SCHOOL OF PROFESSIONAL EDUCATION AND EXECUTIVE DEVELOPMENT

Programme Title	:	Bachelor of Engineering (Honours) in Electrical Engineering (84065 & 84066)			
Subject Title	:	Control System Analysis	**Subject Code**	:	SEHS4653
Semester	:	Semester 2, 2023/24			
Date	:	16 May 2024	**Time**	:	19:30 – 21:30
Time Allowed	:	2 hours	**Subject Examiner(s)**	:	Dr Kenneth LO

This question paper has 6 pages (including this covering page).

Instructions to Candidates:

1. This paper consists of 5 questions. Answer ALL questions in the answer book provided.
2. Begin each question on a fresh page in the answer book provided.
3. Show all your workings clearly and neatly. Reasonable steps should be shown.
4. Candidates are NOT allowed to retain this paper.
5. Metric-size graph paper and semi-log graph papers are available from invigilator.
6. Laplace Transform table and useful formulae are provided on pages 4 to 6.

Authorised Materials:

	YES	NO	Remark
CALCULATOR	[✓]	[ ]	NO programme should be stored in the calculator.
SPECIFICALLY PERMITTED ITEMS	[ ]	[✓]	

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

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Answer ALL questions in the answer book provided.

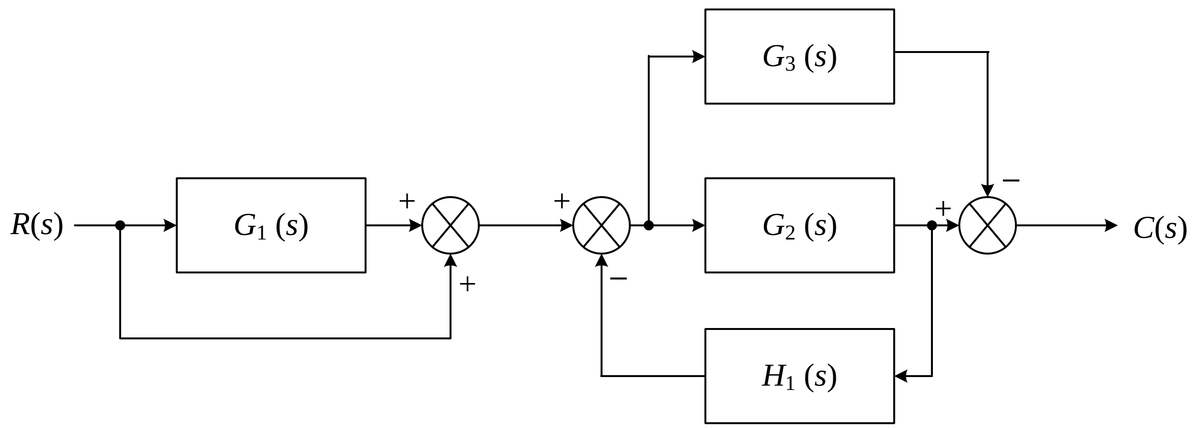
Question 1

Determine the impulse response of the system, $\ddot{y}(t) - \dot{y}(t) - 6y(t) = 4\delta(t)$, with zero initial condition by using Laplace Transform.

(Total: 4 marks)

Question 2

Reduce the following block diagram.



(Total: 14 marks)

Question 3

A unity feedback control system with proportional-derivative (PD) controller is shown in Figure 1 below.

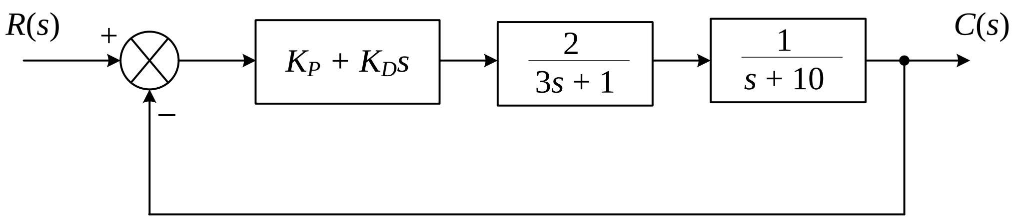


Figure 1

(a) Determine the values of K_P and K_D such that the unit-step response of the control system has a maximum overshoot (percentage) of 3% and an undamped natural frequency of 8 rad/s.

(15 marks)

(b) Hence, find the steady-state error under the condition in part (a). (5 marks)

(Total: 20 marks)

Question 4

A unity negative feedback control system (with $K > 0$) has the following open-loop transfer function,

$$G(s) = \frac{K}{(s+3)(s^2+4s+8)}.$$

(a) Use Routh–Hurwitz stability criterion to find the range of K such that the system is stable. (8 marks)

(b) Draw the root-locus of the system on the metric-size graph paper provided. Show all your steps clearly. (12 marks)

(c) Identify the location of closed-loop pole such that a desired damping ratio of 0.36 is required. (6 marks)

(d) Determine the peak and rise time of the system under unit-step input based on the result obtained in part (c). (6 marks)

(Total: 32 marks)

Question 5

A series phase-lead compensator, $G_c(s)$, is used to control the position of a servo motor as shown in Figure 2 below.

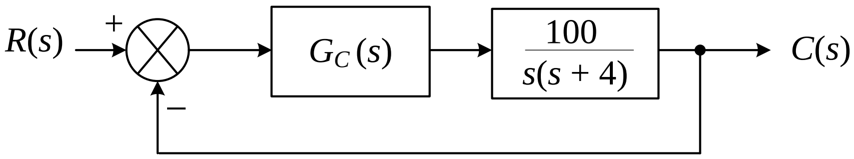


Figure 2

(a) Find the value of the gain, $K_C\alpha$, of the compensator such that the system has statics velocity error constant of 10 sec^{-1}. (5 marks)

(b) With the result obtained in part (a), plot the exact Bode diagrams at angular frequencies of 1 rad/s, 2 rad/s, 5 rad/s, 8 rad/s, and 10 rad/s for $G_1(s) = K_c\alpha\frac{100}{s(s+4)}$. (15 marks)

(c) Hence, find the transfer function of the phase-lead compensator such that the phase margin is at least 60°. (10 marks)

(Total: 30 marks)

Appendix I: Laplace Transform Table

	Time Function $f(t)$	Laplace Transform $F(s)$
1	Unit-impulse function $\delta(t)$	1
2	Unit-step function $u_s(t)$	$\frac{1}{s}$
3	Unit-ramp function *t*	$\frac{1}{s^2}$
4	t^n $(n =$ positive integer$)$	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{s+a}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}$ $(n =$ positive integer$)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
10	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
11	$\frac{1}{b-a}(e^{-at}-e^{-bt})$ $(a \neq b)$	$\frac{1}{(s+a)(s+b)}$
12	$\frac{1}{b-a}(be^{-bt}-ae^{-at})$ $(a \neq b)$	$\frac{s}{(s+a)(s+b)}$
13	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
14	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
15	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
16	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
17	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

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18	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ where $\phi=\cos^{-1}\zeta$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
19	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ where $\phi=\cos^{-1}\zeta$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
20	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
21	$\frac{d}{dt}f(t)$	$sF(s)-f(0)$
22	$\frac{d^2}{dt^2}f(t)$	$s^2F(s)-sf(0)-f'(0)$
23	$\frac{d^n}{dt^n}f(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots-sf^{(n-2)}(0)-f^{(n-1)}(0)$
24	$\int f(t)dt$	$\frac{F(s)}{s}+\frac{1}{s}\left[\int f(t)dt\right]$
25	$f(t-T)$	$e^{-Ts}F(s)$
26	$f(\infty)=\lim_{t\to\infty}f(t)$	$=\lim_{s\to 0}sF(s)$
27	$f(0^+)=\lim_{t\to 0^+}f(t)$	$=\lim_{s\to\infty}sF(s)$

Appendix II: Useful Formulae

Rise Time: $$t_r = \frac{\pi - \beta}{\omega_d}$$

Peak Time: $$t_p = \frac{\pi}{\omega_d}$$

Maximum Overshoot: $$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

Settling Time (2%): $$t_s = \frac{4}{\zeta\omega_n}$$

Settling Time (5%): $$t_s = \frac{3}{\zeta\omega_n}$$

Damped Natural Frequency: $$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

<u>Series-lead Compensator:</u>

Transfer Function: $$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Maximum Phase Lead: $$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

New Gain Crossover Frequency and its Corresponding Gain: $$\omega_m = \frac{1}{\sqrt{\alpha}T} \quad \text{and} \quad |G_1(j\omega)| = -20\log\frac{1}{\sqrt{\alpha}}$$

<u>Series-lag Compensator:</u>

Transfer Function: $$G_c(s) = K_c\beta\frac{Ts+1}{\beta Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

At New Gain Crossover Frequency: $$|G_c(j\omega)| = 20\log\beta$$

- END OF PAPER -

The following academic staff have been involved in the preparation of this examination paper:

Subject Lecturer(s)

Name : **Dr Kenneth LO** **Signature** :

Name : ____________ **Signature** : ____________

Name : ____________ **Signature** : ____________

Subject Leader (if applicable)

Name : **Dr Kenneth LO** **Signature** :

Please ensure that the marking scheme is sent together with the examination paper.

Each student will receive ONE answer book (with 15 pages), please check box if the following arrangement(s) or additional item(s) is/are needed during the examination:

☑ NOT allow students to retain the question papers
❑ Multiple-choice Answer Sheet (no. of sheets per student: ______)
☑ Graph Paper (no. of pieces per student: ___1___)
❑ Single Line Paper (no. of pieces per student: ______)
❑ Additional Answer Book(s) (no. of additional answer book required per student: ______)
❑ Tailor-made answer sheets. Answer books NOT required.
☑ Others (please specify: ____Semi-log graph paper (at least 4 cycles): 2 pieces per student)

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