1-02-k- solution

1. a. $\omega_s = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{1\text{ H}(16\ \mu\text{F})}} = 250\text{ rad/s}$

$f_s = \dfrac{\omega_s}{2\pi} = \dfrac{250\text{ rad/s}}{2\pi} = 39.79\text{ Hz}$

b. $\omega_s = \dfrac{1}{\sqrt{(0.5\text{ H})(0.16\ \mu\text{F})}} = 3535.53\text{ rad/s}$

$f_s = \dfrac{\omega_s}{2\pi} = \dfrac{3535.53\text{ rad/s}}{2\pi} = 562.7\text{ Hz}$

c. $\omega_s = \dfrac{1}{\sqrt{(0.28\text{ mH})(7.46\ \mu\text{F})}} = 21{,}880\text{ rad/s}$

$f_s = \dfrac{\omega_s}{2\pi} = \dfrac{21{,}880\text{ rad/s}}{2\pi} = 3482.31\text{ Hz}$

2. a. $X_C = 30\ \Omega$ b. $Z_{T_s} = 10\ \Omega$ c. $I = \dfrac{E}{Z_{T_s}} = \dfrac{50\text{ mV}}{10\ \Omega} = 5\text{ mA}$

d. $V_R = IR = (5\text{ mA})(10\ \Omega) = 50\text{ mV} = \text{E}$

$V_L = IX_L = (5\text{ mA})(30\ \Omega) = 150\text{ mV}$

$V_C = IX_C = (5\text{ mA})(30\ \Omega) = 150\text{ mV}$

$V_L = V_C$

e. $Q_s = \dfrac{X_L}{R} = \dfrac{30\ \Omega}{10\ \Omega} = 3$ (low Q) f. $P = I^2R = (5\text{ mA})^2\ 10\ \Omega = 0.25\text{ mW}$

3. a. $X_L = 40\ \Omega$

b. $I = \dfrac{E}{Z_{T_s}} = \dfrac{20\ \text{mV}}{2\ \Omega} = 10\ \text{mA}$

c. $V_R = IR = (10\ \text{mA})(2\ \Omega) = 20\ \text{mV} = \text{E}$
$V_L = IX_L = (10\ \text{mA})(40\ \Omega) = 400\ \text{mV}$
$V_C = IX_C = (10\ \text{mA})(40\ \Omega) = 400\ \text{mV}$
$V_L = V_C = 20\ V_R$

d. $Q_s = \dfrac{X_L}{R} = \dfrac{40\ \Omega}{2\ \Omega} = 20$ (high Q)

e. $X_L = 2\pi fL,\ L = \dfrac{X_L}{2\pi f} = \dfrac{40\ \Omega}{2\pi(5\ \text{kHz})} = 1.27\ \text{mH}$

$X_C = \dfrac{1}{2\pi fC},\ C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(5\ \text{kHz})(40\ \Omega)} = 0.796\ \mu\text{F}$

f. $BW = \dfrac{f_s}{Q_s} = \dfrac{5\ \text{kHz}}{20} = 250\ \text{Hz}$

g. $f_2 = f_s + \dfrac{BW}{2} = 5\ \text{kHz} + \dfrac{0.25\ \text{kHz}}{2} = 5.125\ \text{kHz}$

$f_1 = f_s - \dfrac{BW}{2} = 5\ \text{kHz} - \dfrac{0.25\ \text{kHz}}{2} = 4.875\ \text{kHz}$

4. a. $f_s = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow L = \dfrac{1}{(2\pi f_s)^2 C} = \dfrac{1}{(2\pi\ 1.8\ \text{kHz})^2\ 2\ \mu\text{F}} = 3.91\ \text{mH}$

b. $X_L = 2\pi fL = 2\pi(1.8\ \text{kHz})(3.91\ \text{mH}) = 44.2\ \Omega$
$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(1.8\ \text{kHz})(2\ \mu\text{F})} = 44.2\ \Omega$
$X_L = X_C$

c. $E_{\text{rms}} = (0.707)(20\ \text{mV}) = 14.14\ \text{mV}$
$I_{\text{rms}} = \dfrac{E_{\text{rms}}}{R} = \dfrac{14.14\ \text{mV}}{4.7\ \Omega} = 3.01\ \text{mA}$

d. $P = I^2R = (3.01\ \text{mA})^2\ 4.7\ \Omega = 42.58\ \mu\text{W}$

e. $S_T = P_T = 42.58\ \mu\text{VA}$

f. $F_p = 1$

g. $$Q_s = \frac{X_L}{R} = \frac{44.2\ \Omega}{4.7\ \Omega} = 9.4$$

$$BW = \frac{f_s}{Q_s} = \frac{1.8\ \text{kHz}}{9.4} = 191.49\ \text{Hz}$$

h. $$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{4.7\ \Omega}{2(3.91\ \text{mH})} + \frac{1}{2}\sqrt{\left(\frac{4.7\ \Omega}{3.91\ \text{mH}}\right)^2 + \frac{4}{(3.91\ \text{mH})(2\ \mu\text{F})}}\right]$$

$$= \frac{1}{2\pi}[601.02 + 11.324 \times 10^3]$$

$$= 1897.93\ \text{Hz}$$

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

$$= \frac{1}{2\pi}[-601.02 + 11.324 \times 10^3]$$

$$= 1.707\ \text{kHz}$$

$$P_{\text{HPF}} = \frac{1}{2}P_{\text{max}} = \frac{1}{2}(42.58\ \mu\text{W}) = 21.29\ \mu\text{W}$$