THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

COLLEGE OF PROFESSIONAL AND CONTINUING EDUCATION
專業及持續教育學院

School of Professional Education and Executive Development
專業進修學院
SPEED

SEHS
Division of Science, Engineering and Health Studies
科技、工程及健康學部

SEHS4653 Control System Analysis

Past Paper Revision (Part 3)

Dr Kenneth LO

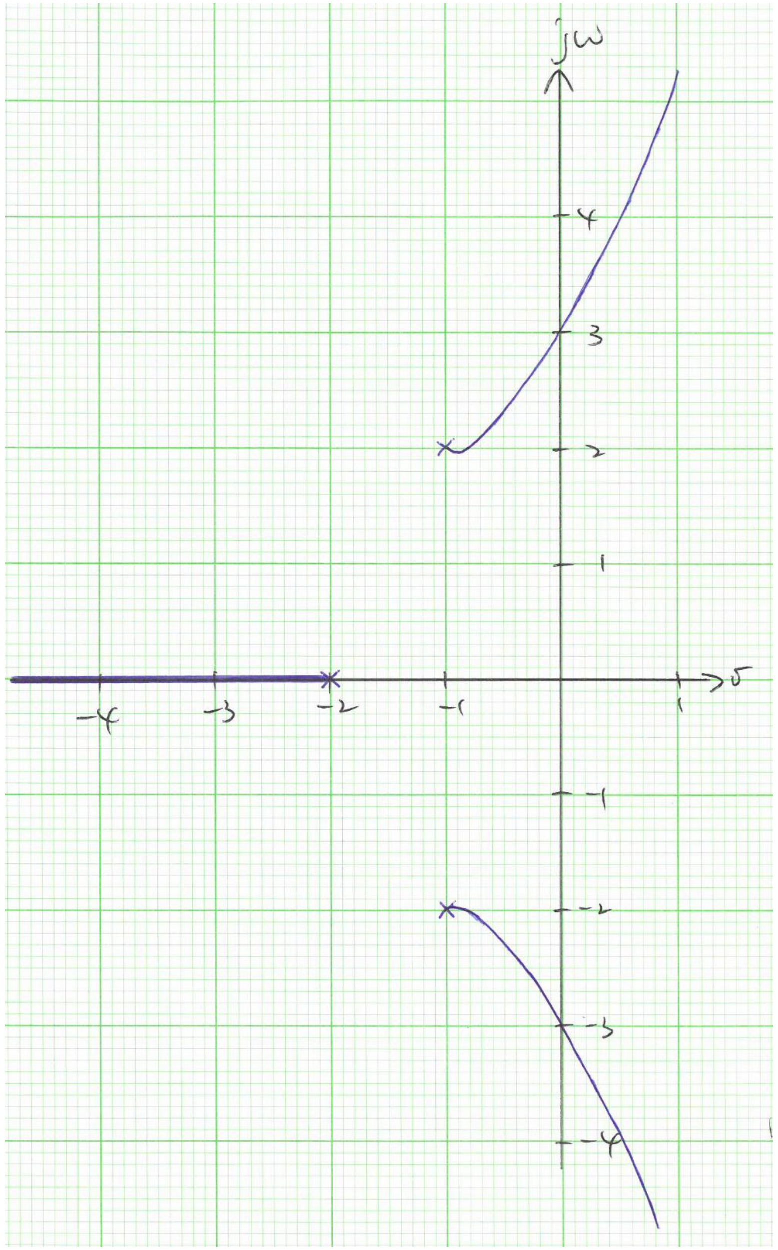
Question 4 Sem 2, 2022/23

Figure 3 shows the root locus plot of a negative unity feedback control system with the following open-loop transfer function,

$$G(s)H(s) = \frac{K}{den(s)}, K > 0 .$$

A larger version of the root locus plot (Figure 3) is placed at the last page of this paper. Tear off the last page and insert it into your answer book for answering this question,

(a) Find the equation of *den*(*s*). (5 marks)



From the root locus plot, the open-loop poles are: $s = -2, -1 \pm j2$

Hence, the equation of den(s) will be,

$$den(s) = (s + 2)(s + 1 + j2)(s + 1 - j2) = (s + 2)(s^2 + 2s + 5)$$

$$= s^3 + 4s^2 + 9s + 10$$

Question 4 Sem 2, 2022/23

$$G(s)H(s) = \frac{K}{den(s)}, K > 0$$

(b) Hence, determine the range of K such that the system is stable. (6 marks)

$$G(s)H(s) = \frac{K}{s^3 + 4s^2 + 9s + 10}$$

The characteristic equation of the system is,

$$\Delta(s) = 1 + \frac{K}{s^3 + 4s^2 + 9s + 10} = s^3 + 4s^2 + 9s + 10 + K = 0$$

s^3	1	9
s^2	4	$10 + K$
s^1	$\frac{(4)(9) - (1)(K + 10)}{4}$	
s^0	$K + 10$	

$$\therefore 0 < K < 26$$

Question 4 Sem 2, 2022/23

(c) Locate the closed-loop poles of the system if the damping ratio is required to be 0.2. Hence, compute the value of K. (8 marks)

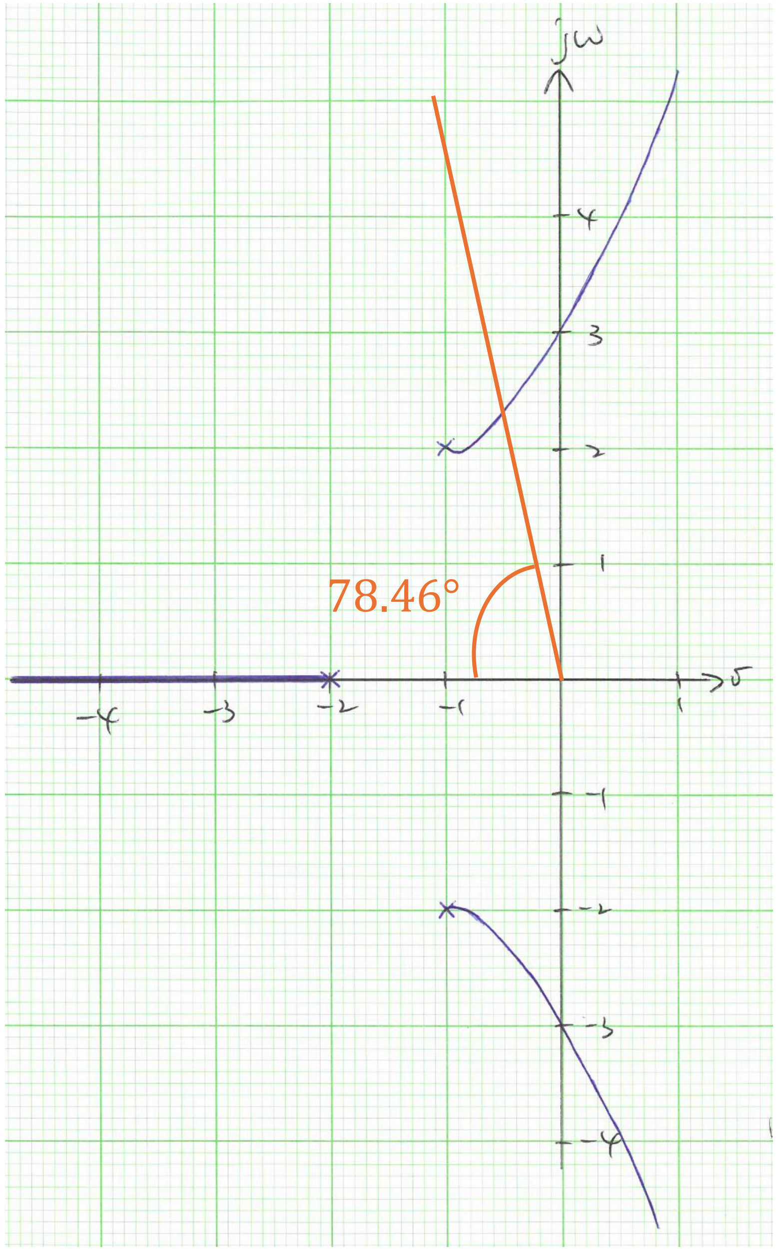
Draw a line from origin with angle, θ, which is calculated by,
$\cos\theta = \zeta \Rightarrow \theta = \cos^{-1} 0.2 = 78.46°$.

Hence, the closed-loop poles are, $s = -0.5 \pm j2.3$

$$K = -(s^3 + 4s^2 + 9s + 10)$$

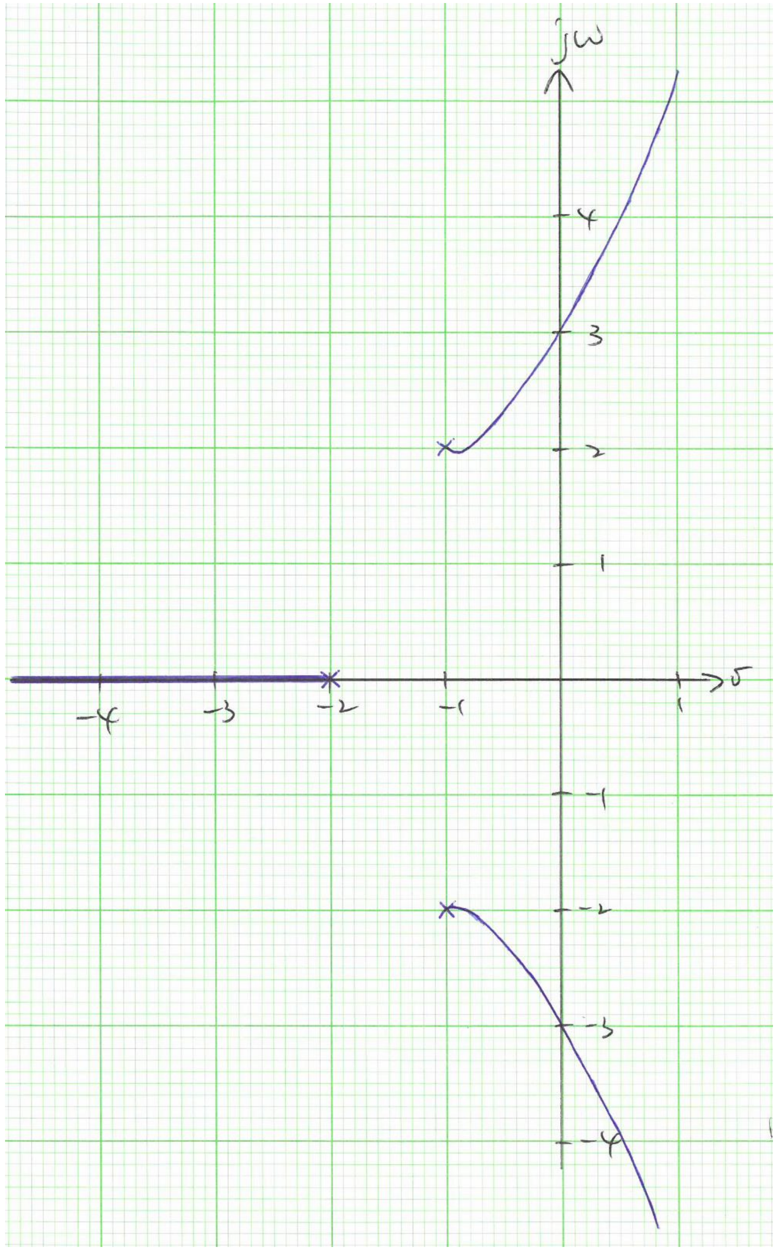
$$= -(-0.5 + j2.3)^3 - 4(-0.5 + j2.3)^2 - 9(-0.5 + j2.3) + 10$$

$\therefore K = 6.93$



Question 4 Sem 2, 2022/23

(d) Compute the maximum (percentage) overshoot, rise time and peak time of the system with unit-step input at the K value obtained in part (c). (6 marks)
(Total: 25 marks)



Maximum percentage overshoot: $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$$M_P = e^{\frac{-0.2\pi}{\sqrt{1-0.2^2}}} = 52.66\ \%$$

Rise Time: $t_r = \dfrac{\pi - \beta}{\omega_d}$

$$t_r = \frac{\pi - 78.46°}{2.3} = 0.771s$$

Peak Time: $t_p = \dfrac{\pi}{\omega_d}$

$$t_p = \frac{\pi}{2.3} = 1.32s$$

Question 4 Sem 1, 2023/24

A unity negative feedback control system (with $K > 0$) has the following open-loop transfer function,

$$G(s) = \frac{K}{(s+1)(s+2)(s+4)}.$$

(a) Use Routh–Hurwitz stability criterion to find the range of K such that the system is stable.
(8 marks)

The characteristic equation of the system is, $\Delta(s) = s^3 + 7s^2 + 14s + 8 + K = 0$

s^3	1	14
s^2	7	$8+K$
s^1	$\dfrac{(7)(14)-(1)(8+K)}{7}$	
s^0	$8+K$	

$\therefore 0 < K < 90$

back

$$G(s) = \frac{K}{(s+1)(s+2)(s+4)}$$

Question 4 Sem 1, 2023/24

(b) Draw the root-locus of the system on the metric-size graph paper provided. Show all your steps clearly. (14 marks)

Open-loop poles: $s = -1, -2, -4$

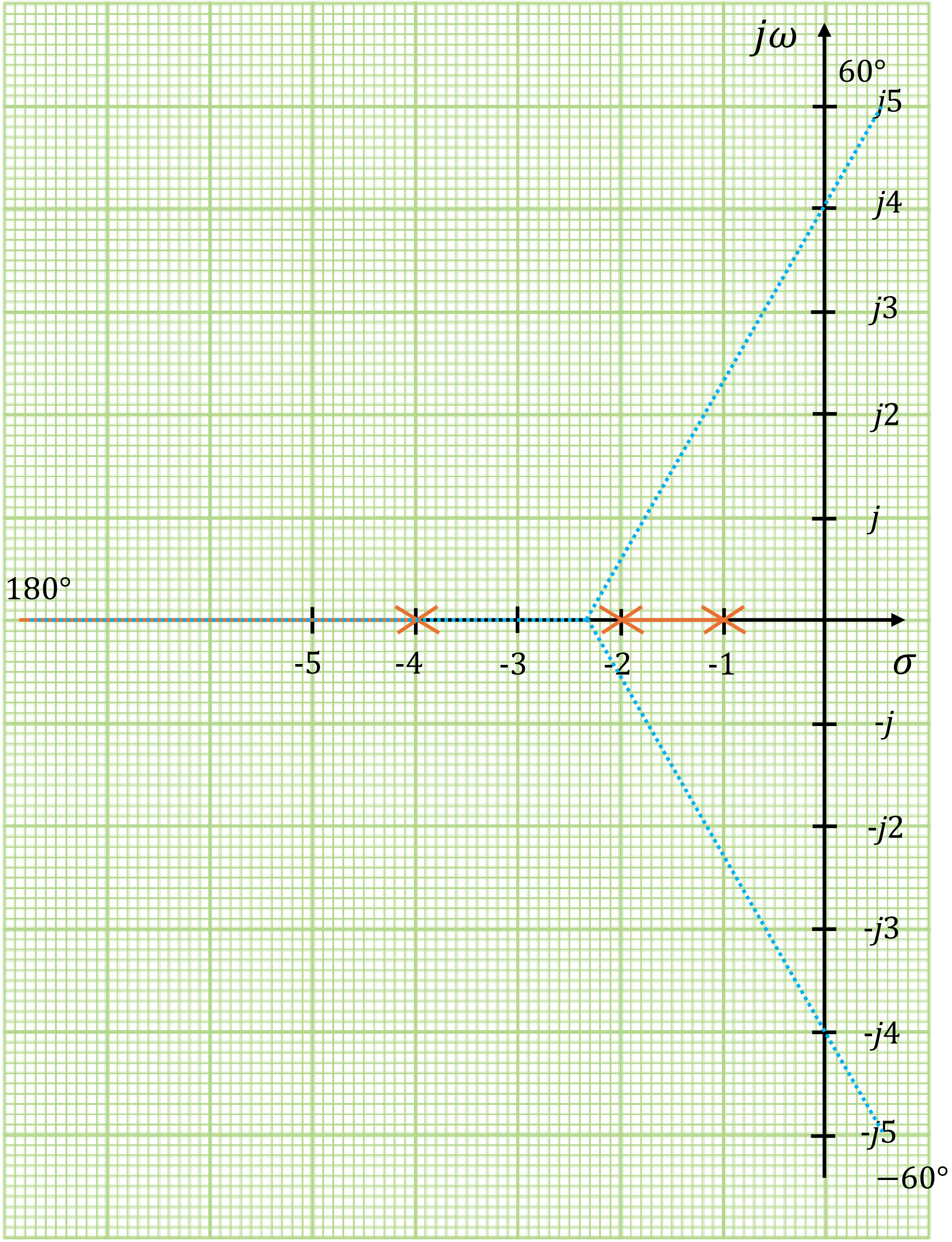
Root locus branch on the real axis: $(-\infty, -4)$, $(-2, -1)$

Angle of asymptote:

$$\phi_A = \frac{\pm 180°(2k+1)}{3-0} = \pm 60°, \pm 180°$$

Intersection point of asymptotes on the real-axis:

$$\sigma_A = \frac{(-1)+(-2)+(-4)}{3-0} = -\frac{7}{3} = -2.333$$



$$G(s) = \frac{K}{(s+1)(s+2)(s+4)}$$

Question 4 Sem 1, 2023/24

Breakaway point on the real-axis: Part(a)

$$\Delta(s) = s^3 + 7s^2 + 14s + 8 + K = 0$$

$$K = -s^3 - 7s^2 - 14s - 8$$

$$\frac{dK}{ds} = -3s^2 - 14s - 14 = 0$$

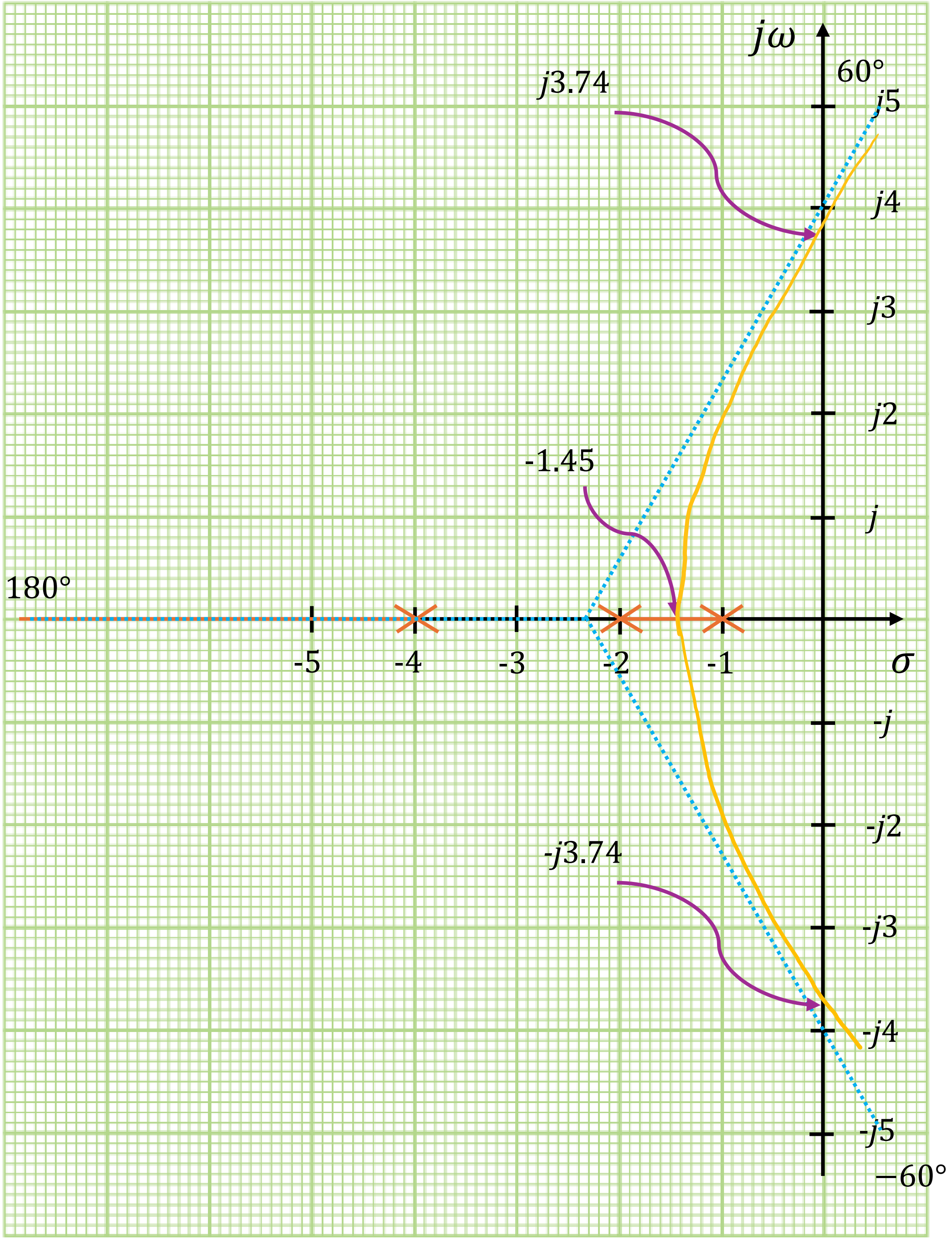
$$s = -1.45, \quad s = -3.22 \text{ (rejected)}$$

Angle of departure (or arrival): NO

Intersection point on the $j\omega$-axis:

$$7s^2 + 98 = 0$$

$$s = \pm j\sqrt{14} = \pm j3.74$$



Question 4 Sem 1, 2023/24

$$G(s) = \frac{K}{(s+1)(s+2)(s+4)}$$

(c) Identify the location of closed-loop pole such that a desired damping ratio of 0.5 is required.

(6 marks)

(Total: 28 marks)

Draw a line from origin with angle, θ, which is calculated by, $\cos\theta = \zeta \Rightarrow \theta = \cos^{-1} 0.5 = 60°$.

Hence, the closed-loop poles are,

$$s = -1.05 \pm j1.85$$