Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-j

AC network theorems

Contents

Reference:

Introductory Circuit Analysis 14th edition, Boylesad & Olivari
Basic Circuit Analysis – Schaum's Outline Series

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1. Independent vs dependent sources

The term independent specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.

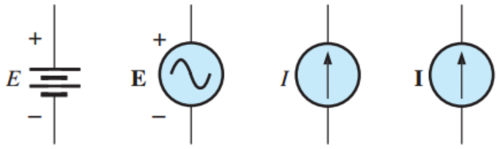

FIG. 18.1
Independent sources.

A dependent or controlled source is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.

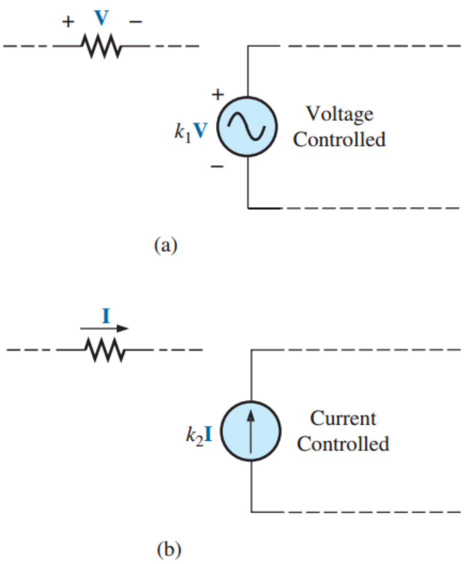

FIG. 18.2
Controlled or dependent sources.

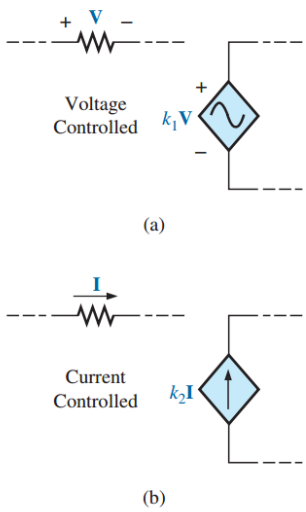

FIG. 18.3
Special notation for controlled or dependent sources.

EXAMPLE 18.3 Convert the voltage source in Fig. 18.8(a) to a current source.

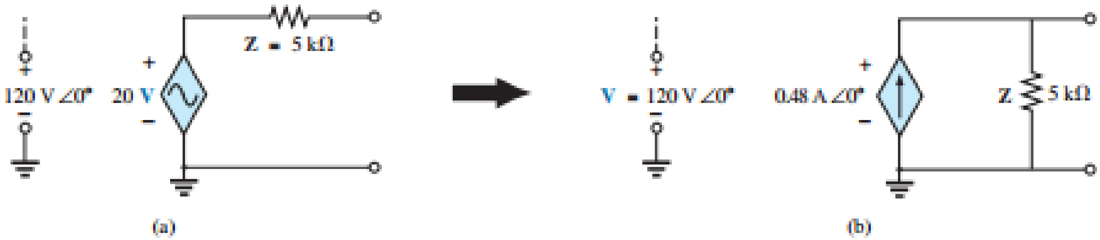

FIG. 18.8
Source conversion with a voltage-controlled voltage source.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{20\ \mathbf{V}}{5\ \text{k}\Omega \angle 0^\circ} = \frac{20(120\ \text{V} \angle 0^\circ)}{5\ \text{k}\Omega \angle 0^\circ} = \frac{2.4\ \text{kV} \angle 0^\circ}{5\ \text{k}\Omega \angle 0^\circ}$$

$$= \mathbf{0.48\ A \angle 0^\circ} \qquad \text{[Fig. 18.8(b)]}$$

2. Mesh Analysis

1. *Assign a distinct current in the clockwise direction to each independent closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. However, it eliminates the need to have to choose a direction for each application. Any direction can be chosen for each loop current with no loss in accuracy as long as the remaining steps are followed properly.*
2. *Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.*
3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the format approach to follow.*
   a. *If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents passing through in the opposite direction.*
   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*
4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

EXAMPLE 18.5 Using the general approach to mesh analysis, find the current $\mathbf{I}_1$ in Fig. 18.10.

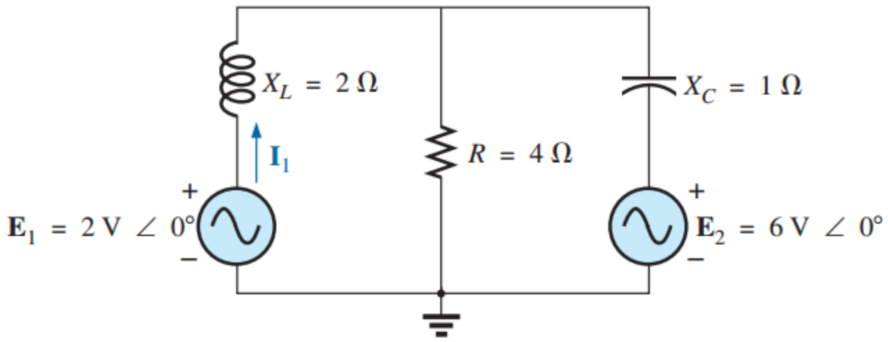

FIG. 18.10
Example 18.5.

The network is redrawn in Fig. 18.11 with subscripted impedances:

$$\mathbf{Z}_1 = +jX_L = +j2\ \Omega \qquad \mathbf{E}_1 = 2\ \text{V}\angle 0°$$
$$\mathbf{Z}_2 = R = 4\ \Omega \qquad \mathbf{E}_2 = 6\ \text{V}\angle 0°$$
$$\mathbf{Z}_3 = -jX_C = -j1\ \Omega$$

Steps 1 and 2 are as indicated in Fig. 18.11.

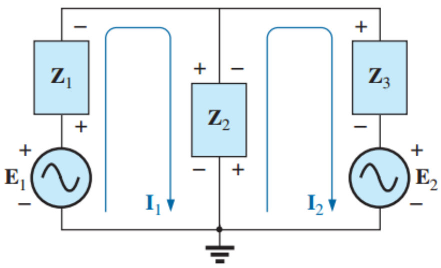

FIG. 18.11
Assigning the mesh currents and subscripted impedances for the network in Fig. 18.10.

Step 3:

$$\begin{aligned} +\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) &= 0 \\ -\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 &= 0 \end{aligned}$$

or

$$\begin{aligned} \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 &= 0 \\ -\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 &= 0 \end{aligned}$$

so that

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 \\ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 &= -\mathbf{E}_2 \end{aligned}$$

which are rewritten as

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\quad &= \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 \qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

Step 4:

Determinants

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$

$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_1 = \frac{(2\text{ V} - 6\text{ V})(4\ \Omega) + (2\text{ V})(-j1\ \Omega)}{(+j2\ \Omega)(4\ \Omega) + (+j2\ \Omega)(-j2\ \Omega) + (4\ \Omega)(-j2\ \Omega)}$$

$$= \frac{-16 - j2}{j8 - j^2 2 - j4} = \frac{-16 - j2}{2 + j4} = \frac{16.12\text{ A}\angle -172.87^\circ}{4.47\angle 63.43^\circ}$$

$$= \mathbf{3.61\ A\angle -236.30^\circ} \quad \text{or} \quad \mathbf{3.61\ A\angle 123.70^\circ}$$

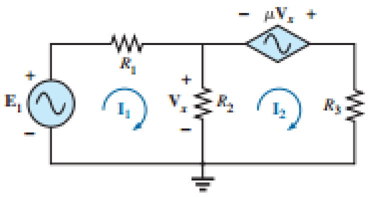

FIG. 18.13
Applying mesh analysis to a network with a voltage-controlled voltage source.

EXAMPLE 18.6 Write the mesh currents for the network in Fig. 18.13 having a dependent voltage source.

Solution:

Steps 1 and 2 are defined in Fig. 18.13.

Step 3:

$$\begin{aligned} \mathbf{E}_1 - \mathbf{I}_1\mathbf{R}_1 - \mathbf{R}_2(\mathbf{I}_1 - \mathbf{I}_2) &= 0 \\ \mathbf{R}_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{V}_x - \mathbf{I}_2\mathbf{R}_3 &= 0 \end{aligned}$$

Then substitute $\mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$.

The result is two equations and two unknowns:

$$\begin{aligned} \mathbf{E}_1 - \mathbf{I}_1\mathbf{R}_1 - \mathbf{R}_2(\mathbf{I} - \mathbf{I}_2) &= 0 \\ \mathbf{R}_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{R}_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2\mathbf{R}_3 &= 0 \end{aligned}$$

EXAMPLE 18.8 Write the mesh currents for the network in Fig. 18.15 having a dependent current source.

Solution:

Steps 1 and 2 are defined in Fig. 18.15.

Step 3: $\quad \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0$

and $\quad k\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$

Now $\mathbf{I} = \mathbf{I}_1$ so that $\quad k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2 \quad$ or $\quad \mathbf{I}_2 = \mathbf{I}_1(1 - k)$

The result is two equations and two unknowns.

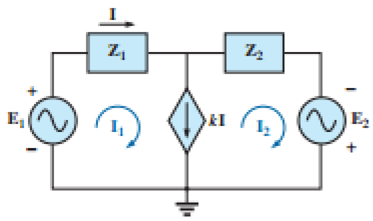

FIG. 18.15
Applying mesh analysis to a network with a current-controlled current source.

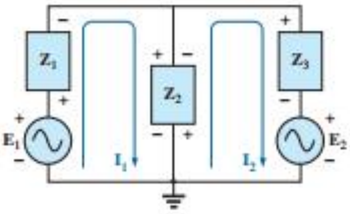

FIG. 18.16
Assigning the mesh currents and subscripted impedances for the network in Fig. 18.10 (repeated).

EXAMPLE 18.9 Using the format approach to mesh analysis, repeat Example 18.5. The block impedance diagram is repeated as Fig. 18.16 for convenience.

Solution:

Step 1 is as indicated in Fig. 18.16.

Steps 2 through 4 result in the following:

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

which can be rewritten as

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$-\mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = -\mathbf{E}_2$$

and we have the same set of equations as in Example 18.5 resulting in the same solution of

$$\mathbf{I}_1 = 3.61\ \text{A}\ \angle -236.30°$$

EXAMPLE 18.10 Using the format approach to mesh analysis, find the current $\mathbf{I}_2$ in Fig. 18.17.

Solution: The network is redrawn in Fig. 18.18:

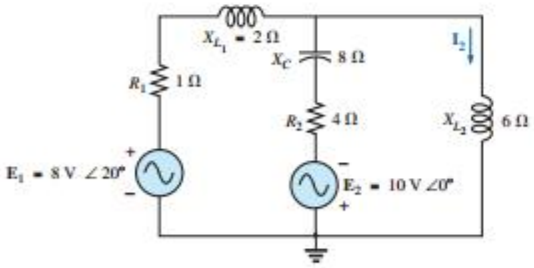

FIG. 18.17
Example 18.10.

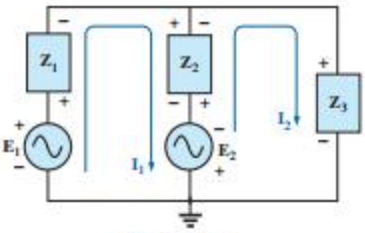

FIG. 18.18
Assigning the mesh currents and subscripted impedances for the network in Fig. 18.17.

$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 1\ \Omega + j2\ \Omega \qquad \mathbf{E}_1 = 8\ \text{V}\angle 20°$$
$$\mathbf{Z}_2 = R_2 - jX_C = 4\ \Omega - j8\ \Omega \qquad \mathbf{E}_2 = 10\ \text{V}\angle 0°$$
$$\mathbf{Z}_3 = +jX_{L_2} = +j6\ \Omega$$

EXAMPLE 18.11 Write the mesh equations for the network in Fig. 18.20. Do not solve.

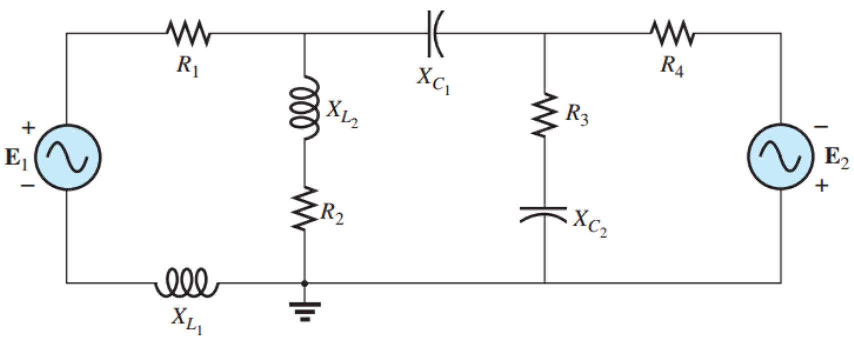

FIG. 18.20
Example 18.11.

Solution: The network is redrawn in Fig. 18.21. Again note the reduced complexity and increased clarity provided by the use of subscripted impedances:

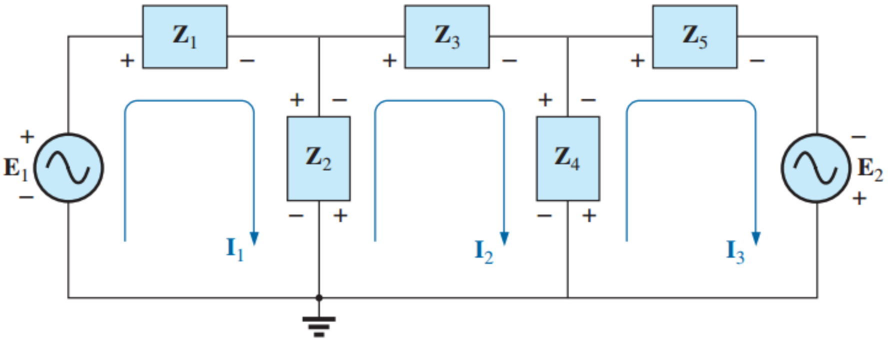

FIG. 18.21
Assigning the mesh currents and subscripted impedances for the network in Fig. 18.20.

$$\mathbf{Z}_1 = R_1 + jX_{L_1} \qquad \mathbf{Z}_4 = R_3 - jX_{C_2}$$
$$\mathbf{Z}_2 = R_2 + jX_{L_2} \qquad \mathbf{Z}_5 = R_4$$
$$\mathbf{Z}_3 = jX_{C_1}$$

and
$$\begin{aligned}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 \\
\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_3\mathbf{Z}_4 &= 0 \\
\mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_4 &= \mathbf{E}_2
\end{aligned}$$

or
$$\begin{array}{llll}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2(\mathbf{Z}_2) & & +\,0 & = \mathbf{E}_1 \\
\mathbf{I}_1\mathbf{Z}_2 & -\,\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_3(\mathbf{Z}_4) & & = 0 \\
0 & -\,\mathbf{I}_2(\mathbf{Z}_4) & -\,\mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) & = \mathbf{E}_2
\end{array}$$

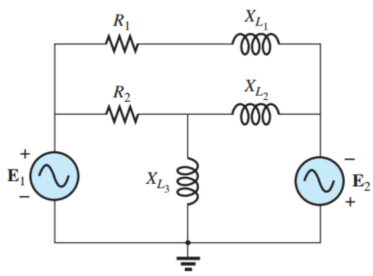

FIG. 18.22
Example 18.12.

EXAMPLE 18.12 Using the format approach, write the mesh equations for the network in Fig. 18.23.

Solution: The network is redrawn as shown in Fig. 18.23, where

$$\mathbf{Z}_1 = R_1 + jX_{L_1} \qquad \mathbf{Z}_3 = jX_{L_2}$$
$$\mathbf{Z}_2 = R_2 \qquad \mathbf{Z}_4 = jX_{L_3}$$

and
$$\begin{aligned}
\mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 &= \mathbf{E}_1 \\
\mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_3 &= 0 \\
\mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_1\mathbf{Z}_4 &= \mathbf{E}_2
\end{aligned}$$

or
$$\begin{array}{llll}
\mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) & -\mathbf{I}_2\mathbf{Z}_2 & -\mathbf{I}_3\mathbf{Z}_4 & = \mathbf{E}_1 \\
-\mathbf{I}_1\mathbf{Z}_2 & +\mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) & -\mathbf{I}_3\mathbf{Z}_3 & = 0 \\
-\mathbf{I}_1\mathbf{Z}_4 & -\mathbf{I}_2\mathbf{Z}_3 & +\mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) & = \mathbf{E}_2
\end{array}$$

Note the symmetry *about* the diagonal axis; that is, note the location of $-\mathbf{Z}_2$, $-\mathbf{Z}_4$, and $-\mathbf{Z}_3$ off the diagonal.

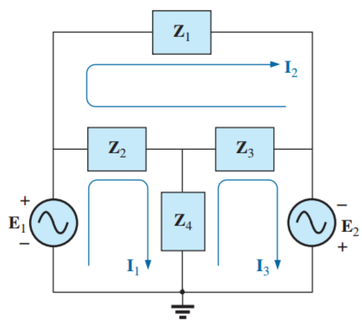

FIG. 18.23
Assigning the mesh currents and subscripted impedances for the network in Fig. 18.22.

3. Node Analysis

1. ***Determine the number of nodes within the network.***
2. ***Pick a reference node and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.***
3. ***Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.***
4. ***Solve the resulting equations for the nodal voltages.***

EXAMPLE 18.13 Determine the voltage across the inductor for the network in Fig. 18.24.

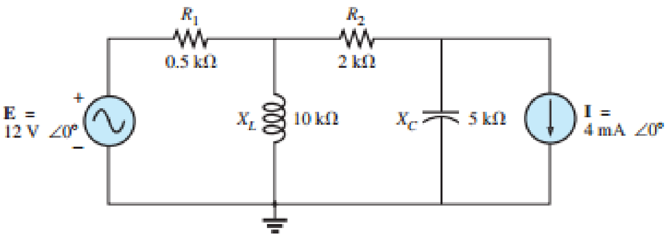

FIG. 18.24
Example 18.13.

Solution:

Steps 1 and 2 are as indicated in Fig. 18.25.

Step 3: Note Fig. 18.26 for the application of Kirchhoff's current law to node $\mathbf{V}_1$:

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$
$$0 = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$
$$\frac{\mathbf{V}_1 - \mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} = 0$$

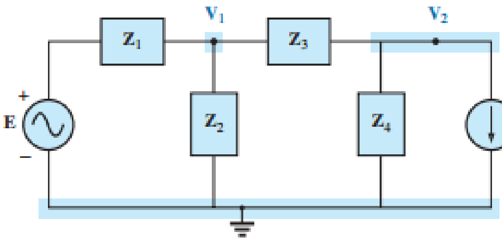

FIG. 18.25
Assigning the nodal voltages and subscripted impedances to the network in Fig. 18.24.

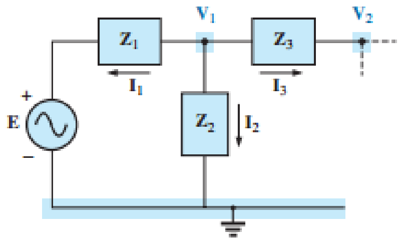

FIG. 18.26
Applying Kirchhoff's current law to the node $\mathbf{V}_1$ in Fig. 18.25.

Rearranging terms gives

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right] = \frac{\mathbf{E}_1}{\mathbf{Z}_1} \quad \textbf{(18.1)}$$

Note Fig. 18.27 for the application of Kirchhoff's current law to node $\mathbf{V}_2$.

$$0 = \mathbf{I}_3 + \mathbf{I}_4 + \mathbf{I}$$
$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_3} + \frac{\mathbf{V}_2}{\mathbf{Z}_4} + \mathbf{I} = 0$$

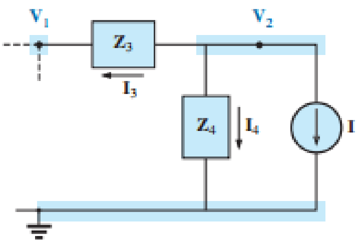

FIG. 18.27
Applying Kirchhoff's current law to the node $\mathbf{V}_2$ in Fig. 18.25.

Rearranging terms gives

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4}\right] - \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right] = -\mathbf{I} \quad \textbf{(18.2)}$$

Grouping equations 18.1 and 18.2 gives

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right] = \frac{\mathbf{E}}{\mathbf{Z}_1}$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right] = \mathbf{I}$$

$$\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3} = \frac{1}{0.5\text{ k}\Omega}+\frac{1}{j10\text{ k}\Omega}+\frac{1}{2\text{ k}\Omega} = 2.5\text{ mS}\angle -2.29^\circ$$

$$\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4} = \frac{1}{2\text{ k}\Omega}+\frac{1}{-j5\text{ k}\Omega} = 0.539\text{ mS}\angle 21.80^\circ$$

and

$$\mathbf{V}_1[2.5\text{ mS}\angle -2.29^\circ] - \mathbf{V}_2[0.5\text{ mS}\angle 0^\circ] = 24\text{ mA}\angle 0^\circ$$

$$\mathbf{V}_1[0.5\text{ mS}\angle 0^\circ] - \mathbf{V}_2[0.539\text{ mS}\angle 21.80^\circ] = 4\text{ mA}\angle 0^\circ$$

with

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 24\text{ mA}\angle 0^\circ & -0.5\text{ mS}\angle 0^\circ \\ 4\text{ mA}\angle 0^\circ & -0.539\text{ mS}\angle 21.80^\circ \end{vmatrix}}{\begin{vmatrix} 2.5\text{ mS}\angle -2.29^\circ & -0.5\text{ mS}\angle 0^\circ \\ 0.5\text{ mS}\angle 0^\circ & -0.539\text{ mS}\angle 21.80^\circ \end{vmatrix}}$$

$$= \frac{(24\text{ mA}\angle 0^\circ)(-0.539\text{ mS}\angle 21.80^\circ)+(0.5\text{ mS}\angle 0^\circ)(4\text{ mA}\angle 0^\circ)}{(2.5\text{ mS}\angle -2.29^\circ)(-0.539\text{ mS}\angle 21.80^\circ)+(0.5\text{ mS}\angle 0^\circ)(0.5\text{ mS}\angle 0^\circ)}$$

$$= \frac{-12.94\times 10^{-6}\text{ V}\angle 21.80^\circ + 2\times 10^{-6}\text{ V}\angle 0^\circ}{-1.348\times 10^{-6}\angle 19.51^\circ + 0.25\times 10^{-6}\angle 0^\circ}$$

$$= \frac{-(12.01+j4.81)\times 10^{-6}\text{ V} + 2\times 10^{-6}\text{ V}}{-(1.271+j0.45)\times 10^{-6} + 0.25\times 10^{-6}}$$

$$= \frac{-10.01\text{ V} - j4.81\text{ V}}{-1.021 - j0.45} = \frac{11.106\text{ V}\angle -154.33^\circ}{1.116\angle -156.21^\circ}$$

$$\mathbf{V}_1 = \mathbf{9.95\text{ V}\angle 1.88^\circ}$$

EXAMPLE 18.14 Write the nodal equations for the network in Fig. 18.28 having a dependent current source.

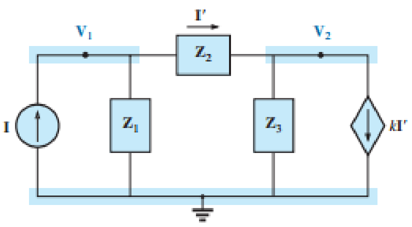

FIG. 18.28
Applying nodal analysis to a network with a current-controlled current source.

Solution:

Steps 1 and 2 are as defined in Fig. 18.28.

Step 3: At node $\mathbf{V}_1$,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}$$

At node $\mathbf{V}_2$,

$$\mathbf{I}_2 + \mathbf{I}_3 + k\mathbf{I} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\left[\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}\right] = 0$$

and
$$\mathbf{V}_1\left[\frac{1-k}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1-k}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right] = 0$$

resulting in two equations and two unknowns.

4. Star-delta conversion

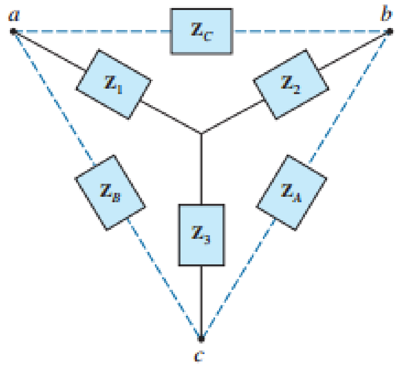

FIG. 18.49
Δ-Y configuration.

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} \quad (18.18)$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} \quad (18.19)$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} \quad (18.20)$$

For the impedances of the Δ in terms of those for the Y, the equations are

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2} \quad (18.21)$$

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} \quad (18.22)$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3} \quad (18.23)$$

In the study of dc networks, we found that if all of the resistors of the Δ or Y were the same, the conversion from one to the other could be accomplished using the equation

$$R_\Delta = 3R_Y \qquad \text{or} \qquad R_Y = \frac{R_\Delta}{3}$$

For ac networks,

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} \quad (18.24)$$

5. Superposition Theorem

The sum of the powers delivered by each of two or more ac sources of the same frequency is not equal to the power delivered by all the sources. However, for a network with a dc source and an ac source the total power can be determined by the sum of the powers delivered by each source.

EXAMPLE 19.1 Using the superposition theorem, find the current **I** through the 4 Ω reactance (X_{L_2}) in Fig. 19.1.

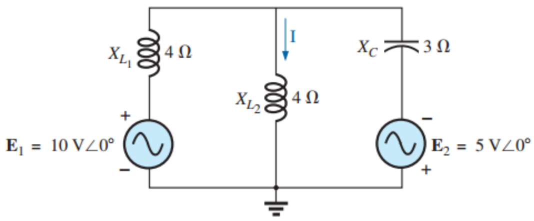

FIG. 19.1
Example 19.1.

Solution: For the redrawn circuit (Fig. 19.2),

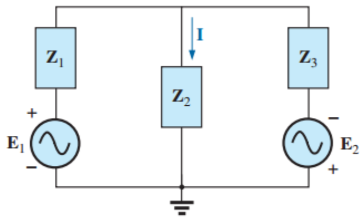

FIG. 19.2
Assigning the subscripted impedances to the network in Fig. 19.1.

$$\mathbf{Z}_1 = +jX_{L_1} = j4\Omega$$
$$\mathbf{Z}_2 = +jX_{L_2} = j4\Omega$$
$$\mathbf{Z}_3 = -jX_C = -j3\Omega$$

Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 19.3) by replacing $\mathbf{E}_2$ by a short circuit, we have

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j4\ \Omega)(-j3\ \Omega)}{j4\ \Omega - j3\ \Omega} = \frac{12\ \Omega}{j}$$
$$= -j12\ \Omega = 12\ \Omega\angle -90°$$

$$\mathbf{I}_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\ V\angle 0°}{-j12\ \Omega + j4\ \Omega} = \frac{10\ V\angle 0°}{8\ \Omega\angle -90°}$$
$$= 1.25\ A\angle 90°$$

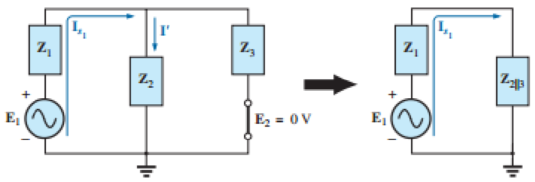

FIG. 19.3
*Determining the effect of the voltage source $\mathbf{E}_1$ on the current **I** of the network in Fig. 19.1 by replacing $\mathbf{E}_2$ by a short circuit.*

and

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \quad \text{(current divider rule)}$$
$$= \frac{(-j3\ \Omega)(j1.25\ A)}{j4\ \Omega - j3\ \Omega} = \frac{3.75\ A}{j1} = 3.75\ A\angle -90°$$

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 19.4), we have

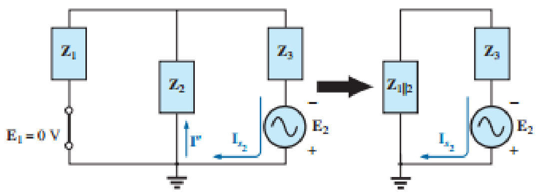

FIG. 19.4
*Determining the effect of the voltage source $\mathbf{E}_2$ on the current **I** of the network in Fig. 19.1 by replacing $\mathbf{E}_1$ by a short circuit.*

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j4\ \Omega}{2} = j2\ \Omega \quad (\text{because } \mathbf{Z}_1 = \mathbf{Z}_2)$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\ V\angle 0°}{j2\ \Omega - j3\ \Omega} = \frac{5\ V\angle 0°}{1\ \Omega\angle -90°} = 5\ A\angle 90°$$

and $$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\ A\angle 90°$$

The resultant current through the 4 Ω reactance X_{L_2} (Fig. 19.5) is

$$\mathbf{I} = \mathbf{I}' - \mathbf{I}''$$
$$= 3.75\ A\angle -90° - (2.50\ A\angle 90°) = -j3.75\ A - j2.50\ A$$
$$= -j6.25\ A$$
$$\mathbf{I} = \mathbf{6.25\ A\angle -90°}$$

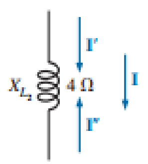

FIG. 19.5
Determining the resultant current for the network in Fig. 19.1.

6. Thevenin's Theorem

Any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Fig. 19.23.

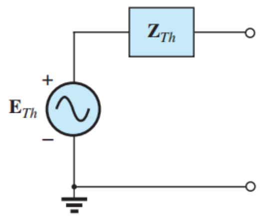

FIG. 19.23

Since the reactances of a circuit are frequency dependent, the Thévenin circuit found for a particular network is applicable only at *one* frequency.

The steps required to apply this method to dc circuits are repeated here with changes for sinusoidal ac circuits. As before, the only change is the replacement of the term *resistance* with *impedance*. Again, dependent and independent sources are treated separately.

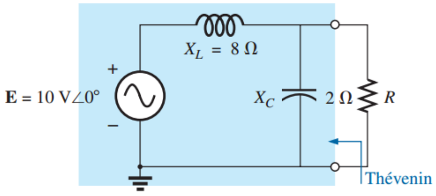

FIG. 19.24
Example 19.7.

EXAMPLE 19.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 19.24.

Solution:

Steps 1 and *2* (Fig. 19.25):

$$\mathbf{Z}_1 = jX_L = j8\ \Omega \quad \mathbf{Z}_2 = -jX_C = -j2\ \Omega$$

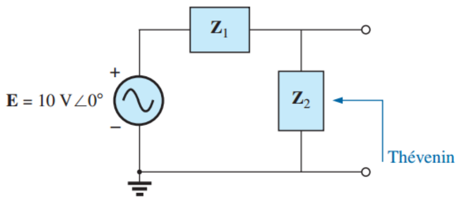

FIG. 19.25
Assigning the subscripted impedances to the network in Fig. 19.24.

Step 3 (Fig. 19.26):

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j8\ \Omega)(-j2\ \Omega)}{j8\ \Omega - j2\ \Omega} = \frac{-j^2 16\ \Omega}{j6} = \frac{16\ \Omega}{6\angle 90^\circ}$$
$$= \mathbf{2.67\ \Omega\angle -90^\circ}$$

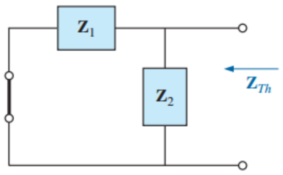

FIG. 19.26
Determining the Thévenin impedance for the network in Fig. 19.24.

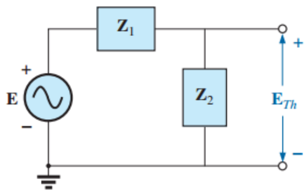

FIG. 19.27
Determining the open-circuit Thévenin voltage for the network in Fig. 19.24.

Step 4 (Fig. 19.27):

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{(voltage divider rule)}$$
$$= \frac{(-j2\ \Omega)(10\ \text{V})}{j8\ \Omega - j2\ \Omega} = \frac{-j20\ \text{V}}{j6} = \mathbf{3.33\ V\angle -180^\circ}$$

Step 5: The Thévenin equivalent circuit is shown in Fig. 19.28.

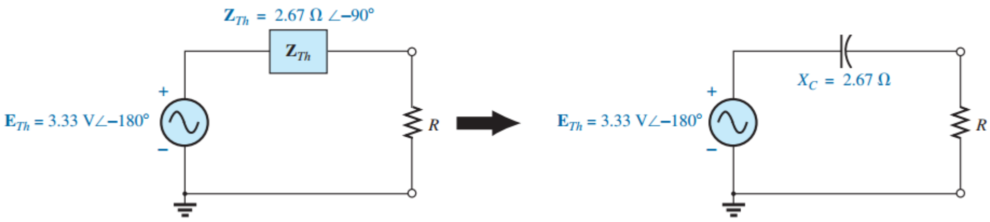

FIG. 19.28
The Thévenin equivalent circuit for the network in Fig. 19.24.

7. Norton's Theorem

1. ***Remove that portion of the network across which the Norton equivalent circuit is to be found.***
2. ***Mark (◦, •, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate Z_N by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate I_N by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.***
5. ***Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.***

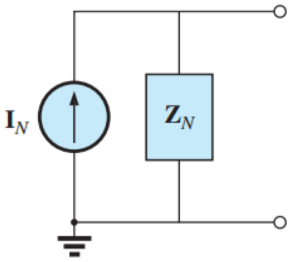

FIG. 19.60
The Norton equivalent circuit for ac networks.

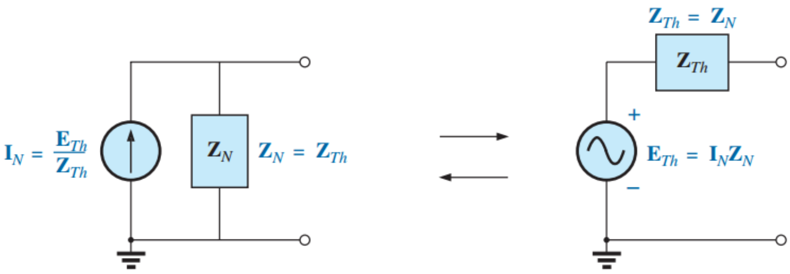

FIG. 19.61
Conversion between the Thévenin and Norton equivalent circuits.

EXAMPLE 19.14 Determine the Norton equivalent circuit for the network external to the 6 Ω resistor in Fig. 19.62.

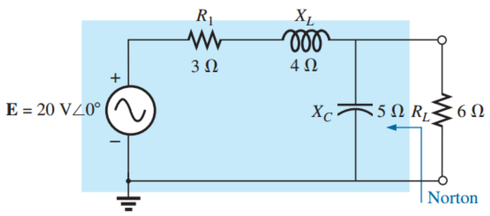

FIG. 19.62
Example 19.14.

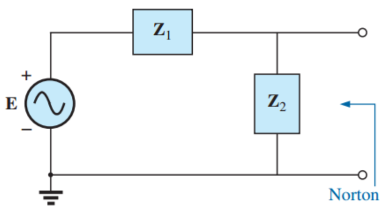

FIG. 19.63
Assigning the subscripted impedances to the network in Fig. 19.62.

Solution:

Steps 1 and *2* (Fig. 19.63):

$$\mathbf{Z}_1 = R_1 + jX_L = 3\ \Omega + j4\ \Omega = 5\ \Omega\angle 53.13^\circ$$

$$\mathbf{Z}_2 = -jX_C = -j5\ \Omega$$

Step 3 (Fig. 19.64):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega\angle 53.13^\circ)(5\ \Omega\angle -90^\circ)}{3\ \Omega + j4\ \Omega - j5\ \Omega} = \frac{25\ \Omega\angle -36.87^\circ}{3 - j1}$$

$$= \frac{25\ \Omega\angle -36.87^\circ}{3.16\angle -18.43^\circ} = 7.91\ \Omega\angle -18.44^\circ = \mathbf{7.50\ \Omega - \mathit{j}2.50\ \Omega}$$

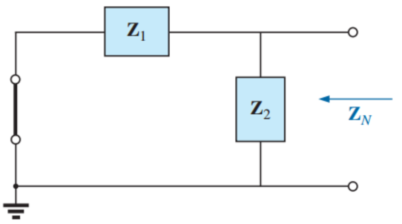

FIG. 19.64
Determining the Norton impedance for the network in Fig. 19.62.

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\ \text{V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = \mathbf{4A\angle -53.13^\circ}$$

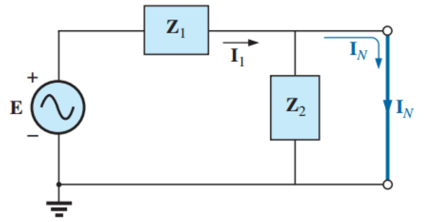

FIG. 19.65
Determining $\mathbf{I}_N$ for the network in Fig. 19.62.

Step 5: The Norton equivalent circuit is shown in Fig. 19.66.

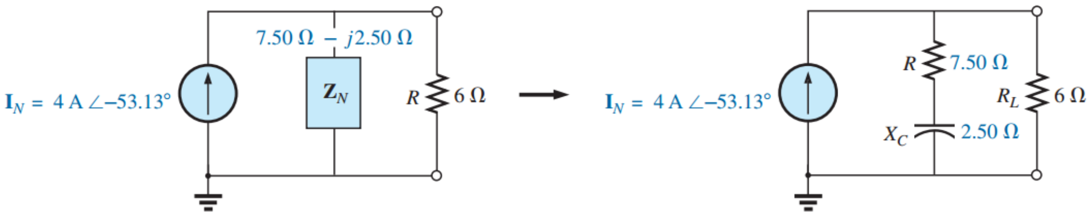

FIG. 19.66
The Norton equivalent circuit for the network in Fig. 19.62.

5. Glossary – English/Chinese Translation

English	Chinese
dependent power sources	依赖电源
ac mesh analysis	交流网格分析
ac node analysis	交流节点分析
star delta conversion	Star Delta 转换
ac superposition theorem	交流叠加定理
Thevenin's theorem	戴维南定理
Norton's theorem	诺顿定理