THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

School of Professional Education and Executive Development
專業進修學院
SPEED

SEHS4653 Control System Analysis

Laplace Transform Table

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	Time Function $f(t)$	Laplace Transform $F(s)$
1	Unit-impulse function $\delta(t)$	1
2	Unit-step function $u_s(t)$	$\frac{1}{s}$
3	Unit-ramp function t	$\frac{1}{s^2}$
4	$t^n \ \ (n = \text{positive integer})$	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{s+a}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at} \ \ (n = \text{positive integer})$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
10	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
11	$\frac{1}{b-a}(e^{-at}-e^{-bt}) \ \ (a \neq b)$	$\frac{1}{(s+a)(s+b)}$
12	$\frac{1}{b-a}(be^{-bt}-ae^{-at}) \ \ (a \neq b)$	$\frac{s}{(s+a)(s+b)}$
13	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
14	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
15	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
16	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

17	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
18	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ where $\phi = \cos^{-1} \zeta$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
19	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ where $\phi = \cos^{-1} \zeta$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
20	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
21	$\frac{d}{dt} f(t)$	$sF(s) - f(0)$
22	$\frac{d^2}{dt^2} f(t)$	$s^2 F(s) - sf(0) - f'(0)$
23	$\frac{d^n}{dt^n} f(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
24	$\int f(t)dt$	$\frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)dt\right]$
25	$f(t-T)$	$e^{-Ts} F(s)$
26	$f(\infty) = \lim_{t\to\infty} f(t)$	$= \lim_{s\to 0} sF(s)$
27	$f(0^+) = \lim_{t\to 0^+} f(t)$	$= \lim_{s\to\infty} sF(s)$