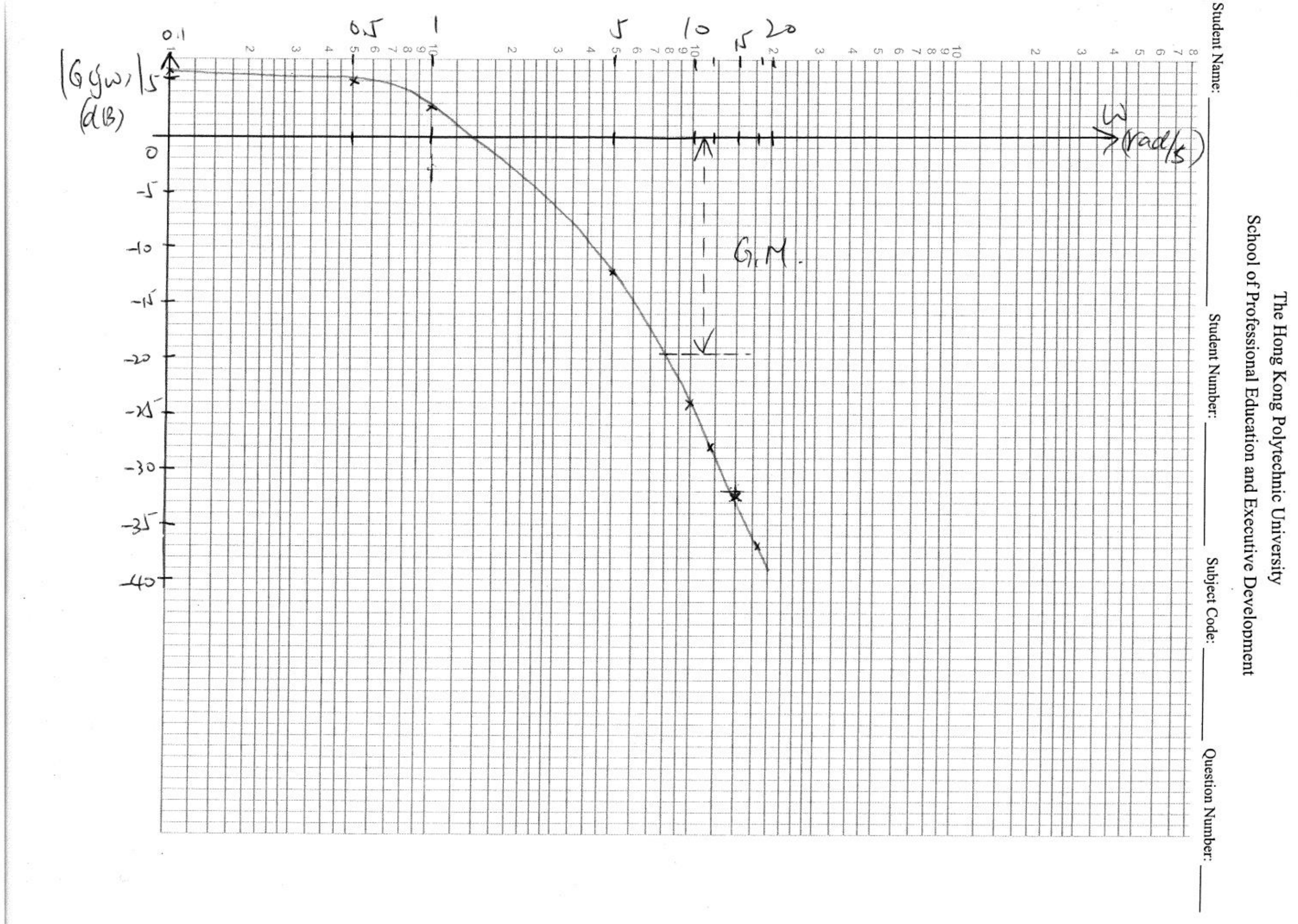
ω (rad/s)	$G(j\omega)$ (dB)	$\angle G(j\omega)$ (°)
0.1	5.98	–7.43
0.5	5.0	–35.14
1	2.8	–62.02
5	–12.11	–150.26
10	–24.02	–192.72
12	–27.77	–202.81
15	–32.64	–214.06
18	–36.82	–222.24

Question 6 Sem 2, 2020/21

b) Plot the exact Bode diagrams in the semi-log graph paper provided. (6 marks)

c) Hence, determine the gain and phase margins of the system. (4 marks)

Gain Margin
$= 0 - (-19.5) = 19.5$ dB



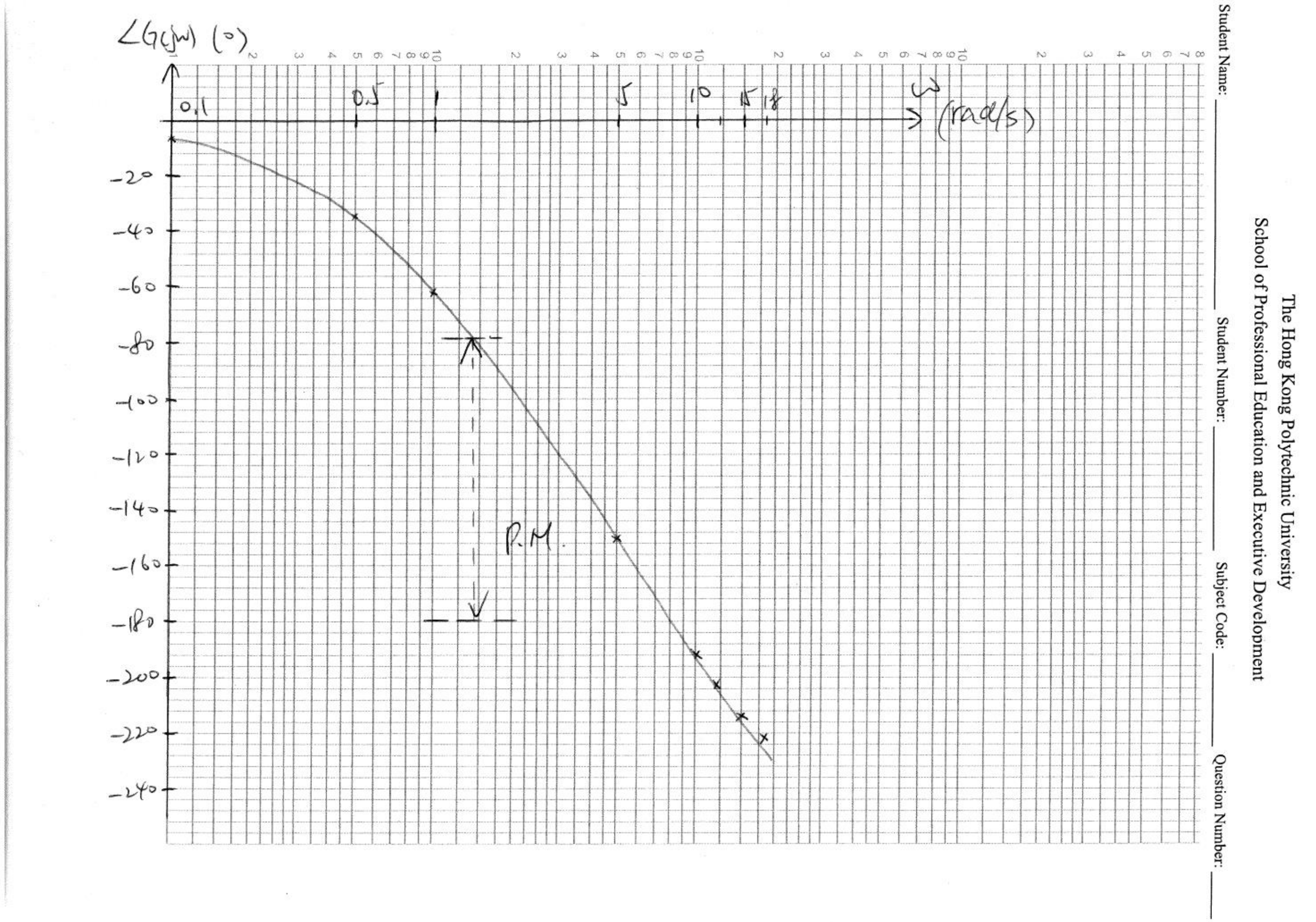
Question 6 Sem 2, 2020/21

b) Plot the exact Bode diagrams in the semi-log graph paper provided. (6 marks)

c) Hence, determine the gain and phase margins of the system. (4 marks)

Phase Margin

$= 180° + (-78°) = 102°$



Question 6   Sem 2, 2020/21

The open-loop transfer function of a pressure process control system is given by $G(s) = \dfrac{100}{(s+1)(s+5)(s+10)}$.

d) Design a proportional controller for the system such that it has a gain margin of 8 dB. (6 marks)

Gain Margin = 19.5 dB (from part c)

Hence, the proportional controller needs to contribute **11.5 dB** to the overall system.

$$20\log K = 11.5$$

$$K = 3.758$$