SCHOOL OF PROFESSIONAL EDUCATION AND EXECUTIVE DEVELOPMENT

Programme Title	:	Bachelor of Engineering (Honours) in Electrical Engineering (84065 & 84066)			
Subject Title	:	Control System Analysis	**Subject Code**	:	SEHS4653
Semester	:	Semester 1, 2023/24			
Date	:	11 December 2023	**Time**	:	19:30 – 21:30
Time Allowed	:	2 hours	**Subject Examiner(s)**	:	Dr Kenneth LO

This question paper has 6 pages (including this covering page).

Instructions to Candidates:

1. This paper consists of 5 questions. Answer ALL questions in the answer book provided.
2. Begin each question on a fresh page in the answer book provided.
3. Show all your workings clearly and neatly. Reasonable steps should be shown.
4. Candidates are NOT allowed to retain this paper.
5. Metric-size graph paper and semi-log graph papers are available from invigilator.
6. Laplace Transform table is provided on pages 6 to 7.

Authorised Materials:

	YES	NO	Remark
CALCULATOR	[✓]	[ ]	NO programme should be stored in the calculator.
SPECIFICALLY PERMITTED ITEMS	[ ]	[✓]	

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

Answer ALL questions in the answer book provided.

Question 1

The differential equation of a control system is, $\ddot{y}(t) + 7\dot{y}(t) + 10y(t) = 2r(t)$ where $r(t)$ and $y(t)$ are the system input and output respectively.

(a) (i) Determine the unit-impulse response of the system with zero initial condition by using Laplace transform. (5 marks)

(ii) Comment on the stability of the system under unit-impulse input. [Hint: Use final value theorem.] (3 marks)

(b) The above differential equation can also be described by the following state-space model,

$$\dot{x}(t) = Ax(t) + Br(t)$$
$$y(t) = Cx(t)$$

(i) Find the matrices A and B if $C = [1 \quad 0]$. (6 marks)

(ii) Hence, find the state transition matrix in s-domain. (4 marks)

(Total: 18 marks)

Question 2

Use Mason's rule to find the transfer function, $G = \frac{C}{R}$, for the following signal flow graph.

(Total: 10 marks)

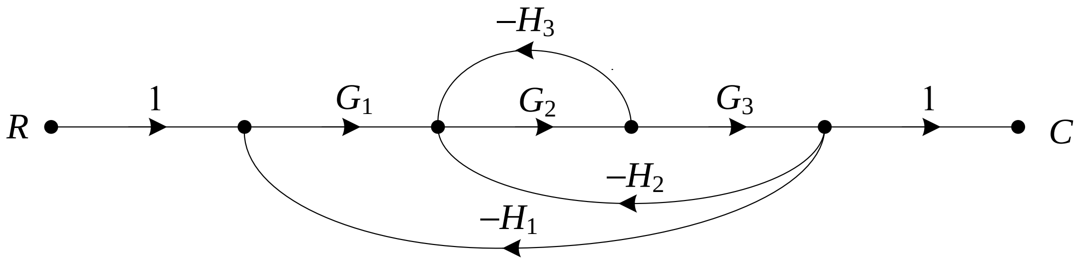

Figure Q2

Question 3

A unity feedback control system with rate-feedback loop is shown in Figure Q3 below.

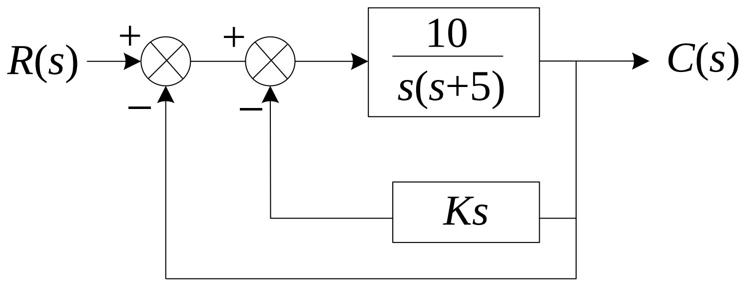

Figure Q3

(a) In the absence of the rate-feedback (i.e. $K = 0$), determine the maximum overshoot (%) under unit-step input, and steady-state error under unit-ramp input. (10 marks)

(b) Compute the rate-feedback constant (K) if the maximum overshoot is now reduced to 1% under unit-step input. Hence, find the steady-state error under unit-ramp input. (14 marks)

(Total: 24 marks)

Question 4

A unity negative feedback control system (with $K > 0$) has the following open-loop transfer function,

$$G(s) = \frac{K}{(s+1)(s+2)(s+4)}.$$

(a) Use Routh–Hurwitz stability criterion to find the range of K such that the system is stable. (8 marks)

(b) Draw the root-locus of the system on the metric-size graph paper provided. Show all your steps clearly. (14 marks)

(c) Identify the location of closed-loop pole such that a desired damping ratio of 0.5 is required. (6 marks)

(Total: 28 marks)

Question 5

A series phase-lag compensator is added in the forward path of the position control system, as shown in Figure Q5 below, to enhance the stability of the system. The transfer function of the compensated system is listed as,

$$G_c(s)G(s) = \left(\frac{s+0.4}{s+4}\right)\left(\frac{100}{s(s+3)(s+5)}\right).$$

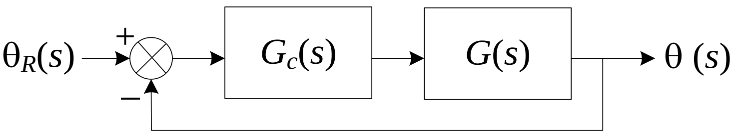

Figure Q5

Given the following open-loop frequency response of the compensated system ($G_c(s)G(s)$).

Table 1

$\boldsymbol{\omega}$ (rad/ s)	Gain (dB)	Phase (°)
0.2	11.4	–72.4
0.4	7.3	–62.9
0.8	4.8	–61.9
1	4.2	–65.6
3	–1.8	–120.4
5	–8.4	–160
8	–17.1	–193.7
10	–22	–207.2
20	–38.6	–237.3

(a) Plot the Bode diagrams of the compensated system in the semi-log graph paper provided. (8 marks)

(b) Hence, find the gain margin and phase margin of the compensated system. (4 marks)

(c) Identify the new gain margin and phase margin if a gain of magnitude 2 is added before $G_c(s)$. How will be the system stability affected? (8 marks)

(Total: 20 marks)

Appendix I: Laplace Transform Table

	Time Function $f(t)$	Laplace Transform $F(s)$
1	Unit-impulse function $\delta(t)$	1
2	Unit-step function $u_s(t)$	$\frac{1}{s}$
3	Unit-ramp function t	$\frac{1}{s^2}$
4	$t^n \ \ (n = \text{positive integer})$	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{s+a}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at} \ \ (n = \text{positive integer})$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
10	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
11	$\frac{1}{b-a}(e^{-at}-e^{-bt}) \ \ (a \neq b)$	$\frac{1}{(s+a)(s+b)}$
12	$\frac{1}{b-a}(be^{-bt}-ae^{-at}) \ \ (a \neq b)$	$\frac{s}{(s+a)(s+b)}$
13	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
14	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
15	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
16	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
17	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

18	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ where $\phi=\cos^{-1}\zeta$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
19	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ where $\phi=\cos^{-1}\zeta$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
20	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
21	$\frac{d}{dt}f(t)$	$sF(s)-f(0)$
22	$\frac{d^2}{dt^2}f(t)$	$s^2F(s)-sf(0)-f'(0)$
23	$\frac{d^n}{dt^n}f(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots$ $-sf^{(n-2)}(0)$ $-f^{(n-1)}(0)$
24	$\int f(t)dt$	$\frac{F(s)}{s}+\frac{1}{s}\left[\int f(t)dt\right]$
25	$f(t-T)$	$e^{-Ts}F(s)$
26	$f(\infty)=\lim_{t\to\infty}f(t)$	$=\lim_{s\to 0}sF(s)$
27	$f(0^+)=\lim_{t\to 0^+}f(t)$	$=\lim_{s\to\infty}sF(s)$

- END OF PAPER -

The following academic staff have been involved in the preparation of this examination paper:

Subject Lecturer(s)

Name : **Dr Kenneth LO** **Signature** : ____________

Name : ____________ **Signature** : ____________

Name : ____________ **Signature** : ____________

Subject Leader (if applicable)

Name : **Dr Kenneth LO** **Signature** : ____________

Please ensure that the marking scheme is sent together with the examination paper.

Each student will receive ONE answer book (with 15 pages), please check box if the following arrangement(s) or additional item(s) is/are needed during the examination:

☑ NOT allow students to retain the question papers
❑ Multiple-choice Answer Sheet (no. of sheets per student: _______)
☑ Graph Paper (no. of pieces per student: ___1___)
❑ Single Line Paper (no. of pieces per student: _______)
❑ Additional Answer Book(s) (no. of additional answer book required per student: _______)
❑ Tailor-made answer sheets. Answer books NOT required.
☑ Others (please specify: ____Semi-log graph paper (at least 4 cycles): 4 pieces per student)

Note: The examination paper will be sent to PolyU and CPCE Libraries for display after the examination. Multiple-choice/ True or False questions, if any, will not be provided to the libraries.

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