Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-1

Balance Three Phase Circuits

Contents

Reference:

Introductory Circuit Analysis 14th edition, Boylesad & Olivari

Basic Circuit Analysis – Schaum's Outline Series

Email: norbertcheung@szu.edu.cn

Web Site: http://norbert.idv.hk

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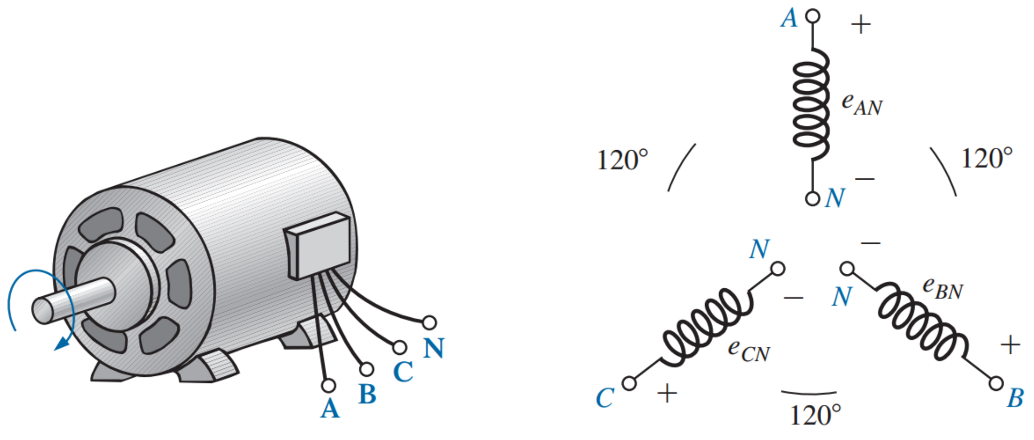
1. Introduction

Advantages Associated with a Three-Phase Power Distribution over a Single-Phase System

1. *Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less) and in turn reduces construction and maintenance costs.*
2. *The lighter power lines are easier to install, and the supporting structures can be less massive and farther apart.*
3. *Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.*
4. *In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.*

2. 3 Phase Generator

The three-phase generator in Fig. 24.1(a) has three induction coils placed 120° apart on the stator, as shown symbolically by Fig. 24.1(b). Since the three coils have an equal number of turns, and each coil rotates with the same angular velocity, the voltage induced across each coil has the same peak value, shape, and frequency. As the shaft of the generator is turned by some external means, the induced voltages e_{AN}, e_{BN}, and e_{CN} are generated simultaneously, as shown in Fig. 24.2. Note the 120° phase shift between waveforms and the similarities in appearance of the three sinusoidal functions.



(a) (b)

FIG. 24.1
(a) Three-phase generator; (b) induced voltages of a three-phase generator.

at any instant of time, the algebraic sum of the three-phase voltages of a three-phase generator is zero.

This is shown at $\omega t = 0$ in Fig. 24.2, where it is also evident that *when one induced voltage is zero, the other two are 86.6% of their positive or negative maximums. In addition, when any two are equal in magnitude and sign (at $0.5E_m$), the remaining induced voltage has the opposite polarity and a peak value.*

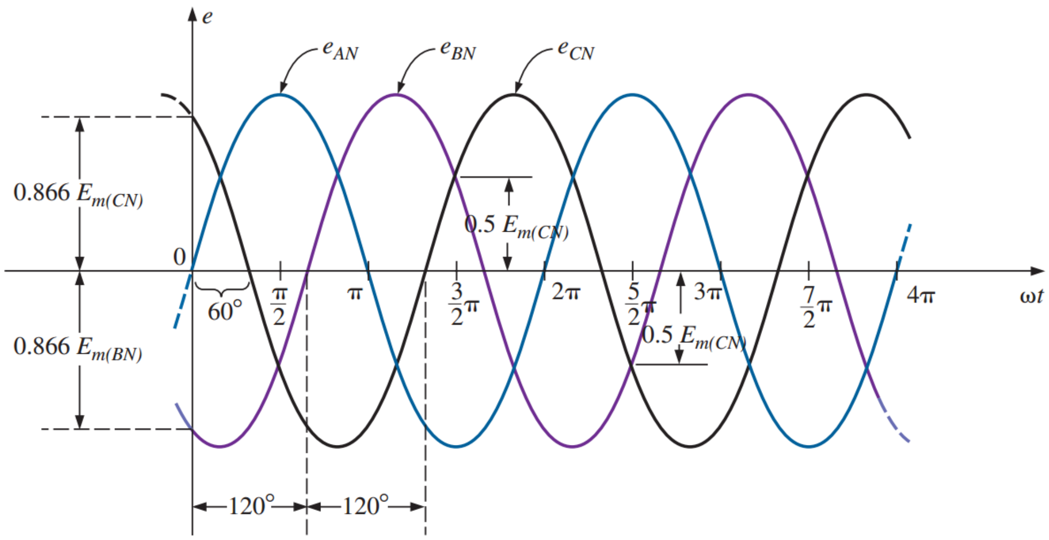


FIG. 24.2
Phase voltages of a three-phase generator.

The respective sinusoidal expressions for the induced voltages in Fig. 24.2 are

$$\begin{aligned} e_{AN} &= E_{m(AN)} \sin \omega t \\ e_{BN} &= E_{m(BN)} \sin(\omega t - 120°), \\ e_{CN} &= E_{m(CN)} \sin(\omega t - 240°) = E_{m(CN)} \sin(\omega t + 120°) \end{aligned} \quad \textbf{(24.1)}$$

The phasor diagram of the induced voltages is shown in Fig. 24.3, where the effective value of each is determined by

$$E_{AN} = 0.707 E_{m(AN)}$$
$$E_{BN} = 0.707 E_{m(BN)}$$
$$E_{CN} = 0.707 E_{m(CN)}$$

and

$$\begin{aligned} \mathbf{E}_{AN} &= E_{AN} \angle 0° \\ \mathbf{E}_{BN} &= E_{BN} \angle -120° \\ \mathbf{E}_{CN} &= E_{CN} \angle +120° \end{aligned} \quad \textbf{(24.2)}$$

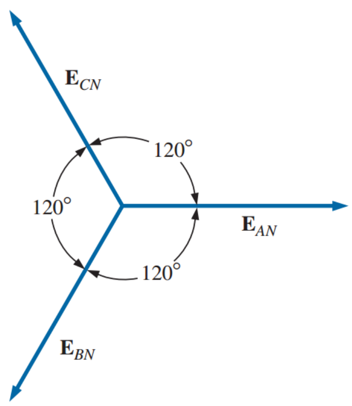


FIG. 24.3
Phasor diagram for the phase voltages of a three-phase generator.

3. Y Generator

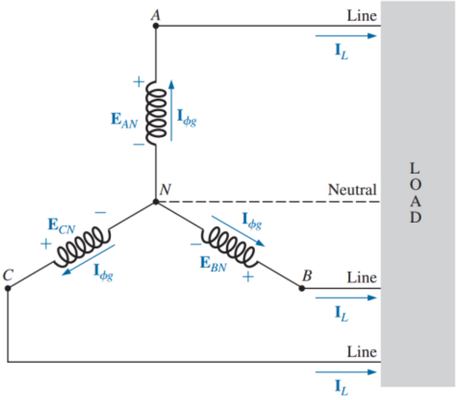


FIG. 24.4
Y-connected generator.

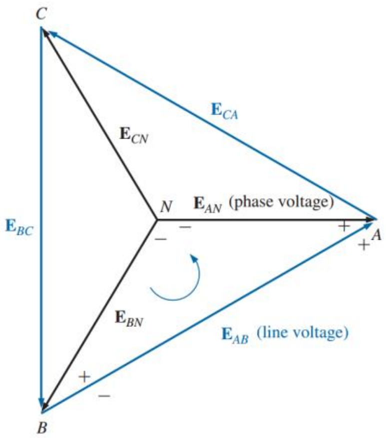


FIG. 24.5
Line and phase voltages of the Y-connected three-phase generator.

The point at which all the terminals are connected is called the *neutral point*. If a conductor is not attached from this point to the load, the system is called a *Y-connected, three-phase, three-wire generator.* If the neutral is connected, the system is a *Y-connected, three-phase, four-wire generator.* The function of the neutral will be discussed in detail when we consider the load circuit.

The three conductors connected from *A*, *B*, and *C* to the load are called *lines*. For the Y-connected system, it should be obvious from Fig. 24.4 that the **line current** equals the **phase current** for each phase; that is,

$$\mathbf{I}_L = \mathbf{I}_{\phi g} \tag{24.3}$$

where ϕ is used to denote a phase quantity, and g is a generator parameter.

The voltage from one line to another is called a **line voltage**. On the phasor diagram (Fig. 24.5), it is the phasor drawn from the end of one phase to another in the counterclockwise direction.

Applying Kirchhoff's voltage law around the indicated loop in Fig. 24.5, we obtain

$$\mathbf{E}_{AB} - \mathbf{E}_{AN} + \mathbf{E}_{BN} = 0$$

or

$$\mathbf{E}_{AB} = \mathbf{E}_{AN} - \mathbf{E}_{BN}$$

Sketching $-\mathbf{E}_{BN}$ as shown in Fig. 24.6 we can then vectorially add $\mathbf{E}_{AN}$ and $-\mathbf{E}_{BN}$ to obtain $\mathbf{E}_{AB}$ as shown in the figure.

The result is that the dimension

$$x = E_{AN} \cos 30° = \frac{\sqrt{3}}{2} E_{AN}$$

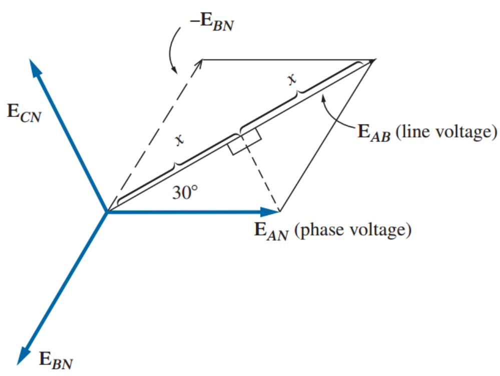


FIG. 24.6
Determining the relationship between line and phase voltages.

and $$E_{AB} = 2x = (2)\frac{\sqrt{3}}{2}E_{AN} = \sqrt{3}E_{AN}$$

with $$\mathbf{E}_{AB} = \sqrt{3}E_{AN}\angle 30^\circ$$

Similarly $$\mathbf{E}_{CA} = \sqrt{3}E_{CN}\angle 150^\circ$$

and $$\mathbf{E}_{BC} = \sqrt{3}E_{BN}\angle 270^\circ$$

In words, the magnitude of the line voltage of a balanced Y-connected generator is $\sqrt{3}$ times the phase voltage:

$$E_L = \sqrt{3}E_\phi \qquad \textbf{(24.4)}$$

with the phase angle between any line voltage and the nearest phase voltage at 30°.

In sinusoidal notation,

$$e_{AB} = \sqrt{2}E_{AB}\sin(\omega t + 30^\circ)$$

$$e_{CA} = \sqrt{2}E_{CA}\sin(\omega t + 150^\circ)$$

and $$e_{BC} = \sqrt{2}E_{BC}\sin(\omega t + 270^\circ)$$

Phase Sequence

The **phase sequence** can be determined by the order in which the phasors representing the phase voltages pass through a fixed point on the phasor diagram if the phasors are rotated in a counterclockwise direction. For example, in Fig. 24.7 the phase sequence is *ABC*. However, since

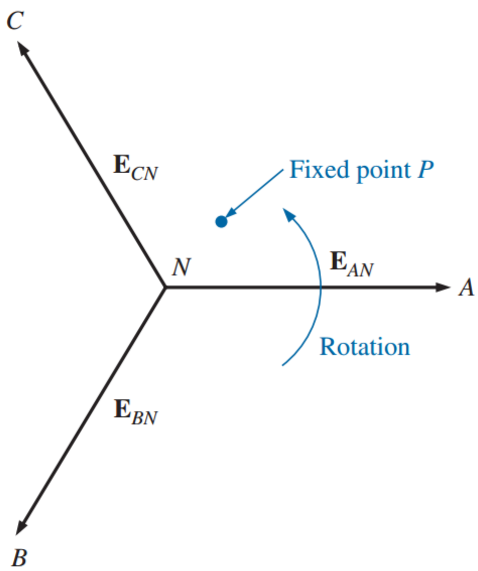


FIG. 24.7
Determining the phase sequence from the phase voltages of a three-phase generator.

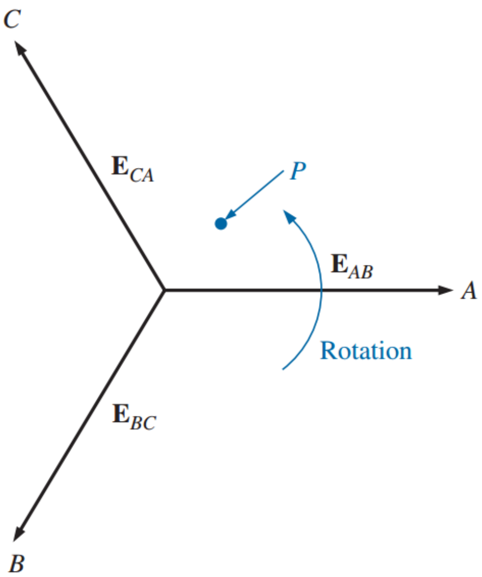


FIG. 24.8
Determining the phase sequence from the line voltages of a three-phase generator.

the fixed point can be chosen anywhere on the phasor diagram, the sequence can also be written as *BCA* or *CAB*. The phase sequence is quite important in the three-phase distribution of power. In a three-phase motor, for example, if two-phase voltages are interchanged, the sequence will change, and the direction of rotation of the motor will be reversed. Other effects will be described when we consider the loaded three-phase system.

If the sequence is given, the phasor diagram can be drawn by simply picking a reference voltage, placing it on the reference axis, and then drawing the other voltages at the proper angular position. For a sequence of *ACB*, for example, we might choose E_{AB} to be the reference Fig. 24.9(a) if we wanted the phasor diagram of the line voltages, or E_{AN} for the phase voltages Fig. 24.9(b). For the sequence indicated, the phasor diagrams would be as in Fig. 24.9. In phasor notation,

$$\text{Line voltages}\ \begin{cases} \mathbf{E}_{AB} = E_{AB}\angle 0^\circ & \text{(reference)} \\ \mathbf{E}_{CA} = E_{CA}\angle -120^\circ \\ \mathbf{E}_{BC} = E_{BC}\angle +120^\circ \end{cases}$$

$$\text{Phase voltages}\ \begin{cases} \mathbf{E}_{AN} = E_{AB}\angle 0^\circ & \text{(reference)} \\ \mathbf{E}_{CN} = E_{CN}\angle -120^\circ \\ \mathbf{E}_{BN} = E_{BN}\angle +120^\circ \end{cases}$$

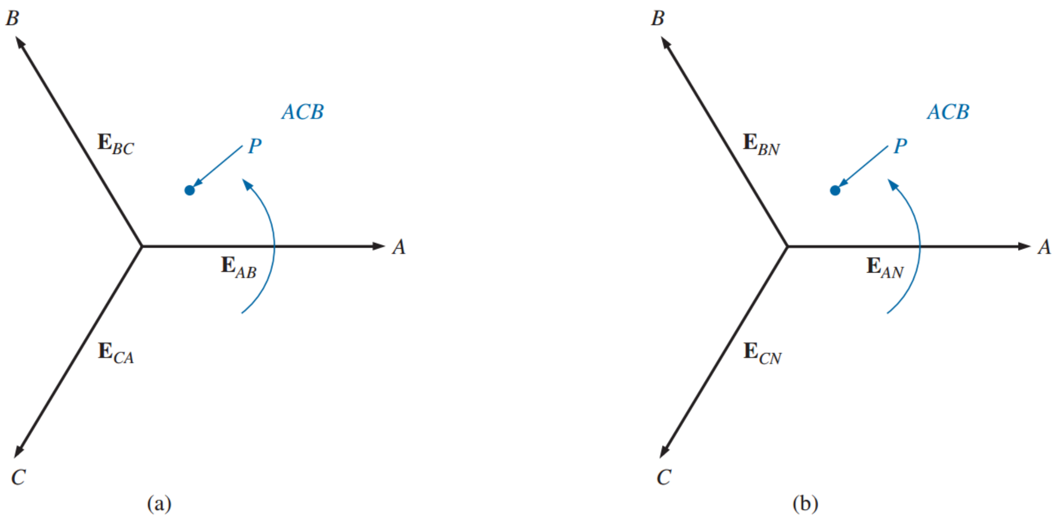


FIG. 24.9
Drawing the phasor diagram from the phase sequence.

4. Y-Y and Y-Δ systems

Y-Y connection

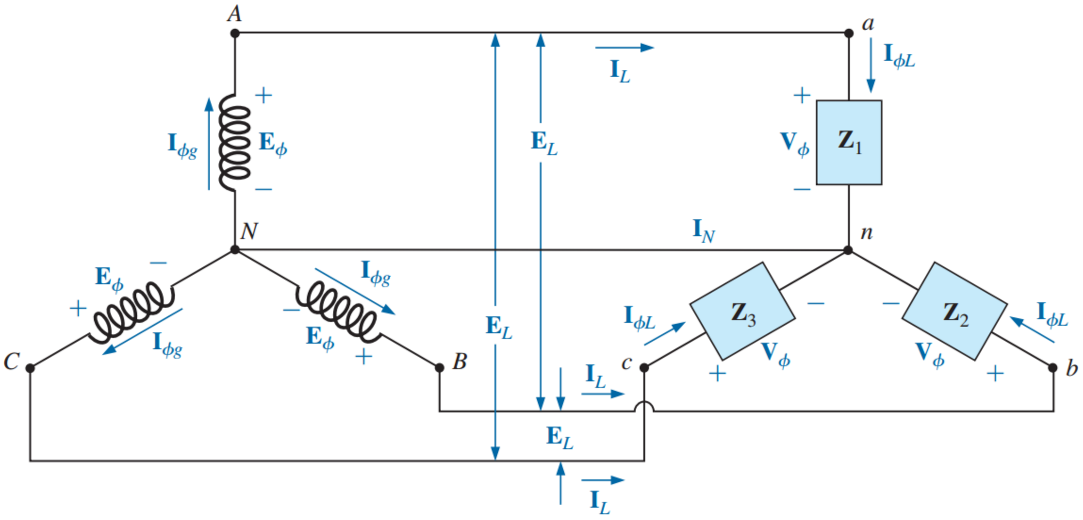


FIG. 24.10
Y-connected generator with a Y-connected load.

If the load is balanced, the **neutral connection** can be removed without affecting the circuit in any manner; that is, if

$$Z_1 = Z_2 = Z_3$$

We shall now examine the *four-wire Y-Y-connected system.* The current passing through each phase of the generator is the same as its corresponding line current, which in turn for a Y-connected load is equal to the current in the phase of the load to which it is attached:

$$\mathbf{I}_{\phi g} = \mathbf{I}_L = \mathbf{I}_{\phi L} \qquad (24.5)$$

For a balanced or an unbalanced load, since the generator and load have a common neutral point, then

$$\mathbf{V}_\phi = \mathbf{E}_\phi \qquad (24.6)$$

In addition, since $\mathbf{I}_{\phi L} = \mathbf{V}_\phi / \mathbf{Z}_\phi$, the magnitude of the current in each phase is equal for a balanced load and unequal for an unbalanced load. Recall that for the Y-connected generator, the magnitude of the line voltage is equal to $\sqrt{3}$ times the phase voltage. This same relationship can be applied to a balanced or an unbalanced four-wire Y-connected load:

$$E_L = \sqrt{3} V_\phi \qquad (24.7)$$

EXAMPLE 24.1 The phase sequence of the Y-connected generator in Fig. 24.11 is *ABC*.

a. Find the phase angles θ_2 and θ_3.
b. Find the magnitude of the line voltages.
c. Find the line currents.
d. Verify that, since the load is balanced, $\mathbf{I}_N = 0$.

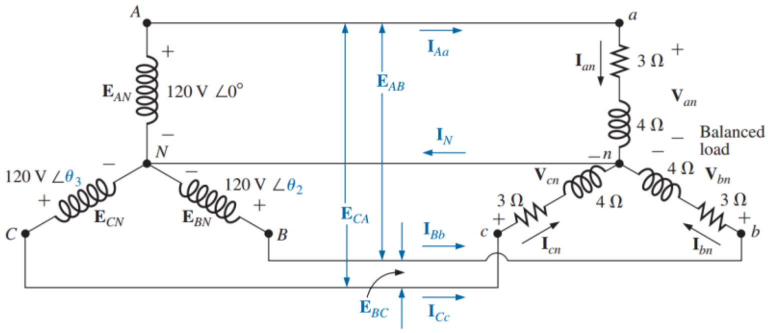


FIG. 24.11
Example 24.1.

Solutions:

a. For an *ABC* phase sequence,

$$\theta_2 = \mathbf{-120^\circ} \quad \text{and} \quad \theta_3 = \mathbf{+120^\circ}$$

b. $E_L = \sqrt{3}E_\phi = (1.73)(120\text{ V}) = 208\text{ V}$. Therefore,

$$E_{AB} = E_{BC} = E_{CA} = \mathbf{208\,V}$$

c. $\mathbf{V}_\phi = \mathbf{E}_\phi$. Therefore,

$$\mathbf{V}_{an} = \mathbf{E}_{AN} \qquad \mathbf{V}_{bn} = \mathbf{E}_{BN} \qquad \mathbf{V}_{cn} = \mathbf{E}_{CN}$$

$$\mathbf{I}_{\phi L} = \mathbf{I}_{an} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{an}} = \frac{120\text{ V}\angle 0^\circ}{3\,\Omega + j4\,\Omega} = \frac{120\text{ V}\angle 0^\circ}{5\,\Omega\angle 53.13^\circ}$$
$$= 24\text{ A}\angle -53.13^\circ$$

$$\mathbf{I}_{bn} = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{bn}} = \frac{120\text{ V}\angle -120^\circ}{5\,\Omega\angle 53.13^\circ} = 24\text{ A}\angle -173.13^\circ$$

$$\mathbf{I}_{cn} = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{cn}} = \frac{120\text{ V}\angle +120^\circ}{5\,\Omega\angle 53.13^\circ} = 24\text{ A}\angle 66.87^\circ$$

and, since $\mathbf{I}_L = \mathbf{I}_{\phi L}$,

$$\mathbf{I}_{Aa} = \mathbf{I}_{an} = \mathbf{24\,A\angle -53.13^\circ}$$
$$\mathbf{I}_{Bb} = \mathbf{I}_{bn} = \mathbf{24\,A\angle -173.13^\circ}$$
$$\mathbf{I}_{Cc} = \mathbf{I}_{cn} = \mathbf{24\,A\angle 66.87^\circ}$$

d. Applying Kirchhoff's current law, we have

$$\mathbf{I}_N = \mathbf{I}_{Aa} + \mathbf{I}_{Bb} + \mathbf{I}_{Cc}$$

In rectangular form,

$$\begin{aligned}
\mathbf{I}_{Aa} &= 24\text{ A}\angle -53.13^\circ &&= 14.40\text{ A} - j19.20\text{ A}\\
\mathbf{I}_{Bb} &= 24\text{ A}\angle -173.13^\circ &&= -22.83\text{ A} - j2.87\text{ A}\\
\mathbf{I}_{Cc} &= 24\text{ A}\angle 66.87^\circ &&= \underline{9.43\text{ A} + j22.07\text{ A}}\\
\Sigma(\mathbf{I}_{Aa} + \mathbf{I}_{Bb} + \mathbf{I}_{Cc}) &&&= 0 + j0
\end{aligned}$$

and $\mathbf{I}_N$ is in fact equals to **zero,** as required for a balanced load.

Y-Δ connection

There is no neutral connection for the Y-Δ system in Fig. 24.12. Any variation in the impedance of a phase that produces an unbalanced system simply varies the line and phase currents of the system.

For a balanced load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 \tag{24.8}$$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$\mathbf{V}_\phi = \mathbf{E}_L \tag{24.9}$$

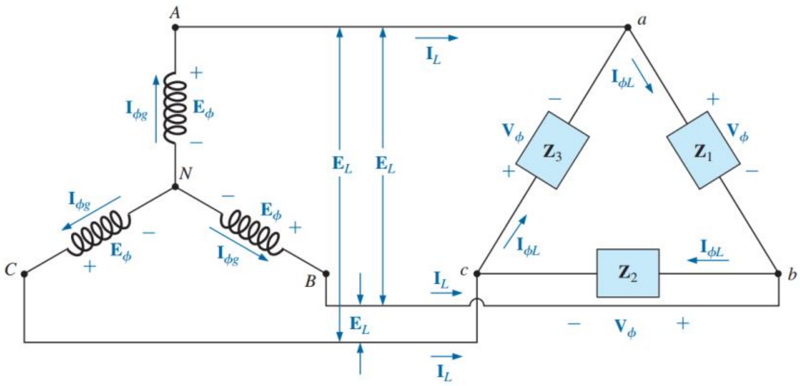


FIG. 24.12
Y-connected generator with a Δ-connected load.

The relationship between the line currents and phase currents of a balanced Δ load can be found using an approach very similar to that used in Section 24.3 to find the relationship between the line voltages and phase voltages of a Y-connected generator. For this case, however, Kirchhoff's current law is used instead of Kirchhoff's voltage law.

The result is

$$I_L = \sqrt{3} I_\phi \tag{24.10}$$

EXAMPLE 24.2 For the three-phase system in Fig. 24.13:

a. Find the phase angles θ_2 and θ_3.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.

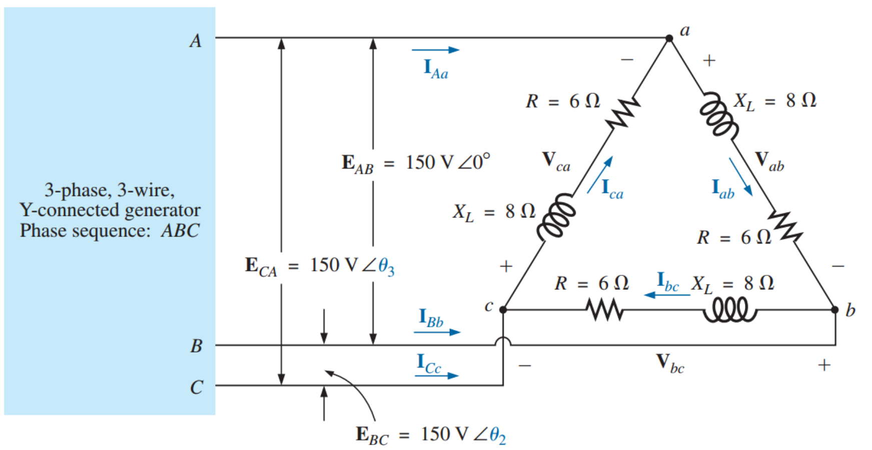


FIG. 24.13

Solutions:

a. For an *ABC* sequence,

$$\theta_2 = -120° \quad \text{and} \quad \theta_3 = +120°$$

b. $\mathbf{V}_\phi = \mathbf{E}_L$. Therefore,

$$\mathbf{V}_{ab} = \mathbf{E}_{AB} \qquad \mathbf{V}_{ca} = \mathbf{E}_{CA} \qquad \mathbf{V}_{bc} = \mathbf{E}_{BC}$$

The phase currents are

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{150 \text{ V} \angle 0°}{6\ \Omega + j8\ \Omega} = \frac{150 \text{ V} \angle 0°}{10\ \Omega \angle 53.13°} = \mathbf{15\ A \angle -53.13°}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \frac{150 \text{ V} \angle -120°}{10\ \Omega \angle 53.13°} = \mathbf{15\ A \angle -173.13°}$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \frac{150 \text{ V} \angle +120°}{10\ \Omega \angle 53.13°} = \mathbf{15\ A \angle 66.87°}$$

c. $I_L = \sqrt{3} I_\phi = (1.73)(15 \text{ A}) = 25.95$ A. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = \mathbf{25.95\ A}$$

5. Δ Generator

If we rearrange the coils of the generator in Fig. 24.14(a) as shown in Fig. 24.14(b), the system is referred to as a *three-phase, three-wire,* **Δ-connected ac generator**. In this system, the phase and line voltages are equivalent and equal to the voltage induced across each coil of the generator; that is,

$$\left.\begin{aligned} \mathbf{E}_{AB} &= \mathbf{E}_{AN} \quad \text{and} \quad e_{AN} = \sqrt{2}E_{AN}\sin\omega t \\ \mathbf{E}_{BC} &= \mathbf{E}_{BN} \quad \text{and} \quad e_{BN} = \sqrt{2}E_{BN}\sin(\omega t - 120°) \\ \mathbf{E}_{CA} &= \mathbf{E}_{CN} \quad \text{and} \quad e_{CN} = \sqrt{2}E_{CN}\sin(\omega t + 120°) \end{aligned}\right\} \text{Phase sequence } ABC$$

or

$$\mathbf{E}_L = \mathbf{E}_{\phi g} \tag{24.11}$$

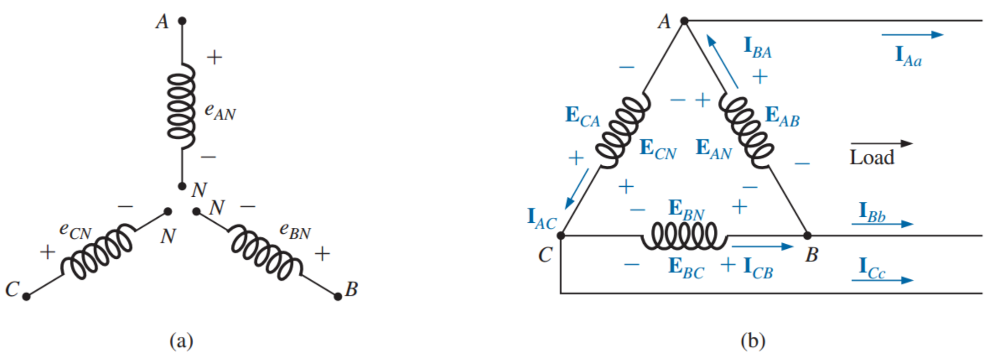


FIG. 24.14
Δ-connected generator.

$$\mathbf{I}_{BA} = \mathbf{I}_{Aa} + \mathbf{I}_{AC}$$

or

$$\mathbf{I}_{Aa} = \mathbf{I}_{BA} - \mathbf{I}_{AC} = \mathbf{I}_{BA} + \mathbf{I}_{CA}$$

Using the same procedure to find the line current as was used to find the line voltage of a Y-connected generator produces the following:

$$\mathbf{I}_{Aa} = \sqrt{3}I_{BA} \angle -30°$$
$$\mathbf{I}_{Bb} = \sqrt{3}I_{CB} \angle -150°$$
$$\mathbf{I}_{Cc} = \sqrt{3}I_{AC} \angle 90°$$

In general,

$$I_L = \sqrt{3}I_{\phi g} \tag{24.12}$$

with the phase angle between a line current and the nearest phase current at 30°.

Phase Sequence

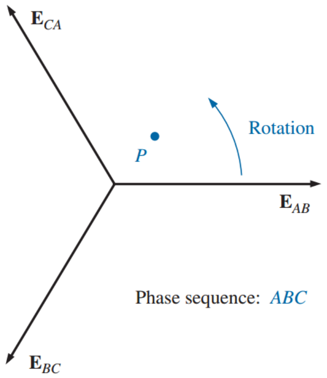


FIG. 24.15

24.8 PHASE SEQUENCE (Δ-CONNECTED GENERATOR)

Even though the line and phase voltages of a Δ-connected system are the same, it is standard practice to describe the phase sequence in terms of the line voltages. The method used is the same as that described for the line voltages of the Y-connected generator. For example, the phasor diagram of the line voltages for a phase sequence *ABC* is shown in Fig. 24.15. In drawing such a diagram, one must take care to have the sequence of the first and second subscripts the same. In phasor notation,

$$\mathbf{E}_{AB} = E_{AB} \angle 0°$$
$$\mathbf{E}_{BC} = E_{BC} \angle -120°$$
$$\mathbf{E}_{CA} = E_{CA} \angle 120°$$

6. Δ-Δ and Δ-Y systems

EXAMPLE 24.3 For the Δ-Δ system shown in Fig. 24.16:

a. Find the phase angles θ_2 and θ_3 for the specified phase sequence.
b. Find the current in each phase of the load.
c. Find the magnitude of the line currents.

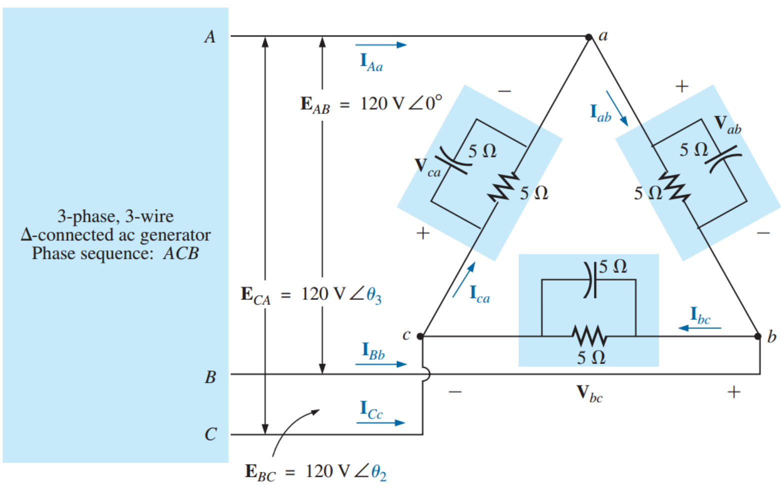


FIG. 24.16

Solutions:

a. For an *ACB* phase sequence,

$$\theta_2 = \mathbf{120°} \qquad \text{and} \qquad \theta_3 = \mathbf{-120°}$$

b. $\mathbf{V}_\phi = \mathbf{E}_L$. Therefore,

$$\mathbf{V}_{ab} = \mathbf{E}_{AB} \qquad \mathbf{V}_{ca} = \mathbf{E}_{CA} \qquad \mathbf{V}_{bc} = \mathbf{E}_{BC}$$

The phase currents are

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{120 \text{ V} \angle 0°}{\dfrac{(5\ \Omega \angle 0°)(5\ \Omega \angle -90°)}{5\ \Omega - j5\ \Omega}} = \frac{120 \text{ V} \angle 0°}{\dfrac{25\ \Omega \angle -90°}{7.071 \angle -45°}}$$

$$= \frac{120 \text{ V} \angle 0°}{3.54\ \Omega \angle -45°} = \mathbf{33.9 \angle 45°}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \frac{120 \text{ V} \angle 120°}{3.54\ \Omega \angle -45°} = \mathbf{33.9\ A \angle 165°}$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \frac{120 \text{ V} \angle -120°}{3.54\ \Omega \angle -45°} = \mathbf{33.9\ A \angle -75°}$$

c. $\mathbf{I}_L = \sqrt{3} I_\phi = (1.73)(34 \text{ A}) = 58.82 \text{ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = \mathbf{58.82\ A}$$

EXAMPLE 24.4 For the Δ-Y system shown in Fig. 24.17:

a. Find the voltage across each phase of the load.
b. Find the magnitude of the line voltages.

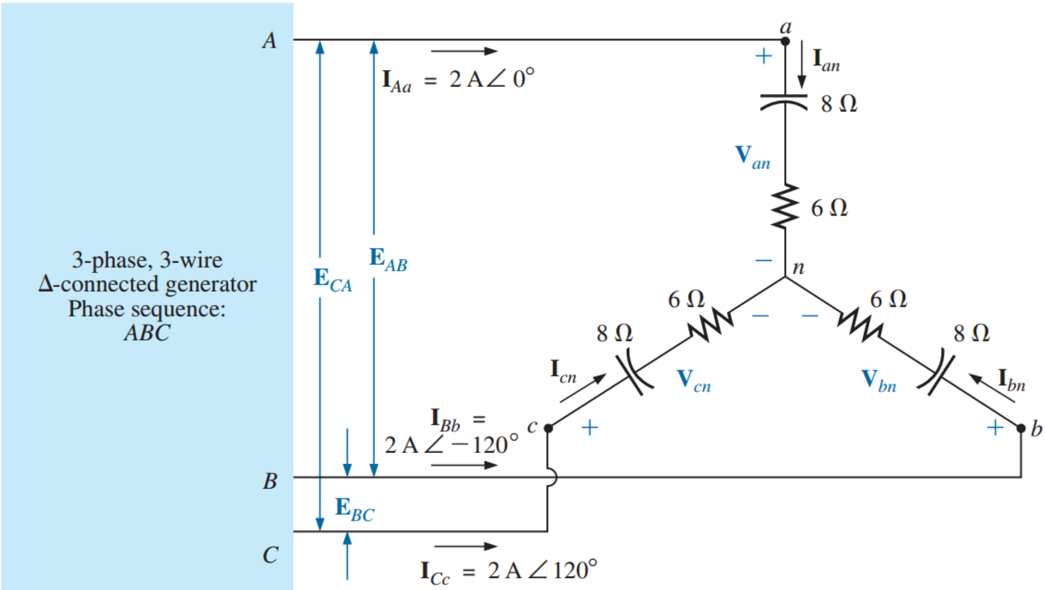


FIG. 24.17
Example 24.4: Δ-Y system.

Solutions:

a. $\mathbf{I}_{\phi L} = \mathbf{I}_L$. Therefore,

$$\mathbf{I}_{an} = \mathbf{I}_{Aa} = 2\text{ A}\angle 0°$$
$$\mathbf{I}_{bn} = \mathbf{I}_{Bb} = 2\text{ A}\angle -120°$$
$$\mathbf{I}_{cn} = \mathbf{I}_{Cc} = 2\text{ A}\angle 120°$$

The phase voltages are

$$\mathbf{V}_{an} = \mathbf{I}_{an}\mathbf{Z}_{an} = (2\text{ A}\angle 0°)(10\ \Omega\angle -53.13°) = \mathbf{20\ V\angle -53.13°}$$
$$\mathbf{V}_{bn} = \mathbf{I}_{bn}\mathbf{Z}_{bn} = (2\text{ A}\angle -120°)(10\ \Omega\angle -53.13°) = \mathbf{20\ V\angle -173.13°}$$
$$\mathbf{V}_{cn} = \mathbf{I}_{cn}\mathbf{Z}_{cn} = (2\text{ A}\angle 120°)(10\ \Omega\angle -53.13°) = \mathbf{20\ V\angle 66.87°}$$

b. $E_L = \sqrt{3}V_\phi = (1.73)(20\text{ V}) = 34.6\text{ V}$. Therefore,

$$E_{BA} = E_{CB} = E_{AC} = \mathbf{34.6\ V}$$

7. Power

Y-Connected Balanced Load

Please refer to Fig. 24.18 for the following discussion.

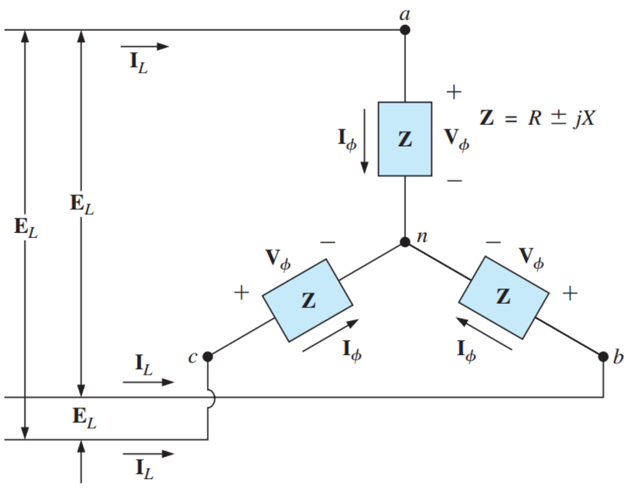


FIG. 24.18
Y-connected balanced load.

Average Power The average power delivered to each phase can be determined by

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \quad \text{(watts, W)} \qquad \textbf{(24.13)}$$

where $\theta_{I_\phi}^{V_\phi}$ indicates that θ is the phase angle between V_ϕ and I_ϕ.

The total power delivered can be determined by Eq. (24.14) or Eq. (24.16):

$$P_T = 3P_\phi \quad \text{(W)} \qquad \textbf{(24.14)}$$

or, since $$V_\phi = \frac{E_L}{\sqrt{3}} \quad \text{and} \quad I_\phi = I_L$$

then $$P_T = 3\frac{E_L}{\sqrt{3}} I_L \cos \theta_{I_\phi}^{V_\phi}$$

But $$\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Therefore,

$$P_T = \sqrt{3}E_L I_L \cos \theta_{I_\phi}^{V_\phi} = 3I_L^2 R_\phi \qquad \text{(W)} \qquad \textbf{(24.15)}$$

Reactive Power The reactive power of each phase (in volt-amperes reactive) is

$$Q_\phi = V_\phi I_\phi \sin \theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_\phi^2}{X_\phi} \qquad \text{(VAR)} \qquad \textbf{(24.16)}$$

The total reactive power of the load is

$$Q_T = 3Q_\phi \qquad \text{(VAR)} \qquad \textbf{(24.17)}$$

or, proceeding in the same manner as above, we have

$$Q_T = \sqrt{3}E_L I_L \sin \theta_{I_\phi}^{V_\phi} = 3I_L^2 X_\phi \qquad \text{(VAR)} \qquad \textbf{(24.18)}$$

Apparent Power The apparent power of each phase is

$$S_\phi = V_\phi I_\phi \qquad \text{(VA)} \qquad \textbf{(24.19)}$$

The total apparent power of the load is

$$S_T = 3S_\phi \qquad \text{(VA)} \qquad \textbf{(24.20)}$$

or, as before,

$$S_T = \sqrt{3}E_L I_L \qquad \text{(VA)} \qquad \textbf{(24.21)}$$

Power Factor The power factor of the system is given by

$$F_p = \frac{P_T}{S_T} = \cos \theta_{I_\phi}^{V_\phi} \qquad \text{(leading or lagging)} \qquad \textbf{(24.22)}$$

EXAMPLE 24.5 For the Y-connected load in Fig. 24.19:

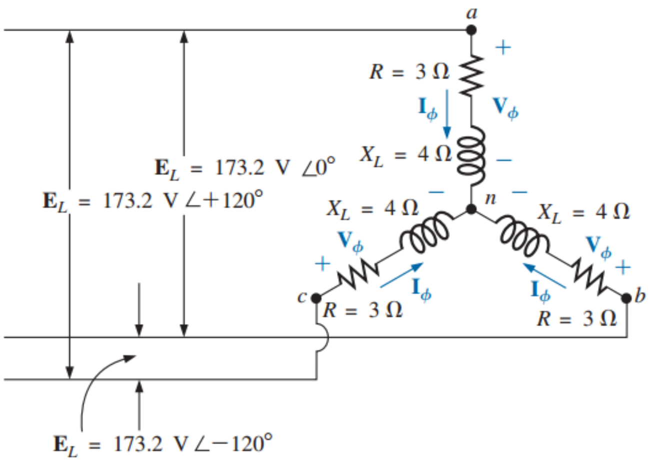


FIG. 24.19
Example 24.5.

a. Find the average power to each phase and the total load.
b. Determine the reactive power to each phase and the total reactive power.
c. Find the apparent power to each phase and the total apparent power.
d. Find the power factor of the load.

Solutions:

a. The *average power* is

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = (100\text{ V})(20\text{ A})\cos 53.13° = (2000)(0.6)$$
$$= \mathbf{1200\ W}$$
$$P_\phi = I_\phi^2 R_\phi = (20\text{ A})^2(3\ \Omega) = (400)(3) = \mathbf{1200\ W}$$
$$P_\phi = \frac{V_R^2}{R_\phi} = \frac{(60\text{ V})^2}{3\ \Omega} = \frac{3600}{3} = \mathbf{1200\ W}$$
$$P_T = 3P_\phi = (3)(1200\text{ W}) = \mathbf{3600\ W}$$

or

$$P_T = \sqrt{3}E_L I_L \cos\theta_{I_\phi}^{V_\phi} = (1.732)(173.2\text{ V})(20\text{ A})(0.6) = \mathbf{3600\ W}$$

b. The *reactive power* is

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = (100\text{ V})(20\text{ A})\sin 53.13° = (2000)(0.8)$$
$$= \mathbf{1600\ VAR}$$

or $$Q_\phi = I_\phi^2 X_\phi = (20\text{ A})^2(4\ \Omega) = (400)(4) = \mathbf{1600\ VAR}$$
$$Q_T = 3Q_\phi = (3)(1600\text{ VAR}) = \mathbf{4800\ VAR}$$

or

$$Q_T = \sqrt{3}E_L I_L \sin\theta_{I_\phi}^{V_\phi} = (1.732)(173.2\text{ V})(20\text{ A})(0.8) = \mathbf{4800\ VAR}$$

c. The *apparent power* is

$$S_\phi = V_\phi I_\phi = (100\text{ V})(20\text{ A}) = \mathbf{2000\ VA}$$

$$S_T = 3S_\phi = (3)(2000\text{ VA}) = \mathbf{6000\ VA}$$

or $$S_T = \sqrt{3}E_L I_L = (1.732)(173.2\text{ V})(20\text{ A}) = \mathbf{6000\ VA}$$

d. The *power factor* is

$$F_p = \frac{P_T}{S_T} = \frac{3600\text{ W}}{6000\text{ VA}} = \mathbf{0.6\ lagging}$$

Δ-Connected Balanced Load

Please refer to Fig. 24.20 for the following discussion.

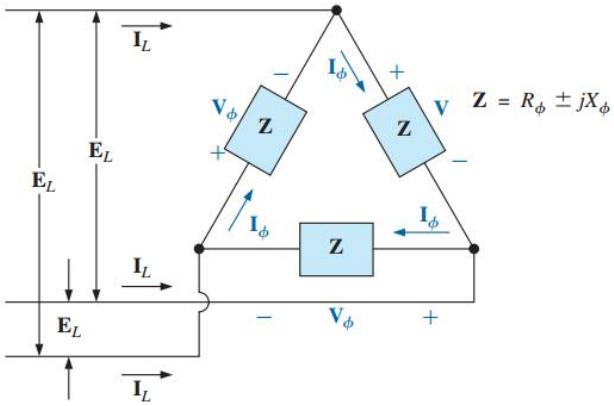


FIG. 24.20
Δ-connected balanced load.

Average Power

$$P_\phi = V_\phi I_\phi \cos\theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \quad (\text{W}) \qquad \mathbf{(24.23)}$$

$$P_T = 3P_\phi \quad (\text{W}) \qquad \mathbf{(24.24)}$$

Reactive Power

$$Q_\phi = V_\phi I_\phi \sin\theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_\phi^2}{X_\phi} \quad (\text{VAR}) \qquad \mathbf{(24.25)}$$

$$Q_T = 3Q_\phi \quad (\text{VAR}) \qquad \mathbf{(24.26)}$$

Apparent Power

$$S_\phi = V_\phi I_\phi \quad (\text{VA}) \qquad \mathbf{(24.27)}$$

$$S_T = 3S_\phi = \sqrt{3}E_L I_L \quad (\text{VA}) \qquad \mathbf{(24.28)}$$

Power Factor

$$F_p = \frac{P_T}{S_T} \qquad \mathbf{(24.29)}$$

EXAMPLE 24.6 For the Δ-Y connected load in Fig. 24.21, find the total average, reactive, and apparent power. In addition, find the power factor of the load.

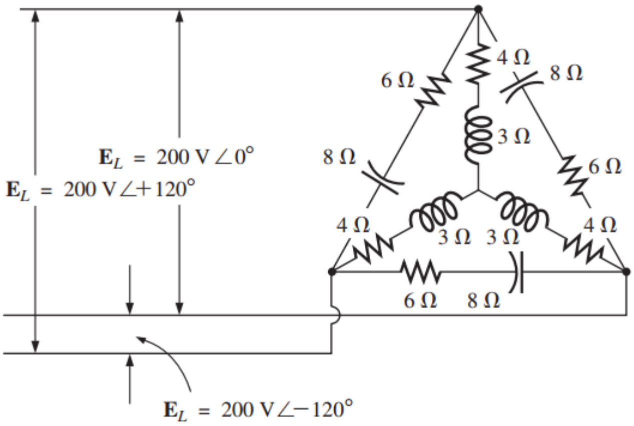


FIG. 24.21
Example 24.6.

Solution: Consider the Δ and Y separately.

For the Δ:

$$Z_\Delta = 6\ \Omega - j8\ \Omega = 10\ \Omega\angle -53.13°$$
$$I_\phi = \frac{E_L}{Z_\Delta} = \frac{200\ V}{10\ \Omega} = 20\ A$$
$$P_{T_\Delta} = 3I_\phi^2R_\phi = (3)(20\ A)^2(6\ \Omega) = \mathbf{7200\ W}$$
$$Q_{T_\Delta} = 3I_\phi^2X_\phi = (3)(20\ A)^2(8\ \Omega) = \mathbf{9600\ VAR\ (C)}$$
$$S_{T_\Delta} = 3V_\phi I_\phi = (3)(200\ V)(20\ A) = \mathbf{12{,}000\ VA}$$

For the Y:

$$Z_Y = 4\ \Omega + j3\ \Omega = 5\ \Omega\angle 36.87°$$
$$I_\phi = \frac{E_L/\sqrt{3}}{Z_Y} = \frac{200\ V/\sqrt{3}}{5\ \Omega} = \frac{116\ V}{5\ \Omega} = 23.12\ A$$
$$P_{T_Y} = 3I_\phi^2R_\phi = (3)(23.12\ A)^2(4\ \Omega) = \mathbf{6414.41\ W}$$
$$Q_{T_Y} = 3I_\phi^2X_\phi = (3)(23.12\ A)^2(3\ \Omega) = \mathbf{4810.81\ VAR\ (L)}$$
$$S_{T_Y} = 3V_\phi I_\phi = (3)(116\ V)(23.12\ A) = \mathbf{8045.76\ VA}$$

For the total load:

$$P_T = P_{T_\Delta} + P_{T_Y} = 7200\ W + 6414.41\ W = \mathbf{13{,}614.41\ W}$$
$$Q_T = Q_{T_\Delta} - Q_{T_Y} = 9600\ VAR\ (C) - 4810.81\ VAR\ (L)$$
$$= \mathbf{4789.19\ VAR\ (C)}$$
$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}614.41\ W)^2 + (4789.19\ VAR)^2}$$
$$= \mathbf{14{,}432.2\ VA}$$
$$F_p = \frac{P_T}{S_T} = \frac{13{,}614.41\ W}{14{,}432.20\ VA} = \mathbf{0.943\ leading}$$

EXAMPLE 24.7 Each transmission line of the three-wire, three-phase system in Fig. 24.22 has an impedance of $15\ \Omega + j20\ \Omega$. The system delivers a total power of 160 kW at 12,000 V to a balanced three-phase load with a lagging power factor of 0.86.

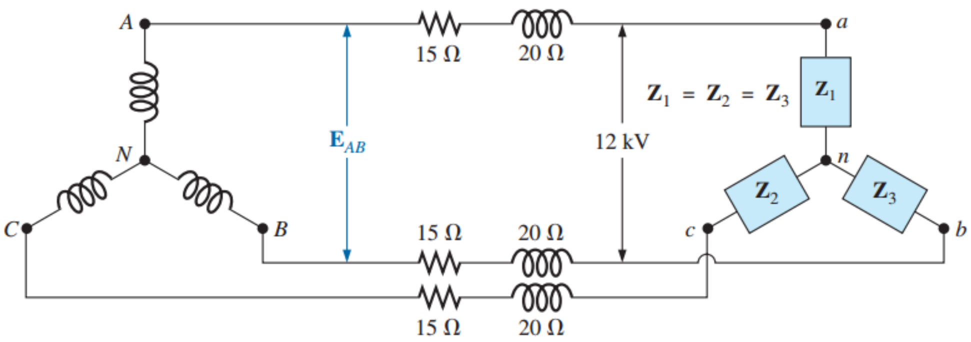


FIG. 24.22
Example 24.7.

a. Determine the magnitude of the line voltage E_{AB} of the generator.
b. Find the power factor of the total load applied to the generator.
c. What is the efficiency of the system?

Solutions:

a. $$V_\phi(\text{load}) = \frac{V_L}{\sqrt{3}} = \frac{12{,}000\ \text{V}}{1.73} = 6936.42\ \text{V}$$

$$P_T(\text{load}) = 3V_\phi I_\phi \cos\theta$$

and

$$I_\phi = \frac{P_T}{3V_\phi \cos\theta} = \frac{160{,}000\ \text{W}}{3(6936.42\ \text{V})(0.86)}$$
$$= \mathbf{8.94\ A}$$

Since $\theta = \cos^{-1} 0.86 = 30.68°$, assigning $\mathbf{V}_\phi$ an angle of 0° or $\mathbf{V}_\phi = V_\phi \angle 0°$, a lagging power factor results in

$$\mathbf{I}_\phi = 8.94\ \text{A} \angle -30.68°$$

For each phase, the system will appear as shown in Fig. 24.23, where

$$\mathbf{E}_{AN} - \mathbf{I}_\phi \mathbf{Z}_{\text{line}} - \mathbf{V}_\phi = 0$$

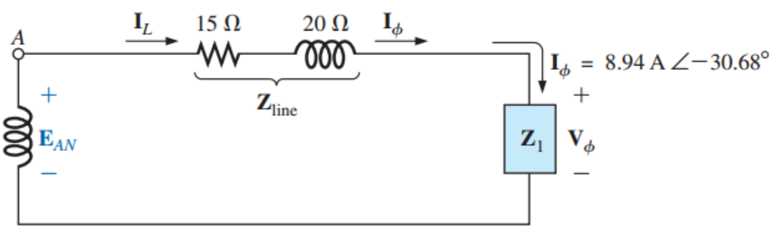


FIG. 24.23

or

$$\begin{aligned}\mathbf{E}_{AN} &= \mathbf{I}_\phi \mathbf{Z}_{\text{line}} + \mathbf{V}_\phi \\ &= (8.94\text{ A}\angle -30.68^\circ)(25\ \Omega\angle 53.13^\circ) + 6936.42\text{ V}\angle 0^\circ \\ &= 223.5\text{ V}\angle 22.45^\circ + 6936.42\text{ V}\angle 0^\circ \\ &= 206.56\text{ V} + j85.35\text{ V} + 6936.42\text{ V} \\ &= 7142.98\text{ V} + j85.35\text{ V} \\ &= 7143.5\text{ V}\angle 0.68^\circ\end{aligned}$$

Then $$\begin{aligned}E_{\text{AB}} &= \sqrt{3}E_{\phi g} = (1.73)(7143.5\text{ V}) \\ &= \mathbf{12,358.26\ V}\end{aligned}$$

b. $$\begin{aligned}P_T &= P_{\text{load}} + P_{\text{lines}} \\ &= 160\text{ kW} + 3(I_L)^2 R_{\text{line}} \\ &= 160\text{ kW} + 3(8.94\text{ A})^2 15\ \Omega \\ &= 160{,}000\text{ W} + 3596.55\text{ W} \\ &= 163{,}596.55\text{ W}\end{aligned}$$

and $$P_T = \sqrt{3}V_L I_L \cos\theta_T$$

or $$\cos\theta_T = \frac{P_T}{\sqrt{3}V_L I_L} = \frac{163{,}596.55\text{ W}}{(1.73)(12{,}358.26\text{ V})(8.94\text{ A})}$$

and $$F_p = \mathbf{0.856} < 0.86 \text{ of load}$$

c. $$\begin{aligned}\eta &= \frac{P_o}{P_i} = \frac{P_o}{P_o + P_{\text{losses}}} = \frac{160\text{ kW}}{160\text{ kW} + 3596.55\text{ W}} = 0.978 \\ &= \mathbf{97.8\%}\end{aligned}$$

5. Glossary – English/Chinese Translation

English	Chinese
three phase	三相
wye and delta connection	星形和三角洲连接
balanced three phase network	平衡三相网络
phase sequence	相序
balanced load	平衡负载
reactive power	无功率
apparent power	视在功率
power factor	功率因数
average power	平均功率