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SEHS4653 Control System Analysis

Past Paper Revision (Part 1)

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Sem 2, 2023/24

Question 1

Determine the impulse response of the system, $\ddot{y}(t) - \dot{y}(t) - 6y(t) = 4\delta(t)$, with zero initial condition by using Laplace Transform.

(Total: 4 marks)

Taking Laplace Transform, we have

$$s^2Y(s) - sY(s) - 6Y(s) = 4 \times 1$$

$$(s^2 - s - 6)Y(s) = 4$$

$$Y(s) = \frac{4}{(s+2)(s-3)}$$

Taking invers Laplace Transform, we have

$$\therefore y(t) = \frac{4}{-3-2}(e^{-2t} - e^{3t}) = \frac{4}{5}(e^{3t} - e^{-2t})$$

Sem 2, 2022/23

Question 1

Given the following differential equations with zero initial conditions, find $x(t)$ using Laplace Transform method.

(a) $\ddot{x}(t) + 7\dot{x}(t) + 6x(t) = 3\delta(t)$ (5 marks)

(b) $2\dot{x}(t) + x(t) = t$ (10 marks)

(Total: 15 marks)

(a) Taking Laplace Transform, we have $s^2X(s) + 7sX(s) + 6X(s) = 3 \times 1$

$$(s^2 + 7s + 6)X(s) = 3 \qquad \therefore X(s) = \frac{3}{s^2 + 7s + 6} = \frac{3}{(s+1)(s+6)}$$

Taking invers Laplace Transform, we have

$$\therefore y(t) = \frac{3}{6-1}(e^{-t} - e^{-6t}) = \frac{3}{5}(e^{-t} - e^{-6t})$$

Sem 2, 2022/23

Question 1

Given the following differential equations with zero initial conditions, find $x(t)$ using Laplace Transform method.

(a) $\ddot{x}(t) + 7\dot{x}(t) + 6x(t) = 3\delta(t)$ (5 marks)

(b) $2\dot{x}(t) + x(t) = t$ (10 marks)

(Total: 15 marks)

(b) Taking Laplace Transform, we have

$$2sX(s) + X(s) = \frac{1}{s^2}$$

$$\therefore X(s) = \frac{1}{s^2(2s+1)}$$

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(b) $X(s) = \dfrac{1}{s^2(2s+1)}$

By partial fraction, $X(s) = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{2s+1}$ $\therefore A(2s+1) + Bs(2s+1) + Cs^2 = 1$

Put $s = 0$, $A(2(0)+1) + B(0)(2(0)+1) + C(0) = 1, A = 1$

Put $s = -\frac{1}{2}$, $A\left((2)\left(-\frac{1}{2}\right)+1\right) + B\left(-\frac{1}{2}\right)\left((2)\left(-\frac{1}{2}\right)+1\right) + C\left(-\frac{1}{2}\right)^2 = 1, C = 4$

Put $s = 1$, $A(2(1)+1) + B(1)(2(1)+1) + C(1)^2 = 1, B = -2$

$$X(s) = \frac{1}{s^2} - \frac{2}{s} + \frac{4}{2s+1} = \frac{1}{s^2} - \frac{2}{s} + \frac{2}{s+\frac{1}{2}}$$

Taking inverse Laplace Transform, we have

$$x(t) = t - 2 + 2e^{-\frac{1}{2}t}$$