SEHS4653 Control System Analysis

Past Paper Revision (Part 2)

Dr Kenneth LO

<u>Question 1</u> <u>Sem 2, 2020/21</u>

Use block diagram reduction to find the transfer function of the control block diagram shown in Figure 1 below. (Total: 10 marks)

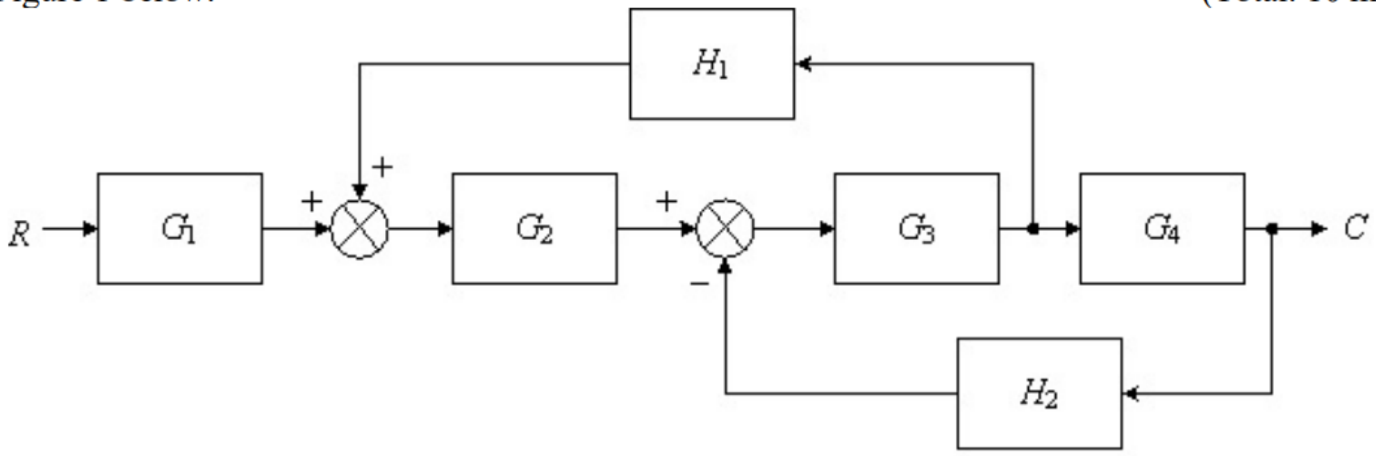
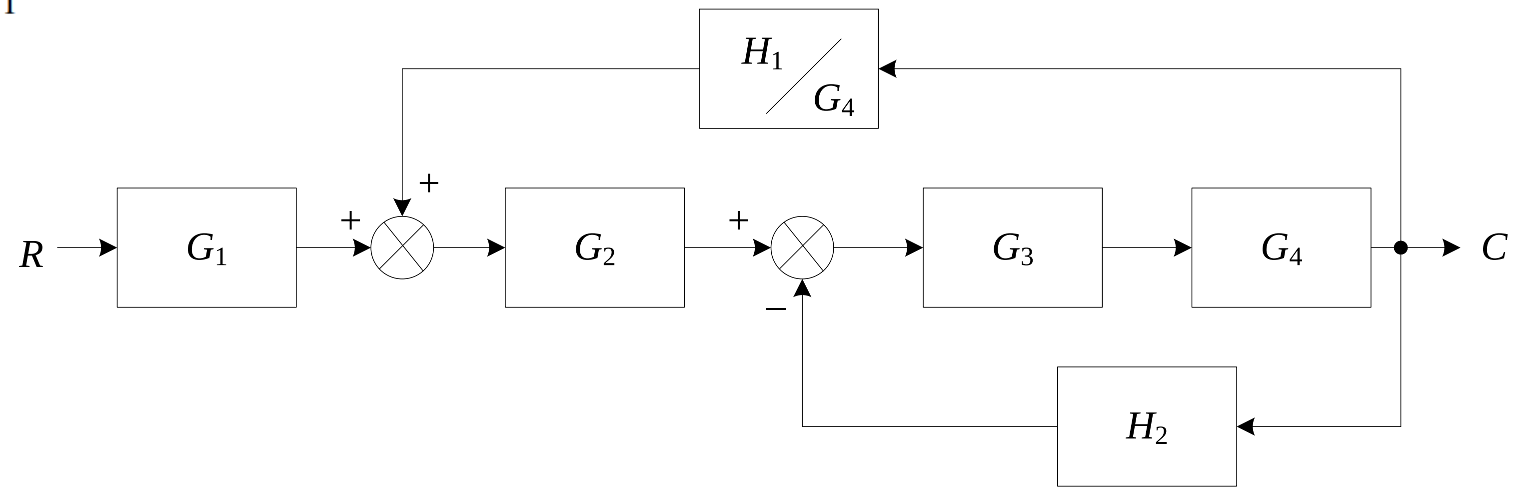
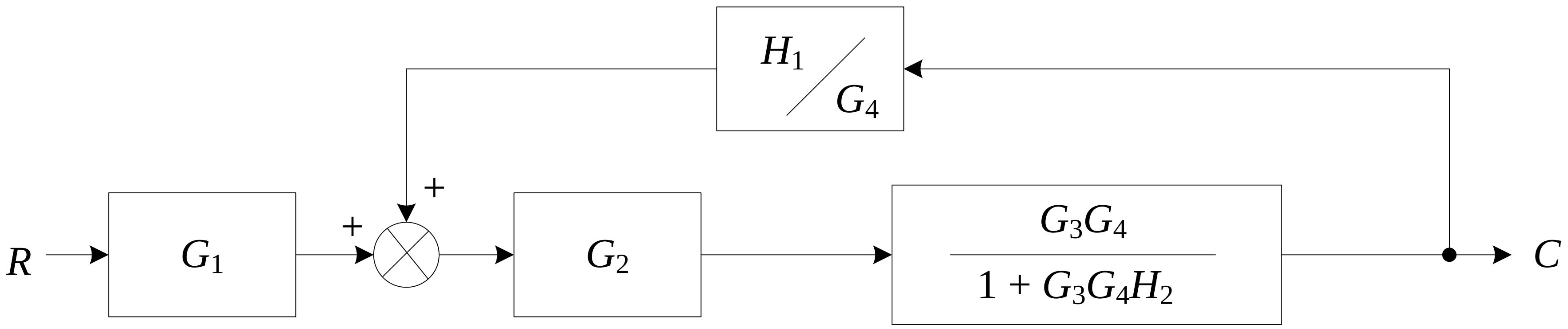
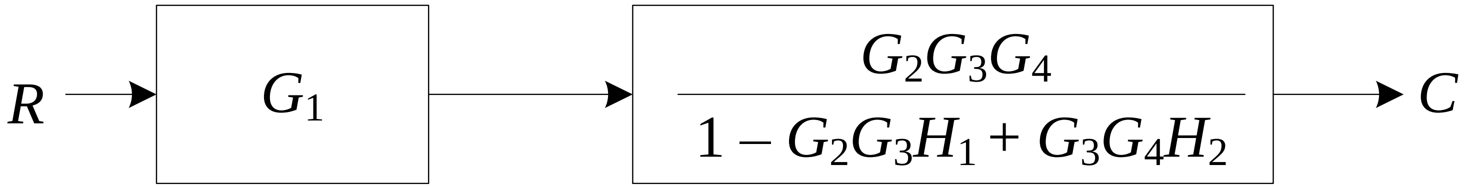


Figure 1







$$\frac{C}{R} = \frac{G_1 G_2 G_3 G_4}{1 - G_2 G_3 H_1 + G_3 G_4 H_2}$$

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Forward Path:

$$P_1 = G_1G_2G_3$$

$$P_2 = G_1G_4$$

Loop:

$$L_1 = H_2$$

$$L_2 = G_2G_3H_1$$

$$L_3 = G_4H_1$$

$$L_4 = -G_1G_2G_3$$

$$L_5 = -G_1G_4$$

Question 2

Use Mason's rule to determine the transfer function of the signal flow graph as shown in Figure 2 below. (Total: 12 marks)

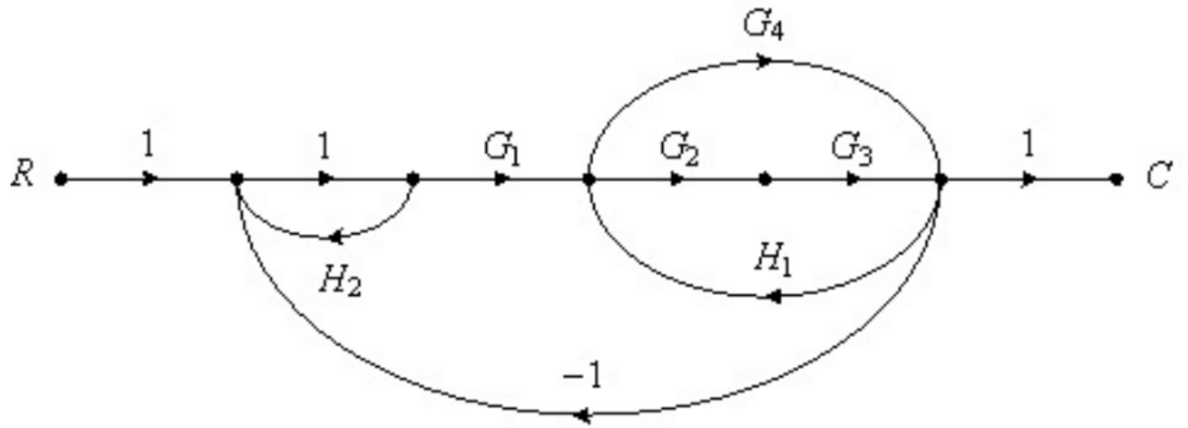


Figure 2

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1L_2 + L_1L_3)$$

$$\therefore \frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 - (H_2 + G_2G_3H_1 + G_4H_1 - G_1G_2G_3 - G_1G_4) + (G_2G_3H_1H_2 + G_4H_1H_2)}$$

Question 3 **Sem 2, 2023/24**

A unity feedback control system with proportional-derivative (PD) controller is shown in Figure 1 below.

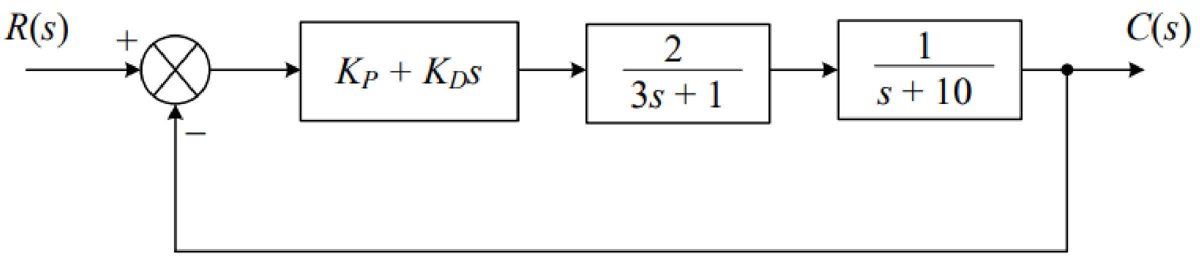


Figure 1

(a) Determine the values of K_P and K_D such that the unit-step response of the control system has a maximum overshoot (percentage) of 3% and an undamped natural frequency of 8 rad/s.
(15 marks)

(b) Hence, find the steady-state error under the condition in part (a). (5 marks)
(Total: 20 marks)

The closed-loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{(K_P + K_D s)\left(\frac{2}{3s+1}\right)\left(\frac{1}{s+10}\right)}{1 + (K_P + K_D s)\left(\frac{2}{3s+1}\right)\left(\frac{1}{s+10}\right)} = \frac{2(K_P + K_D s)}{(3s+1)(s+10) + 2(K_P + K_D s)}$$

$$= \frac{2(K_P + K_D s)}{3s^2 + (31 + 2K_D)s + (10 + 2K_P)} = \frac{\frac{2}{3}(K_P + K_D s)}{s^2 + \frac{(31 + 2K_D)s}{3} + \frac{(10 + 2K_P)}{3}}$$

Equating terms with a second-order system, $s^2 + \frac{(31 + 2K_D)s}{3} + \frac{(10 + 2K_P)}{3} = s^2 + 2\zeta\omega_n s + \omega_n^2$

Given $\omega_n = 8$ rad/s, we have $\frac{10 + 2K_P}{3} = \omega_n^2 = 8^2$

$K_P = 91$

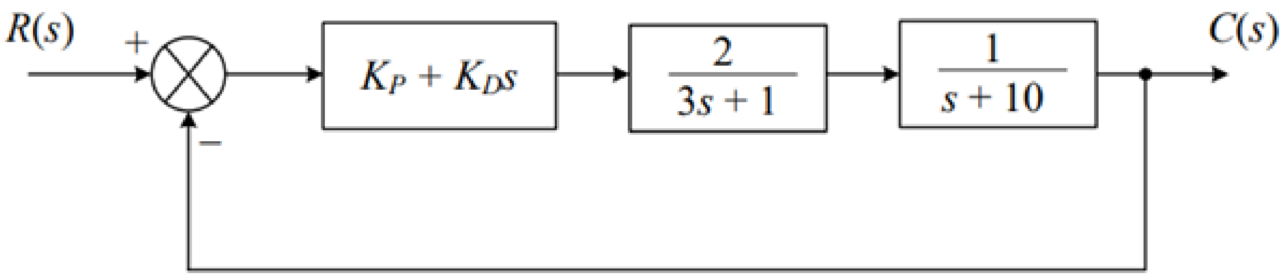
Given $M_P = 3\%$, we have

$0.03 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$$\zeta = \sqrt{\frac{(\ln M_P)^2}{\pi^2 + (\ln M_P)^2}} = \sqrt{\frac{(\ln 0.03)^2}{\pi^2 + (\ln 0.03)^2}} = 0.7448$$

$$2\zeta\omega_n = (2)(0.7448)(8) = \frac{31 + 2K_D}{3}$$

$K_D = 2.375$



$K_P = 91$

$K_D = 2.375$

Static Position Error Constant: $$K_P = \lim_{s\to 0}(91 + 2.375s)\left(\frac{2}{3s+1}\right)\left(\frac{1}{s+10}\right) = (91)\left(\frac{2}{1}\right)\left(\frac{1}{10}\right) = 18.2$$

Hence, steady-state error, $$e(\infty) = \frac{1}{1+K_P} = 0.0521$$