Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-i

Parallel and Serial AC Circuits

Contents

Reference:

Introductory Circuit Analysis 14th edition, Boylesad & Olivari
Basic Circuit Analysis – Schaum's Outline Series

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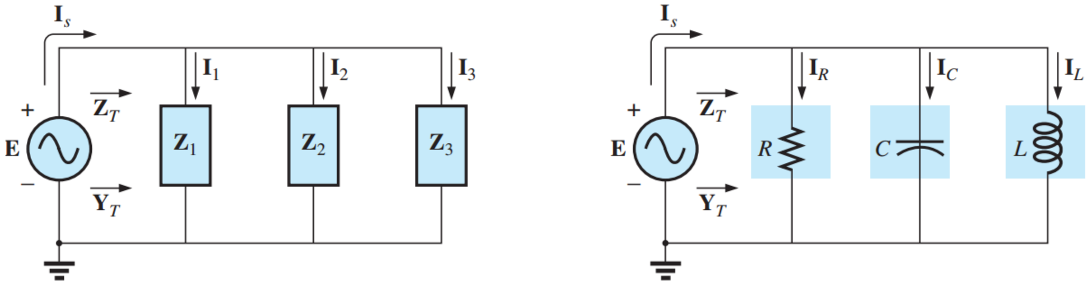
1. Total Impedance and Admittance



(a) (b)

FIG. 16.1
Parallel ac network.

For the network of Fig. 16.2 with any number of parallel elements the total impedance has the same format as encountered for dc networks:

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \dots + \frac{1}{\mathbf{Z}_N} \qquad \textbf{(16.1)}$$

which can be written in the following form:

$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_2} + \dfrac{1}{\mathbf{Z}_3} + \dots + \dfrac{1}{\mathbf{Z}_N}} \qquad \textbf{(16.2)}$$

FIG. 16.4
Impedance diagram for the network in Fig. 16.3.

EXAMPLE 16.2 For the network of Fig. 16.5:

a. Determine the total impedance using Eqs. (16.2) and (16.4).
b. Sketch the impedance diagram.

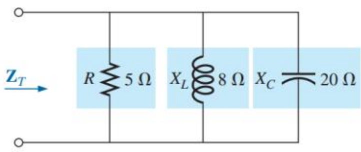


FIG. 16.5
Example 16.2.

Solutions:

a. Eq. (16.2):
$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_R} + \dfrac{1}{\mathbf{Z}_L} + \dfrac{1}{\mathbf{Z}_C}}$$
$$= \frac{1}{\dfrac{1}{5\ \Omega \angle 0^\circ} + \dfrac{1}{8\ \Omega \angle 90^\circ} + \dfrac{1}{20\ \Omega \angle -90^\circ}}$$
$$= \frac{1}{0.2\ \text{S} \angle 0^\circ + 0.125\ \text{S} \angle -90^\circ + 0.05\ \text{S} \angle 90^\circ}$$
$$= \frac{1}{0.2\ \text{S} - j0.075\ \text{S}} = \frac{1}{0.2136\ \text{S} \angle -20.56^\circ}$$
$$= \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

Eq. (16.4)

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R \mathbf{Z}_L \mathbf{Z}_C}{\mathbf{Z}_R \mathbf{Z}_L + \mathbf{Z}_L \mathbf{Z}_C + \mathbf{Z}_R \mathbf{Z}_C}$$
$$= \frac{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ)}{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ) + (8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ) + (5\ \Omega \angle 0^\circ)(20\ \Omega \angle -90^\circ)}$$
$$= \frac{800\ \Omega \angle 0^\circ}{40 \angle 90^\circ + 160 \angle 0^\circ + 100 \angle -90^\circ}$$
$$= \frac{800\ \Omega}{160 + j40 - j100} = \frac{800\ \Omega}{160 - j60}$$
$$= \frac{800\ \Omega}{170.88 \angle -20.56^\circ}$$
$$\mathbf{Z}_T = \mathbf{4.68\ \Omega \angle 20.56^\circ = 4.38\ \Omega + j\,1.64\ \Omega}$$

b. The impedance diagram appears in Fig. 16.6.

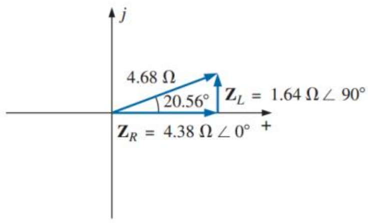


FIG. 16.6
Impedance diagram for the network in Fig. 16.5.

Resistive Elements: For resistors the admittance is defined by

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R\angle 0^\circ} = \frac{1}{R}\angle 0^\circ = G\angle 0^\circ \quad (\text{siemens, S}) \quad \textbf{(16.6)}$$

Inductive Elements: For inductive elements the admittance is defined by

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ \quad (\text{siemens, S}) \quad \textbf{(16.7)}$$

The ratio $1/X_L$ is called the **susceptance** of the inductive element, is given the symbol B_L, and is measured in **siemens (S).** Therefore,

$$B_L = \frac{1}{X_L} \quad (\text{siemens, S}) \quad \textbf{(16.8)}$$

and

$$\mathbf{Y}_L = B_L\angle -90^\circ \quad (\text{siemens, S}) \quad \textbf{(16.9)}$$

Capacitive Elements: For capacitive elements the admittance is defined by

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C\angle -90^\circ} = \frac{1}{X_C}\angle 90^\circ \quad (\text{siemens, S}) \quad \textbf{(16.10)}$$

The ratio $1/X_C$ is also called the **susceptance** of the capacitive element, is given the symbol B_C, and is measured in **siemens (S).** Therefore,

$$B_C = \frac{1}{X_C} \quad (\text{siemens, S}) \quad \textbf{(16.11)}$$

and

$$\mathbf{Y}_C = B_C\angle 90^\circ \quad (\text{siemens, S}) \quad \textbf{(16.12)}$$

For ac parallel networks, the total admittance is simply the sum of the admittance levels of all the parallel branches of Fig. 16.7. That is,

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \ldots + \mathbf{Y}_N \quad (\text{siemens, S}) \quad \textbf{(16.14)}$$

In any case, whether the total impedance or admittance is first found, the other can be found using the simple equation:

$$\mathbf{Y}_T = \frac{1}{\mathbf{Z}_T} \quad (\text{siemens, S}) \quad \textbf{(16.15)}$$

EXAMPLE 16.4 For the parallel *R-L-C* network of Fig. 16.5:

a. Find the admittance for each parallel branch.
b. Calculate the total admittance of the network.
c. Sketch the admittance diagram.
d. Calculate the total impedance using Eq. (16.15) and compare with the solution of Example 16.2.

Solutions:

a. $$\mathbf{Y}_R = G\angle 0° = \frac{1}{R}\angle 0° = \frac{1}{5\ \Omega}\angle 0° = \mathbf{0.2\ S\ \angle 0° = 0.2\ S + j0}$$

$$\mathbf{Y}_L = B_L\angle -90° = \frac{1}{X_L}\angle -90° = \frac{1}{8\ \Omega}\angle -90° = \mathbf{0.125\ S\angle -90° = 0 - j0.125\ S}$$

$$\mathbf{Y}_C = B_C\angle 90° = \frac{1}{X_C}\angle 90° = \frac{1}{20\ \Omega}\angle 90° = \mathbf{0.05\ S\ \angle 90° = 0 + j0.05\ S}$$

b. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = (0.2\ S + j0) + (0 - j0.125\ S) + (0 + j0.05\ S) = 0.2\ S - j0.075\ S = \mathbf{0.214\ S\angle -20.56°}$$

c. The admittance diagram appears in Fig. 16.10.

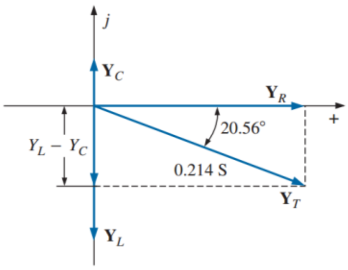


FIG. 16.10
Admittance diagram for the network in Fig. 16.5.

d. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.214\ S\angle -20.56°} = \mathbf{4.68\ \Omega\ \angle 20.56°}\text{—a perfect match}$$

<u>Example in finding admittance diagram and phasor diagram</u>

Phasor notation: As shown in Fig. 16.23.

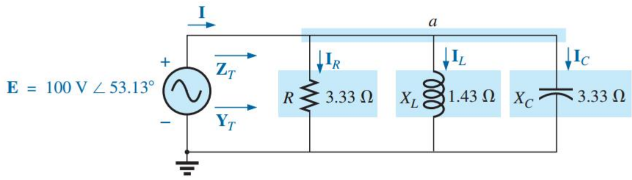


FIG. 16.23
Applying phasor notation to the network in Fig. 16.22.

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\angle 0° + B_L\angle -90° + B_C\angle 90°$$
$$= \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{1.43\ \Omega}\angle -90° + \frac{1}{3.33\ \Omega}\angle 90°$$
$$= 0.3\ S\angle 0° + 0.7\ S\angle -90° + 0.3\ S\angle 90°$$
$$= 0.3\ S - j0.7\ S + j0.3\ S$$
$$\mathbf{Y}_T = 0.3\ S - j0.4\ S = \mathbf{0.5\ S\angle -53.13°}$$
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ S\angle -53.13°} = \mathbf{2\ \Omega\angle 53.13°}$$

Admittance diagram: As shown in Fig. 16.24.

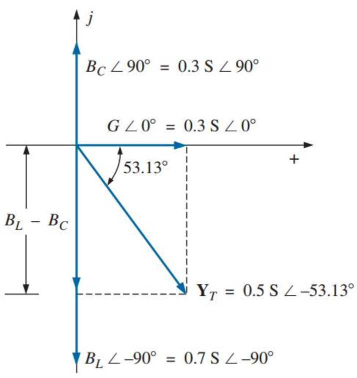


FIG. 16.24
Admittance diagram for the parallel R-L-C network in Fig. 16.22.

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100\ V\angle 53.13°)(0.5\ S\angle -53.13°) = \mathbf{50\ A\angle 0°}$$
$$\mathbf{I}_R = (E\angle\theta)(G\angle 0°) = (100\ V\angle 53.13°)(0.3\ S\angle 0°) = \mathbf{30\ A\angle 53.13°}$$
$$\mathbf{I}_L = (E\angle\theta)(B_L\angle -90°) = (100\ V\angle 53.13°)(0.7\ S\angle -90°) = \mathbf{70\ A\angle -36.87°}$$
$$\mathbf{I}_C = (E\angle\theta)(B_C\angle 90°) = (100\ V\angle 53.13°)(0.3\ S\angle +90°) = \mathbf{30\ A\angle 143.13°}$$

Kirchhoff's current law: At node a,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

Phasor diagram: The phasor diagram in Fig. 16.25 indicates that the impressed voltage $\mathbf{E}$ is in phase with the current $\mathbf{I}_R$ through the resistor, leads the current $\mathbf{I}_L$ through the inductor by 90°, and $\mathbf{E}$ lags the current $\mathbf{I}_C$ of the capacitor by 90°.

Time domain:

$$i = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\ sin\ \omega t}$$

$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13°) = \mathbf{42.42\ sin(\omega t + 53.13°)}$$

$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87°) = \mathbf{98.98\ sin(\omega t - 36.87°)}$$

$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13°) = \mathbf{42.42\ sin(\omega t + 143.13°)}$$

A plot of all of the currents and the impressed voltage appears in Fig. 16.26.

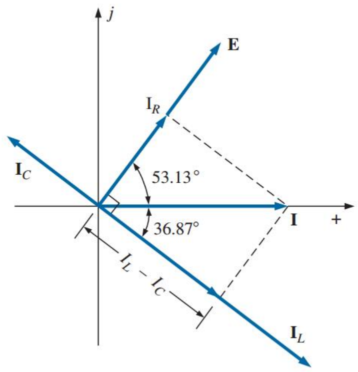


FIG. 16.25
Phasor diagram for the parallel R-L-C network in Fig. 16.22.

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (100\text{ V})(50\text{ A})\cos 53.13° = (5000)(0.6)$$
$$= \mathbf{3000\ W}$$

or $\quad P_T = E^2G = (100\text{ V})^2(0.3\text{ S}) = \mathbf{3000\ W}$

or, finally,

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= EI_R\cos\theta_R + EI_L\cos\theta_L + EI_C\cos\theta_C \\ &= (100\text{ V})(30\text{ A})\cos 0° + (100\text{ V})(70\text{ A})\cos 90° \\ &\qquad + (100\text{ V})(30\text{ A})\cos 90° \\ &= 3000\text{ W} + 0 + 0 \\ &= \mathbf{3000\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$ ($\mathbf{Z}_T$ has a positive angle)

2. Current Divider Rule – parallel networks

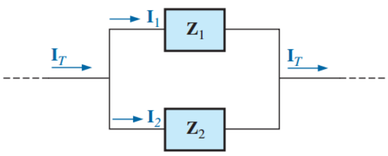


FIG. 16.27
Applying the current divider rule.

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ as shown in Fig. 16.27.

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{16.23}$$

EXAMPLE 16.6 Using the current divider rule, find the current through each impedance in Fig. 16.28.

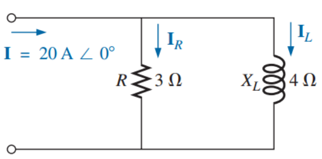


FIG. 16.28
Example 16.6.

Solution:

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega\angle 90°)(20\text{ A}\angle 0°)}{3\ \Omega\angle 0° + 4\ \Omega\angle 90°} = \frac{80\text{ A}\angle 90°}{5\angle 53.13°}$$
$$= \mathbf{16\ A\angle 36.87°}$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega\angle 0°)(20\text{ A}\angle 0°)}{5\ \Omega\angle 53.13°} = \frac{60\text{ A}\angle 0°}{5\angle 53.13°}$$
$$= \mathbf{12\ A\angle -53.13°}$$

3. Frequency Response of Parallel Elements

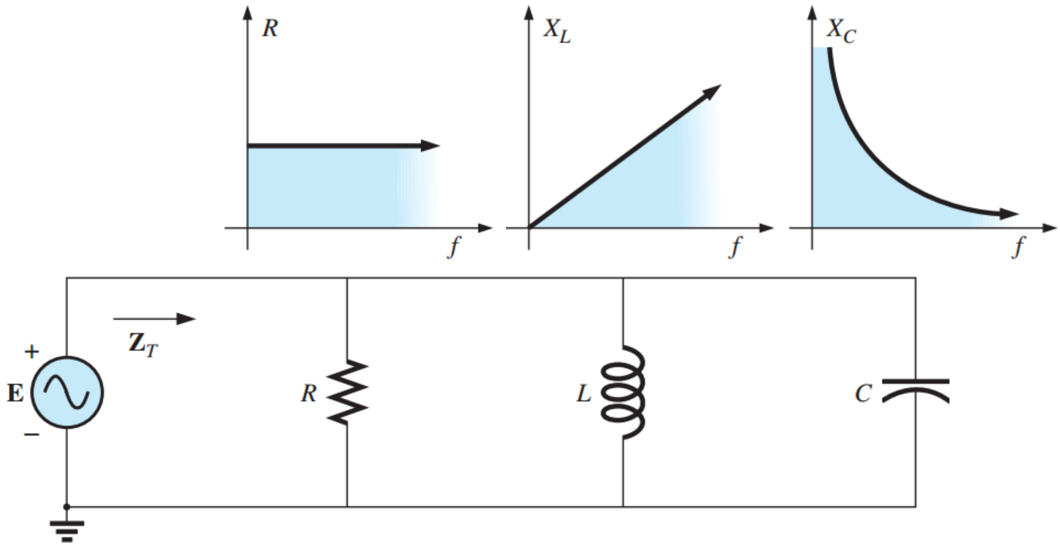


FIG. 16.30
Frequency response for parallel R-L-C elements.

$$\mathbf{Y}_T = \frac{1}{\mathbf{R}} + \frac{1}{\mathbf{X}_L} + \frac{1}{\mathbf{X}_C} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}$$

$$= \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C} = \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$\mathbf{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

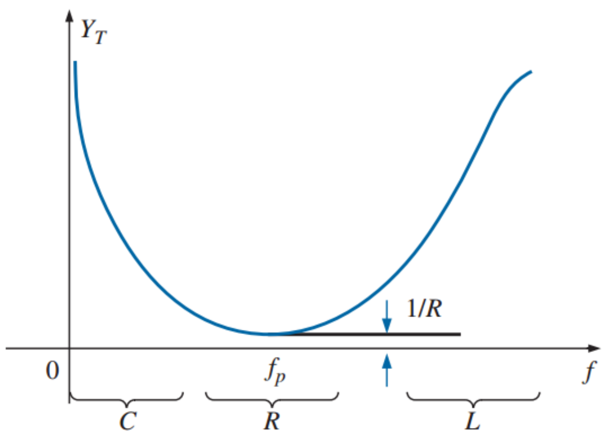


FIG. 16.31
Impedance versus frequency for the network of Fig. 16.30.

When $X_L = X_C$ the total admittance will simply be

$$\boxed{Y_T = \frac{1}{R}}_{\text{at } f_p} \quad \textbf{(16.24)}$$

and the total impedance:

$$\boxed{Z_T = R}_{\text{at } f_p} \quad \textbf{(16.25)}$$

The peak frequency can be found in the same manner as for the series *R-L-C* circuit:

$$X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_p = \frac{1}{2\pi\sqrt{LC}}} \quad \textbf{(16.26)}$$

Parallel *R-L* ac Network

Let us now note the impact of frequency on the total impedance and inductive current for the parallel *R-L* network in Fig. 16.32 for a frequency range of zero through 40 kHz.

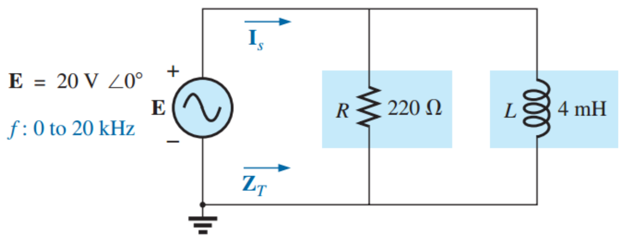


FIG. 16.32

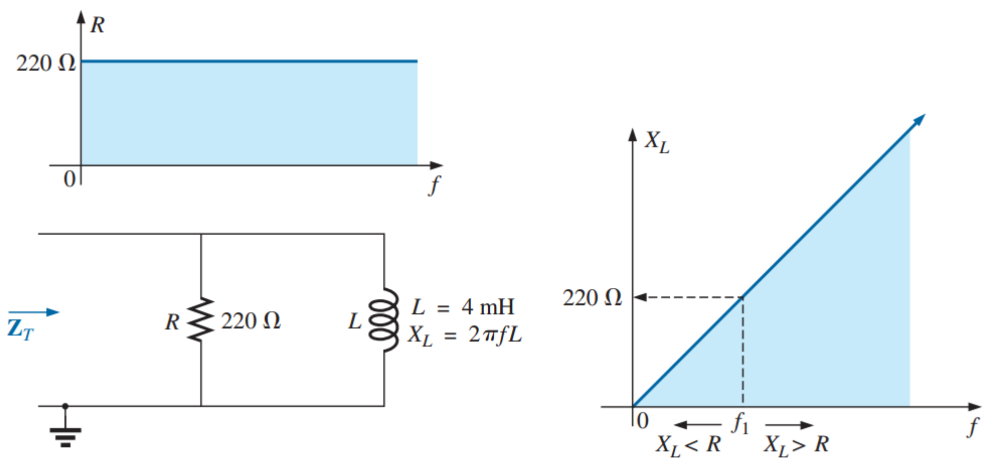


FIG. 16.33
The frequency response of the individual elements of a parallel R-L network.

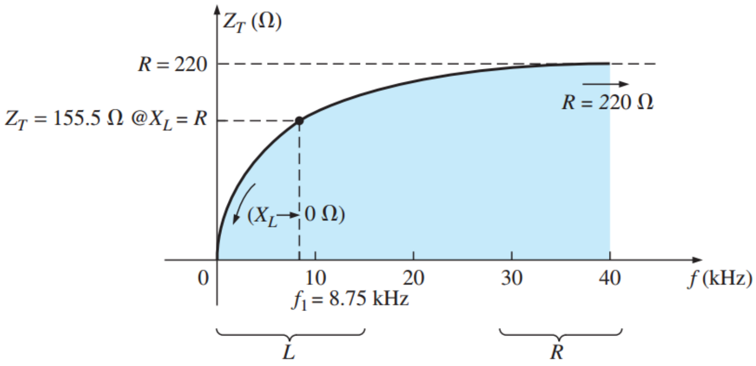


FIG. 16.34
Impedance versus frequency for the parallel ac network of Fig. 16.32.

$$X_L = 2\pi f_1 L = R$$

and $$f_1 = \frac{R}{2\pi L} \qquad \textbf{(16.27)}$$

which for the network in Fig. 16.32 is

$$f_1 = \frac{R}{2\pi L} = \frac{220\ \Omega}{2\pi(4 \times 10^{-3}\,\text{H})} \cong \mathbf{8.75\ kHz}$$

which falls within the frequency range of interest.

and $$\mathbf{Z}_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} \angle 90^\circ - \tan^{-1} X_L/R$$

so that $$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} \qquad \textbf{(16.28)}$$

and $$\theta_T = 90^\circ - \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{R}{X_L} \qquad \textbf{(16.29)}$$

The magnitude and angle of the total impedance can now be found at any frequency of interest simply by substituting into Eqs. (16.28) and (16.29).

$f = 1$ kHz

$$X_L = 2\pi f L = 2\pi(1\ \text{kHz})(4 \times 10^{-3}\,\text{H}) = 25.12\ \Omega$$

and

$$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}} = \frac{(220\ \Omega)(25.12\ \Omega)}{\sqrt{(220\ \Omega)^2 + (25.12\ \Omega)^2}} = \mathbf{24.96\ \Omega}$$

with $$\theta_T = \tan^{-1}\frac{R}{X_L} = \tan^{-1}\frac{220\ \Omega}{25.12\ \Omega} = \tan^{-1}8.76 = 83.49^\circ$$

and $$\mathbf{Z}_T = \mathbf{24.96\ \Omega \angle 83.49^\circ}$$

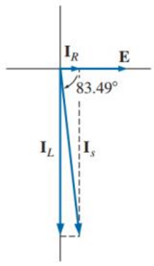


FIG. 16.39
The phasor diagram for the parallel R-L network in Fig. 16.32 at f = 1 kHz.

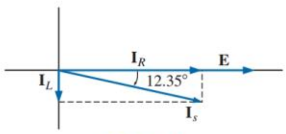


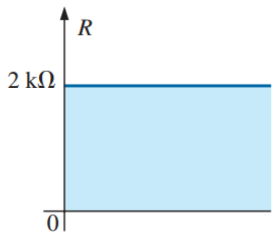
FIG. 16.40
The phasor diagram for the parallel R-L network in Fig. 16.32 at f = 40 kHz.

Parallel *R-C* ac Network

$$R = X_C$$

or $$R = \frac{1}{2\pi fC}$$

and $$f_1 = \frac{1}{2\pi RC} \tag{16.30}$$



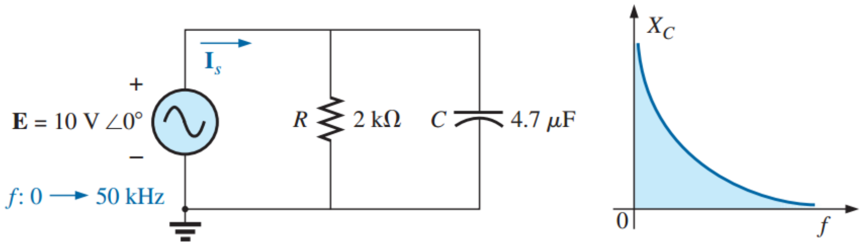


FIG. 16.41

which for the network of Fig. 16.41 is

$$f_1 = \frac{1}{2\pi RC} = \frac{1}{2\pi(2\ \text{k}\Omega)(4.7\ \mu\text{F})} = 16.93\ \text{kHz}$$

$$Z_T = \frac{RX_C}{R + X_C} = \frac{(R\angle 0^\circ)(X_C\angle -90^\circ)}{R - jX_C} = \frac{RX_C\angle -90^\circ}{\sqrt{R^2 + X_C^2}\ \big/ -\tan^{-1}\frac{X_C}{R}}$$

$$= \frac{RX_C}{\sqrt{R^2 + X_C^2}}\ \big/ -90^\circ + \tan^{-1}\frac{X_C}{R} = \frac{RX_C}{\sqrt{R^2 + X_C^2}}\ \big/ -\tan^{-1}\frac{R}{X_C}$$

so that $$Z_T = \frac{RX_C}{\sqrt{R^2 + X_C^2}}$$

and $$\theta_T = -\tan^{-1}\frac{R}{X_C}$$

At $f = f_1$: $$X_C = R = 2\ \text{k}\Omega$$

and $$Z_T = \frac{RX_C}{\sqrt{R^2 + X_C^2}} = \frac{(2\ \text{k}\Omega)(2\ \text{k}\Omega)}{\sqrt{(2\ \text{k}\Omega)^2 + (2\ \text{k}\Omega)^2}} = \mathbf{1.41\ k\Omega}$$

with $$\theta_T = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1}\frac{2\ \text{k}\Omega}{2\ \text{k}\Omega} = -\tan^{-1}1 = \mathbf{-45^\circ}$$

4. Series Parallel Networks

EXAMPLE 17.1 For the network in Fig. 17.1:

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.
e. Compute the power delivered.
f. Find F_p of the network.

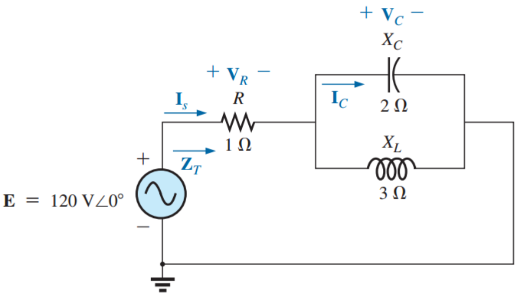


FIG. 17.1
Example 17.1.

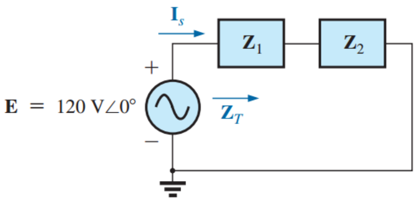


FIG. 17.2
Network in Fig. 17.1 after assigning the block impedances.

The total impedance is defined by

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

with

$$\mathbf{Z}_1 = R\angle 0° = 1\ \Omega\angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \| \mathbf{Z}_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-jX_C + jX_L} = \frac{(2\ \Omega\angle -90°)(3\ \Omega\angle 90°)}{-j2\ \Omega + j3\ \Omega}$$

$$= \frac{6\Omega\angle 0°}{j1} = \frac{6\ \Omega\angle 0°}{1\angle 90°} = 6\ \Omega\angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j6\ \Omega = \mathbf{6.08\ \Omega\angle -80.54°}$$

b. $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\ \text{V}\angle 0°}{6.08\ \Omega\angle -80.54°} = \mathbf{19.74\ A\angle 80.54°}$$

c. Referring to Fig. 17.2, we find that $\mathbf{V}_R$ and $\mathbf{V}_C$ can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\ \text{A}\angle 80.54°)(1\ \Omega\angle 0°) = \mathbf{19.74\ V\angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\ \text{A}\angle 80.54°)(6\ \Omega\angle -90°) = \mathbf{118.44\ V\angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law:

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\ \text{V}\angle -9.46°}{2\ \Omega\angle -90°} = \mathbf{59.22\ A\angle 80.54°}$$

e. $P_{\text{del}} = I_s^2 R = (19.74\ \text{A})^2(1\ \Omega) = \mathbf{389.67\ W}$

f. $F_p = \cos\theta = \cos 80.54° = \mathbf{0.164\ leading}$

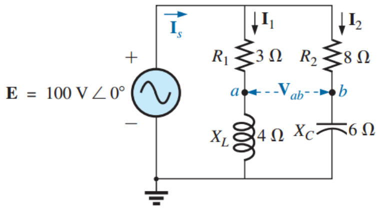


FIG. 17.7
Example 17.4.

EXAMPLE 17.4 For Fig. 17.7:

a. Calculate the current $\mathbf{I}_s$.
b. Find the voltage $\mathbf{V}_{ab}$.

Solutions:

a. Redrawing the circuit as in Fig. 17.8, we obtain

$$\mathbf{Z}_1 = R_1 + jX_L = 3\ \Omega + j4\ \Omega = 5\ \Omega \angle 53.13°$$
$$\mathbf{Z}_2 = R_2 - jX_C = 8\ \Omega - j6\ \Omega = 10\ \Omega \angle -36.87°$$

In this case the voltage $\mathbf{V}_{ab}$ is lost in the redrawn network, but the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ remain defined for future calculations necessary to determine $\mathbf{V}_{ab}$. Fig. 17.8 clearly reveals that the total impedance can be found using the equation for two parallel impedances:

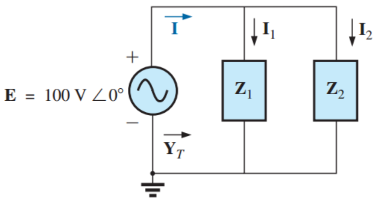


FIG. 17.8
Network in Fig. 17.7 after assigning the block impedances.

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega \angle 53.13°)(10\ \Omega \angle -36.87°)}{(3\ \Omega + j4\ \Omega) + (8\ \Omega - j6\ \Omega)}$$
$$= \frac{50\ \Omega \angle 16.26°}{11 - j2} = \frac{50\ \Omega \angle 16.26°}{11.18 \angle -10.30°}$$
$$= \mathbf{4.47\ \Omega \angle 26.56°}$$

and

$$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V} \angle 0°}{4.47\ \Omega \angle 26.56°} = \mathbf{22.36\ A \angle -26.56°}$$

b. By Ohm's law,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{100\ \text{V} \angle 0°}{5\ \Omega \angle 53.13°} = \mathbf{20\ A \angle -53.13°}$$
$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{100\ \text{V} \angle 0°}{10\ \Omega \angle -36.87°} = \mathbf{10\ A \angle 36.87°}$$

Returning to Fig. 17.7, we have

$$\mathbf{V}_{R_1} = \mathbf{I}_1\mathbf{Z}_{R_1} = (20\ \text{A} \angle -53.13°)(3\ \Omega \angle 0°) = \mathbf{60\ V \angle -53.13°}$$
$$\mathbf{V}_{R_2} = \mathbf{I}_1\mathbf{Z}_{R_2} = (10\ \text{A} \angle +36.87°)(8\ \Omega \angle 0°) = \mathbf{80\ V \angle +36.87°}$$

Instead of using the two steps just shown, we could have determined $\mathbf{V}_{R_1}$ or $\mathbf{V}_{R_2}$ in one step using the voltage divider rule:

$$\mathbf{V}_{R_1} = \frac{(3\ \Omega \angle 0°)(100\ \text{V} \angle 0°)}{3\ \Omega \angle 0° + 4\ \Omega \angle 90°} = \frac{300\ \text{V} \angle 0°}{5 \angle 53.13°} = \mathbf{60\ V \angle -53.13°}$$

To find $\mathbf{V}_{ab}$, Kirchhoff's voltage law must be applied around the loop (Fig. 17.9) consisting of the 3 Ω and 8 Ω resistors. By Kirchhoff's voltage law,

$$\mathbf{V}_{ab} + \mathbf{V}_{R_1} - \mathbf{V}_{R_2} = 0$$

or

$$\mathbf{V}_{ab} = \mathbf{V}_{R_2} - \mathbf{V}_{R_1}$$
$$= 80\ \text{V} \angle 36.87° - 60\ \text{V} \angle -53.13°$$
$$= (64 + j48) - (36 - j48)$$
$$= 28 + j96$$
$$\mathbf{V}_{ab} = \mathbf{100\ V \angle 73.74°}$$

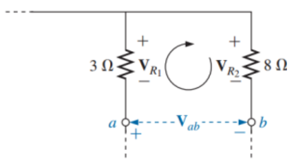


FIG. 17.9
Determining the voltage V_{ab} for the network in Fig. 17.7.

5. Glossary – English/Chinese Translation

English	Chinese
impedance and admittance	阻抗和导纳
resistive inductive and	电阻电感和
capacitive elements	电容元件
reactance and susceptance	电抗和束缚
admittance phasor diagram	导纳相量图
impedance phasor diagram	阻抗相量图
series and parallel networks	串联和并联网络