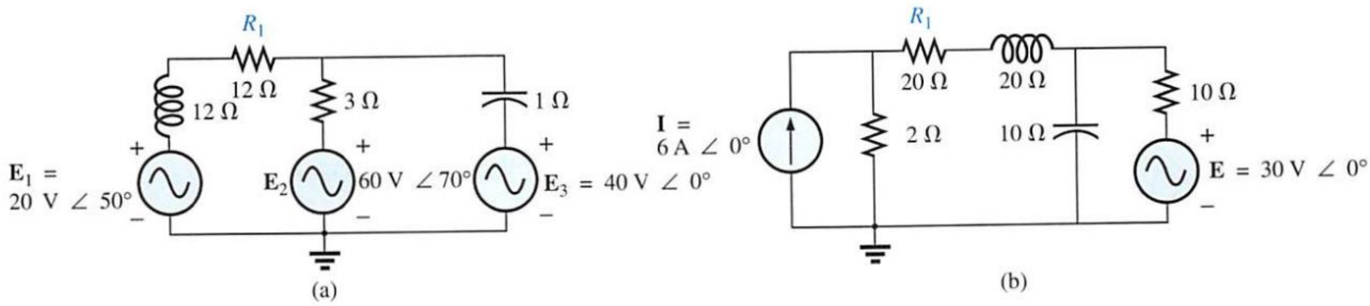
Tutorial - 1-02-h

Question 1 (17-6)

6. Write the mesh equations for the networks of Fig. 17.62. Determine the current through the resistor R_1.



Question 2 (17-8)

***8.** Write the mesh equations for the networks of Fig. 17.64. Determine the current through the resistor R_1.

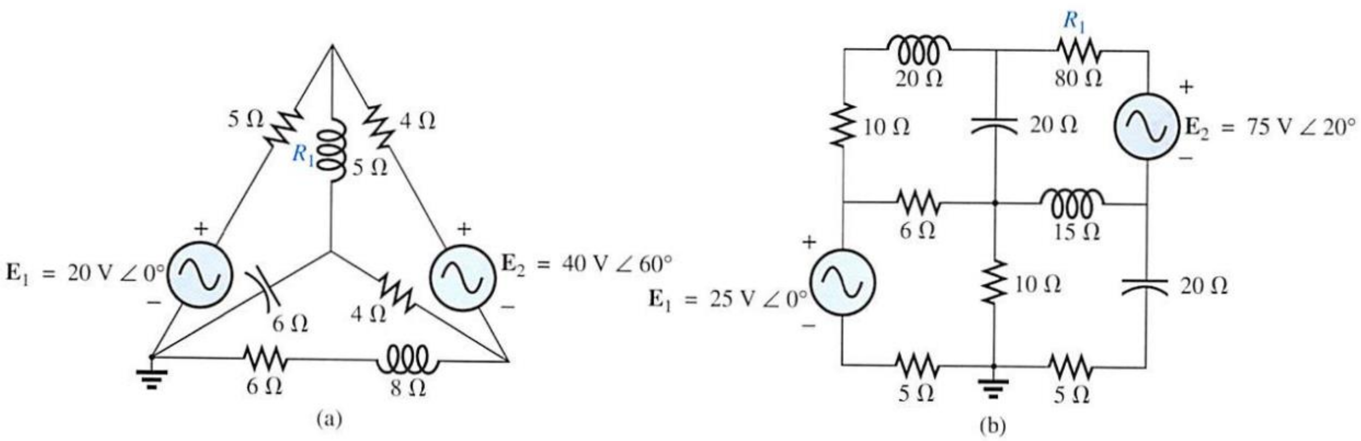
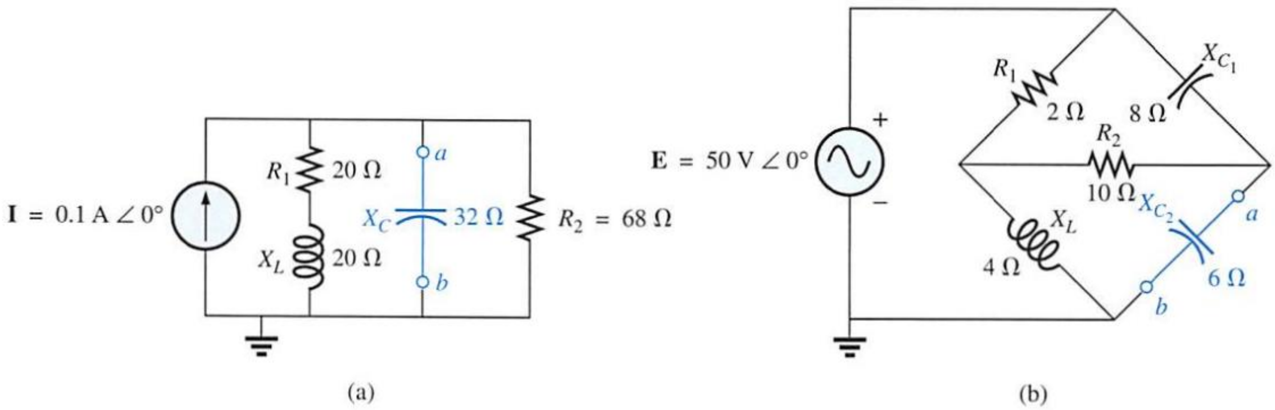


FIG. 17.64

Question 3 (18-13)

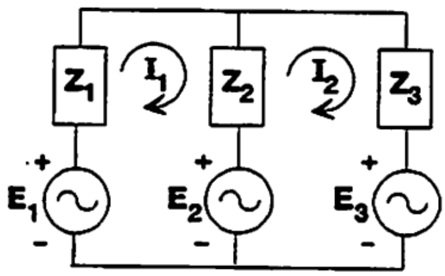
***13.** Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.120 external to the elements between points a and b.



SOLUTION

Q1

6. a.



$Z_1 = 12\ \Omega + j12\ \Omega = 16.971\ \Omega \angle 45°$
$Z_2 = 3\ \Omega \angle 0°$
$Z_3 = -j1\ \Omega$
$E_1 = 20\ V \angle 50°$
$E_2 = 60\ V \angle 70°$
$E_3 = 40\ V \angle 0°$

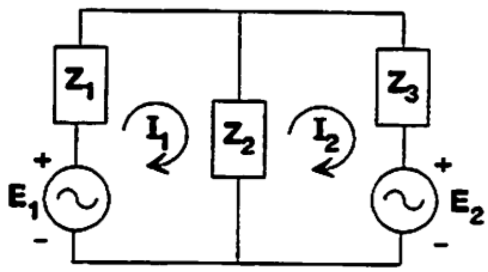
$$I_1[Z_1 + Z_2] - Z_2 I_2 = E_1 - E_2$$
$$I_2[Z_2 + Z_3] - Z_2 I_1 = E_2 - E_3$$

$$(Z_1 + Z_2)I_1 - Z_2 I_2 = E_1 - E_2$$
$$-Z_2 I_1 + (Z_2 + Z_3)I_2 = E_2 - E_3$$

Using determinants:

$$I_{R_1} = I_1 = \frac{(E_1 - E_2)(Z_2 + Z_3) + Z_2(E_2 - E_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 2.552\ A \angle 132.72°$$

b.



Source conversion:

$E_1 = IZ = (6\ A \angle 0°)(2\ \Omega \angle 0°)$
$= 12\ V \angle 0°$
$Z_1 = 2\ \Omega + 20\ \Omega + j20\ \Omega = 22\ \Omega + j20\ \Omega$
$= 29.732\ \Omega \angle 42.274°$
$Z_2 = -j10\ \Omega = 10\ \Omega \angle -90°$
$Z_3 = 10\ \Omega \angle 0°$

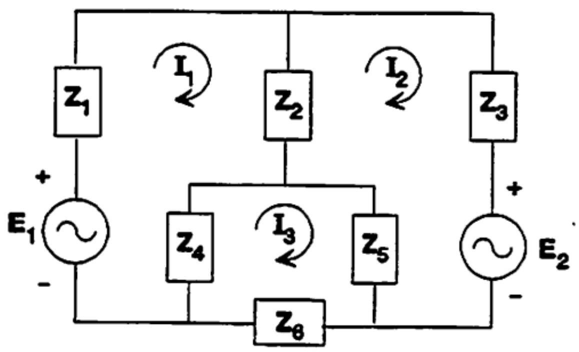
$$I_1[Z_1 + Z_2] - Z_2 I_2 = E_1$$
$$I_2[Z_2 + Z_3] - Z_2 I_1 = -E_2$$

$$(Z_1 + Z_2)I_1 - Z_2 I_2 = E_1$$
$$-Z_2 I_1 + (Z_2 + Z_3)I_2 = -E_2$$

$$I_{R_1} = I_1 = \frac{E_1(Z_2 + Z_3) - Z_2 E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 0.495\ A \angle 72.255°$$

Q2

8. a.



$Z_1 = 5\ \Omega\ \angle 0°,\ Z_2 = 5\ \Omega\ \angle 90°$
$Z_3 = 4\ \Omega\ \angle 0°,\ Z_4 = 6\ \Omega\ \angle -90°$
$Z_5 = 4\ \Omega\ \angle 0°,\ Z_6 = 6\ \Omega + j8\ \Omega$
$E_1 = 20\ V\ \angle 0°,\ E_2 = 40\ V\ \angle 60°$

$$\begin{aligned} I_1(Z_1 + Z_2 + Z_4) - I_2Z_2 - I_3Z_4 &= E_1 \\ I_2(Z_2 + Z_3 + Z_5) - I_1Z_2 - I_3Z_5 &= -E_2 \\ I_3(Z_4 + Z_5 + Z_6) - I_1Z_4 - I_2Z_5 &= 0 \end{aligned}$$

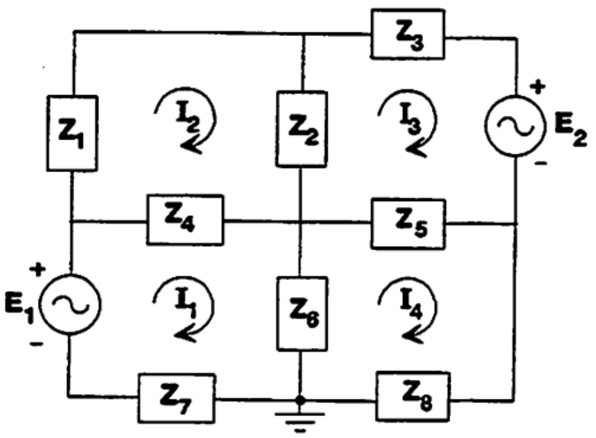
$$\begin{aligned} (Z_1 + Z_2 + Z_4)I_1 \quad - Z_2I_2 \quad - Z_4I_3 &= E_1 \\ -Z_2I_1 + (Z_2 + Z_3 + Z_5)I_2 \quad - Z_5I_3 &= -E_2 \\ -Z_4I_1 \quad - Z_5I_2 + (Z_4 + Z_5 + Z_6)I_3 &= 0 \end{aligned}$$

Using $Z' = Z_1 + Z_2 + Z_4$, $Z'' = Z_2 + Z_3 + Z_5$, $Z''' = Z_4 + Z_5 + Z_6$ and determinants:

$$I_{R_1} = I_1 = \frac{E_1(Z''Z''' - Z_5^2) - E_2(Z_2Z''' + Z_4Z_5)}{Z'(Z''Z''' - Z_5^2) - Z_2(Z_2Z''' + Z_4Z_5) - Z_4(Z_2Z_5 + Z_4Z'')}$$

$$= 3.04\ A\ \angle 169.12°$$

b.



$Z_1 = 10\ \Omega + j20\ \Omega$ | $Z_2 = -j20\ \Omega$
$Z_3 = 80\ \Omega\ \angle 0°$ | $Z_4 = 6\ \Omega\ \angle 0°$
$Z_5 = 15\ \Omega\ \angle 90°$ | $Z_6 = 10\ \Omega\ \angle 0°$
$Z_7 = 5\ \Omega\ \angle 0°$ | $Z_8 = 5\ \Omega - j20\ \Omega$
$E_1 = 25\ V\ \angle 0°$ | $E_2 = 75\ V\ \angle 20°$

$$\begin{aligned} I_1(Z_4 + Z_6 + Z_7) - I_2Z_4 - I_4Z_6 &= E_1 \\ I_2(Z_1 + Z_2 + Z_4) - I_1Z_4 - I_3Z_2 &= 0 \\ I_3(Z_2 + Z_3 + Z_5) - I_2Z_2 - I_4Z_5 &= -E_2 \\ I_4(Z_5 + Z_6 + Z_8) - I_1Z_6 - I_3Z_5 &= 0 \end{aligned}$$

$$\begin{aligned} (Z_4 + Z_6 + Z_7)I_1 \quad - Z_4I_2 \quad + 0 \quad - Z_6I_4 &= E_1 \\ -Z_4I_1 + (Z_1 + Z_2 + Z_4)I_2 \quad - Z_2I_3 \quad + 0 &= 0 \\ 0 \quad - Z_2I_2 + (Z_2 + Z_3 + Z_5)I_3 \quad - Z_5I_4 &= -E_2 \\ -Z_6I_1 \quad + 0 \quad - Z_5I_3 + (Z_5 + Z_6 + Z_7)I_4 &= 0 \end{aligned}$$

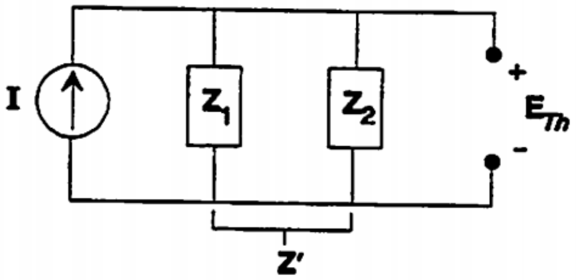
Applying determinants:

$$I_{R_1} = I_{80\Omega} = 0.681\ A\ \angle -162.9°$$

Q3

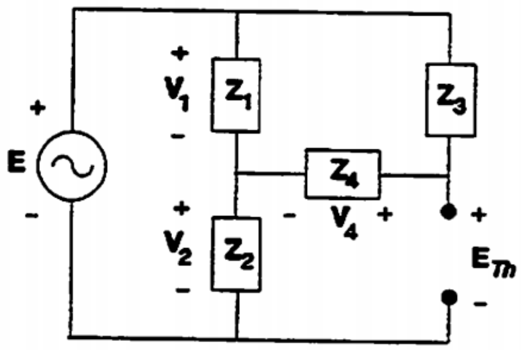
a. From #27. $\mathbf{Z}_{Th} = \mathbf{Z}_1 \parallel \mathbf{Z}_2$

$\mathbf{Z}_{Th} = \mathbf{Z}_N = 21.312\ \Omega\ \angle 32.196°$



$$\mathbf{E}_{Th} = \mathbf{I}\mathbf{Z}' = \mathbf{I}\mathbf{Z}_{Th}$$
$$= (0.1\ \text{A}\ \angle 0°)(21.312\ \Omega\ \angle 32.196°)$$
$$= 2.131\ \text{V} \angle 32.196°$$

b. From #27. $\mathbf{Z}_{Th} = \mathbf{Z}_N = 6.813\ \Omega\ \angle -54.228° = 3.983\ \Omega - j5.528\ \Omega$



$\mathbf{Z}_1 = 2\ \Omega\ \angle 0°$, $\mathbf{Z}_3 = 8\ \Omega \angle -90°$

$\mathbf{Z}_2 = 4\ \Omega\ \angle 90°$, $\mathbf{Z}_4 = 10\ \Omega\ \angle 0°$

$\mathbf{E} = 50\ \text{V}\ \angle 0°$

$\mathbf{E}_{Th} = \mathbf{V}_2 + \mathbf{V}_4$

$$\mathbf{V}_2 = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1 \parallel (\mathbf{Z}_3 + \mathbf{Z}_4)} = \frac{(4\ \Omega\ \angle 90°)(50\ \text{V}\ \angle 0°)}{+j4\ \Omega + 2\ \Omega\ \angle 0° \parallel (10\ \Omega - j8\ \Omega)}$$
$$= 47.248\ \text{V}\ \angle 24.7°$$

$$\mathbf{V}_1 = \mathbf{E} - \mathbf{V}_2 = 50\ \text{V}\ \angle 0° - 47.248\ \text{V}\ \angle 24.7° = 20.972\ \text{V}\ \angle -70.285°$$

$$\mathbf{V}_4 = \frac{\mathbf{Z}_4\mathbf{V}_1}{\mathbf{Z}_4 + \mathbf{Z}_3} = \frac{(10\ \Omega\ \angle 0°)(20.972\ \text{V}\ \angle -70.285°)}{10\ \Omega - j8\ \Omega} = 16.377\ \text{V}\ \angle -31.625°$$

$$\mathbf{E}_{Th} = \mathbf{V}_2 + \mathbf{V}_4 = 47.248\ \text{V}\ \angle 24.7° + 16.377\ \text{V}\ \angle -31.625°$$
$$= (42.925\ \text{V} + j19.743\ \text{V}) + (13.945\ \text{V} - j8.587\ \text{V})$$
$$= 56.870\ \text{V} + j11.156\ \text{V} = 57.954\ \text{V}\ \angle 11.099°$$