Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-g

AC Sinusoidal Waveform

Contents

Reference:

Introductory Circuit Analysis 14th edition, Boylesad & Olivari
Basic Circuit Analysis – Schaum's Outline Series

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1. The Sinusoidal Waveform

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.

The units of measurement Degrees and Radians, are related as shown in Fig. 13.14. The conversions equations between the two are the following:

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees}) \tag{13.6}$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians}) \tag{13.7}$$

The velocity with which the radius vector rotates about the center, called the **angular velocity**, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degree or radians)}}{\text{time (seconds)}} \tag{13.8}$$

Substituting into Eq. (13.8) and assigning the lowercase Greek letter *omega* (ω) to the angular velocity, we have

$$\omega = \frac{\alpha}{t} \tag{13.9}$$

and

$$\alpha = \omega t \tag{13.10}$$

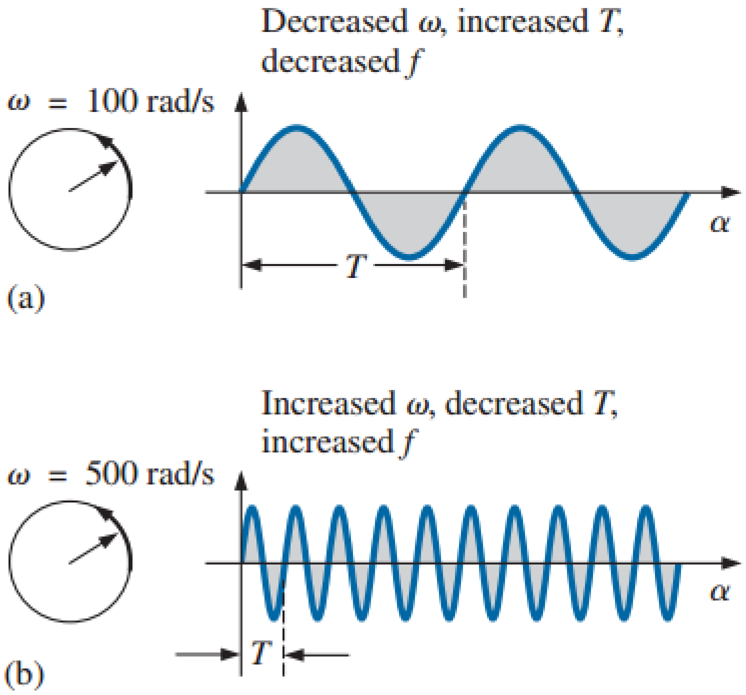

FIG. 13.17
Demonstrating the effect of ω on the frequency and period.

$$\boxed{\omega = \frac{2\pi}{T}} \qquad \text{(rad/s)} \qquad \textbf{(13.11)}$$

In words, this equation states that the smaller the period of the sinusoidal waveform of Fig. 13.16(i), or the smaller the time interval before one complete cycle is generated, the greater must be the angular velocity of the rotating radius vector. Certainly this statement agrees with what we have learned thus far. We can now go one step further and apply the fact that the frequency of the generated waveform is inversely related to the period of the waveform; that is, $f = 1/T$. Thus,

$$\boxed{\omega = 2\pi f} \qquad \text{(rad/s)} \qquad \textbf{(13.12)}$$

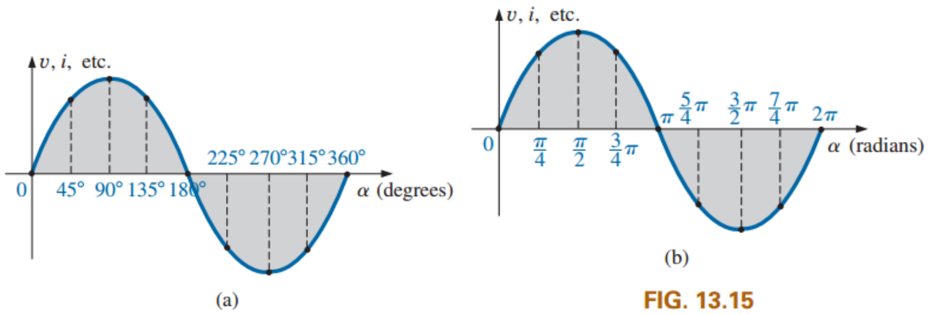

FIG. 13.15

2. Sinusoidal Voltage and Current

The basic mathematical format for the sinusoidal waveform is

$$\boxed{A_m \sin \alpha} \tag{13.13}$$

Due to Eq. (13.10), the general format of a sine wave can also be written

$$\boxed{A_m \sin \omega t} \tag{13.14}$$

with ωt as the horizontal unit of measure.

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$
$$e = E_m \sin \omega t = E_m \sin \alpha$$

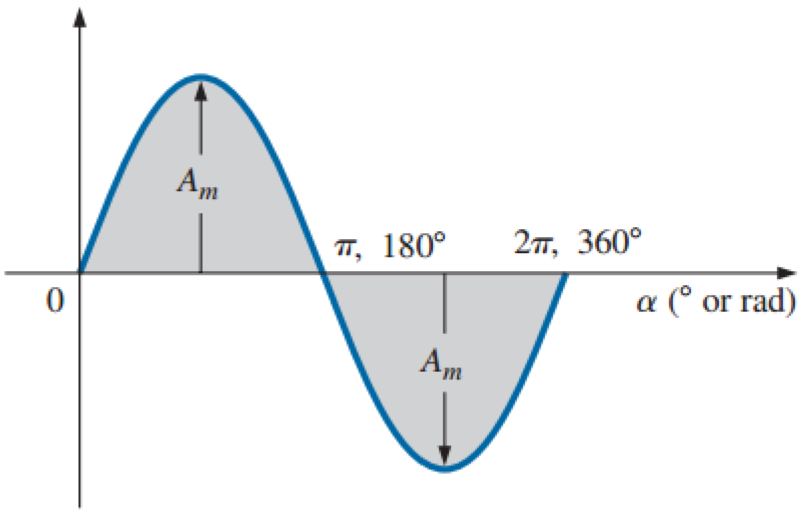

FIG. 13.18
Basic sinusoidal function.

EXAMPLE 13.9

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
b. Determine the time at which the magnitude is attained.

Solutions:

a. Eq. (13.15):

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.58^\circ}$$

However, Fig. 13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.42^\circ}$$

In general, therefore, keep in mind that Eqs. (13.15) and (13.16) will provide an angle with a magnitude between 0° and 90°.

b. Eq. (13.10): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha(\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412\text{ rad}$$

and

$$t_1 = \frac{\alpha}{\omega} = \frac{0.412\text{ rad}}{377\text{ rad/s}} = \mathbf{1.09\text{ ms}}$$

For the second intersection,

$$\alpha(\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73\text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\text{ rad}}{377\text{ rad/s}} = \mathbf{7.24\text{ ms}}$$

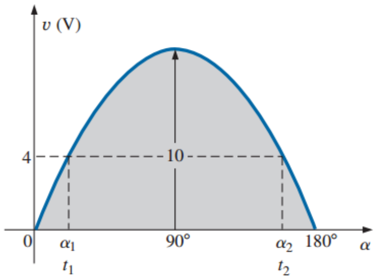

FIG. 13.19
Example 13.9.

13.6 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π, as shown in Fig. 13.25. If the waveform is shifted to the right or left of 0°, the expression becomes

$$\boxed{A_m \sin(\omega t \pm \theta)} \qquad \textbf{(13.17)}$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) slope *before* 0°, as shown in Fig. 13.27, the expression is

$$\boxed{A_m \sin(\omega t + \theta)} \qquad \textbf{(13.18)}$$

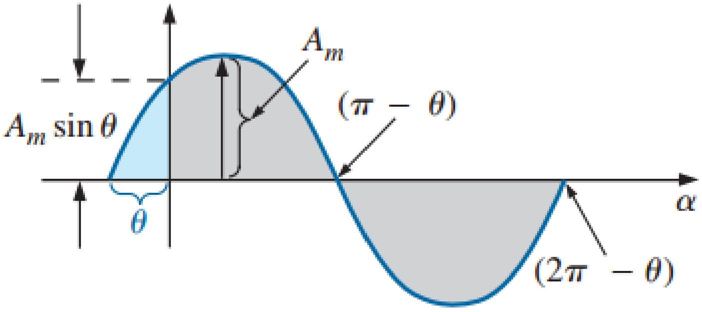

FIG. 13.27
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

$$\boxed{A_m \sin(\omega t - \theta)} \qquad \textbf{(13.19)}$$

Finally, at $\omega t = \alpha = 0°$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin\theta$.

If the waveform crosses the horizontal axis with a positive-going slope 90° $(\pi/2)$ sooner, as shown in Fig. 13.29, it is called a *cosine wave;* that is,

$$\boxed{\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t} \qquad \textbf{(13.20)}$$

or

$$\boxed{\sin\omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)} \qquad \textbf{(13.21)}$$

The terms leading and lagging are used to indicate the relationship between two sinusoidal waveforms of the same frequency plotted on the same set of axes.

$$\boxed{\theta = \frac{\text{phase shift(no. of div.)}}{T\text{(no. of div.)}} \times 360°} \qquad \textbf{(13.24)}$$

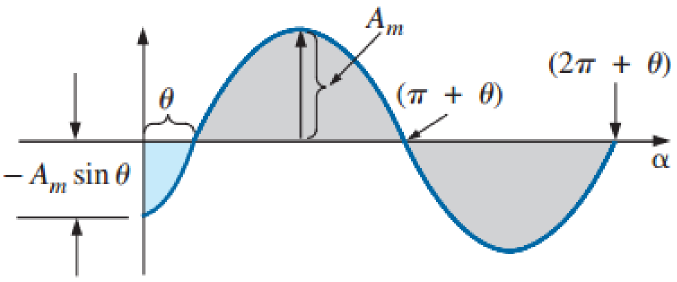

FIG. 13.28
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.

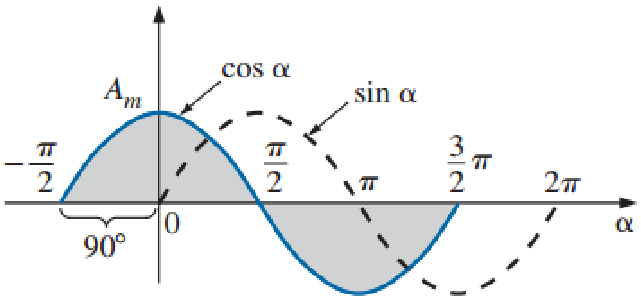

FIG. 13.29
Phase relationship between a sine wave and a cosine wave.

3. Average and rms values

Average value

$$G(\text{average value}) = \frac{\text{algebraic sum of areas}}{\text{length of curve}} \qquad \textbf{(13.26)}$$

The *algebraic* sum of the areas must be determined since some area contributions are from below the horizontal axis. Areas above the axis are assigned a positive sign and those below it a negative sign. A positive average value is then above the axis, and a negative value is below it.

EXAMPLE 13.14 Determine the average value of the waveforms in Fig. 13.44.

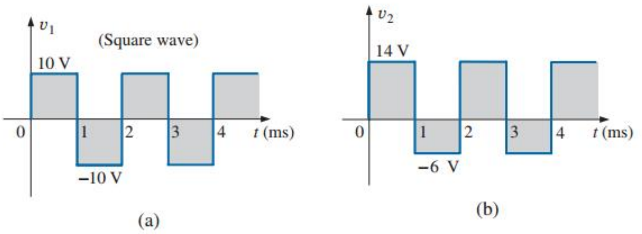

FIG. 13.44
Example 13.14.

Solutions:

a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using Eq. (13.26) gives

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

b. Using Eq. (13.26) gives

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

as shown in Fig. 13.45.

In reality, the waveform in Fig. 13.44(b) is simply the square wave in Fig. 13.44(b) with a dc shift of 4 V; that is,

$$v_2 = v_1 + 4\text{ V}$$

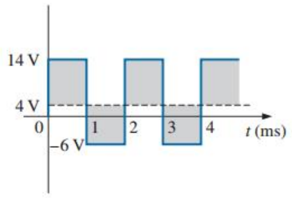

FIG. 13.45
Defining the average value for the waveform in Fig. 13.44(b).

Effective (rms) value

Calculus format:

$$\boxed{I_{\text{rms}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}} \qquad \textbf{(13.31)}$$

which means

$$\boxed{I_{\text{rms}} = \sqrt{\frac{\text{area}\,(i^2(t))}{T}}} \qquad \textbf{(13.32)}$$

In words, Eqs. (13.31) and (13.32) state that to find the rms value, the function $i(t)$ must first be squared. After $i(t)$ is squared, the area under the curve is found by integration. It is then divided by T, the length of the cycle or the period of the waveform, to obtain the average or *mean* value of the squared waveform. The final step is to take the *square root* of the mean value. This procedure is the source for the other designation for the effective value, the **root-mean-square (rms) value**. In fact, since *rms* is the most commonly used term in the educational and industrial communities, it is used throughout this text.

The relationship between the peak value and the rms value is the same for voltages, resulting in the following set of relationships for the examples and text material to follow:

$$\boxed{\begin{aligned} I_{\text{rms}} &= \frac{1}{\sqrt{2}} I_m = 0.707 I_m \\ E_{\text{rms}} &= \frac{1}{\sqrt{2}} E_m = 0.707 E_m \end{aligned}} \tag{13.33}$$

Similarly,

$$\boxed{\begin{aligned} I_m &= \sqrt{2} I_{\text{rms}} = 1.414 I_{\text{rms}} \\ E_m &= \sqrt{2} E_{\text{rms}} = 1.414 E_{\text{rms}} \end{aligned}} \tag{13.34}$$

EXAMPLE 13.21 The 120 V dc source in Fig. 13.61(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13.61(b)] is to deliver the same power to the load.

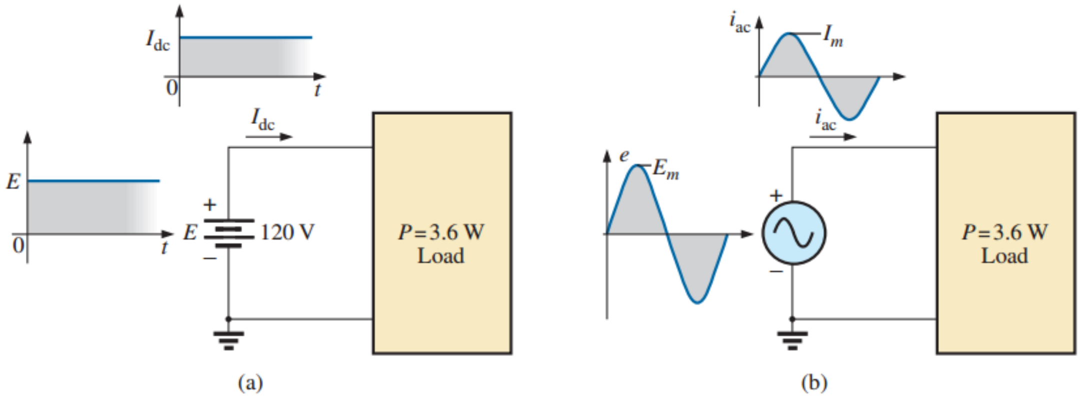

FIG. 13.61
Example 13.21.

Solution:

$$P_{dc} = V_{dc} I_{dc}$$

and
$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2} I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$

$$E_m = \sqrt{2} E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

EXAMPLE 13.23 Calculate the rms value of the voltage in Fig. 13.64.

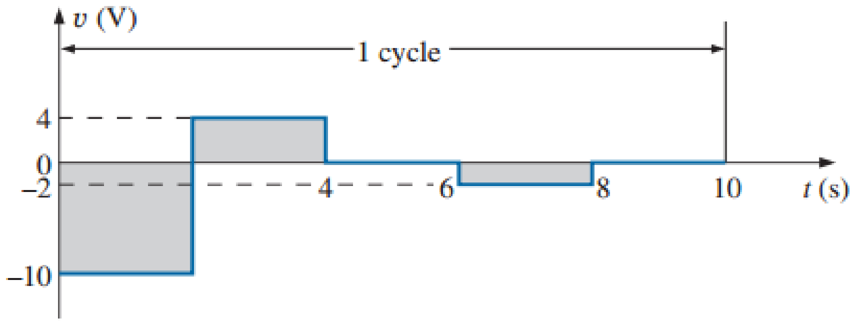

FIG. 13.64
Example 13.23.

Solution: v^2 (Fig. 13.65):

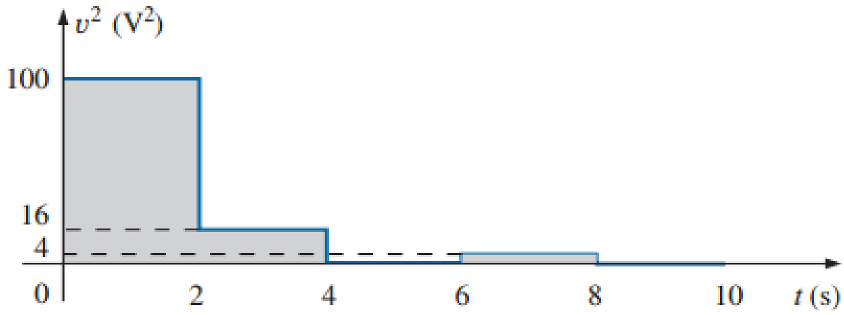

FIG. 13.65
The squared waveform of Fig. 13.64.

$$
\begin{aligned}
V_{\mathrm{rms}} &= \sqrt{\frac{(100\ \mathrm{V}^2)(2\ \mathrm{s}) + (16\ \mathrm{V}^2)(\mathrm{s}) + (4\ \mathrm{V}^2)(2\ \mathrm{s})}{10\ \mathrm{s}}} \\
&= \sqrt{\frac{200\ \mathrm{V}^2\mathrm{s} + 32\ \mathrm{V}^2\mathrm{s} + 8\ \mathrm{V}^2\mathrm{s}}{10\ \mathrm{s}}} \\
&= \sqrt{\frac{240}{10}\mathrm{V}^2} = \sqrt{24\ \mathrm{V}^2} \\
&= \mathbf{4.9\ V}
\end{aligned}
$$

4. Response to L-R-C to AC waveforms

Resistor

For power-line frequencies and frequencies up to a few hundred kilohertz, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current. For this frequency region, the resistor R in Fig. 14.4 can be treated as a constant, and Ohm's law can be applied as follows. For $\upsilon = V_m \sin \omega t$,

$$i = \frac{\upsilon}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where
$$\boxed{I_m = \frac{V_m}{R}} \qquad \textbf{(14.2)}$$

In addition, for a given i,

$$\upsilon = iR = (I_m \sin \omega t) R = I_m R \sin \omega t = V_m \sin \omega t$$

where
$$\boxed{V_m = I_m R} \qquad \textbf{(14.3)}$$

A plot of υ and i in Fig. 14.5 reveals that

For a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

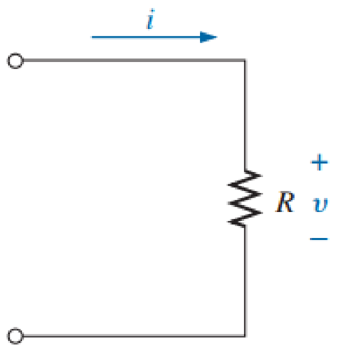

FIG. 14.4

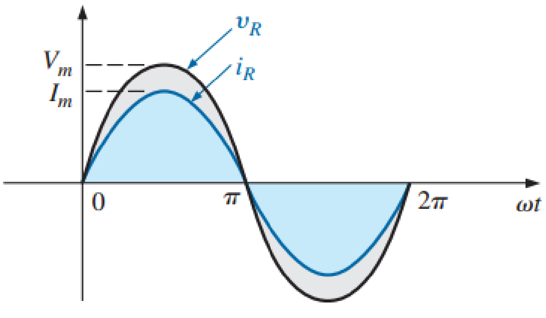

FIG. 14.5
The voltage and current of a resistive element are in phase.

Inductor

We found in Chapter 11 that the voltage across the inductor of Fig. 14.6 is directly related to the inductance of the coil and the rate of change of current through the coil. A relationship defined by the following equation:

$$v_L = L\frac{di_L}{dt}$$

Consequently, the higher the frequency, the greater is the rate of change of current through the coil, and the greater is the magnitude of the voltage. In addition, we found in the same chapter that the inductance of a coil determines the rate of change of the flux linking a coil for a particular change in current through the coil. The higher the inductance, the greater is the rate of change of the flux linkages, and the greater is the resulting voltage across the coil.

For a sinusoidal current defined by

$$i_L = I_m \sin\omega t$$

we can calculate the voltage across the coil by differentiating the current through the coil and substituting into the basic equation above. That is,

$$\begin{aligned} v_L &= L\frac{di_L}{dt} = L\frac{d}{dt}(I_m \sin\omega t) = LI_m\frac{d}{dt}(\sin\omega t) \\ &= LI_m(\omega\cos\omega t) \end{aligned}$$

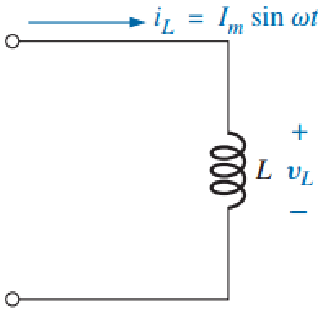

FIG. 14.6

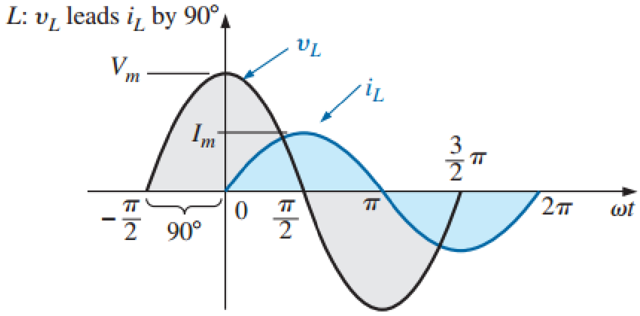

FIG. 14.7
For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

The peak value of the voltage across a coil is directly related to the applied frequency ($\omega = 2\pi f$), the inductance of the coil L, and the peak value of the applied current I_m. A plot of υ_L and i_L in Fig. 14.7 reveals that for an inductor, υ_L leads i_L by 90°, or i_L lags υ_L by 90°.

The quantity ωL, called the **reactance** (from the word *reaction*) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$\boxed{X_L = \omega L} \quad (\text{ohms}, \Omega) \tag{14.4}$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_L = \frac{V_m}{I_m}} \quad (\text{ohms}, \Omega) \tag{14.5}$$

Once the reactance is known, the peak value of the voltage or current can be found from the other by simply applying Ohm's law as follows:

$$\boxed{I_m = \frac{V_m}{X_L}} \tag{14.6}$$

and

$$\boxed{V_m = I_m X_L} \tag{14.7}$$

Capacitor

Let us now examine the capacitive configuration of Fig. 14.8. For the capacitor, we will determine i for a particular voltage across the element rather than the voltage as was determined for the inductive element. When this approach reaches its conclusion, we will know the relationship between the voltage and current and the opposition level to sinusoidally applied emfs.

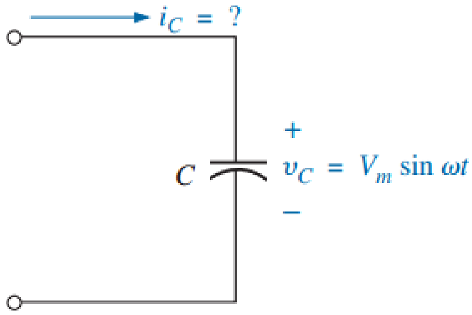

FIG. 14.8

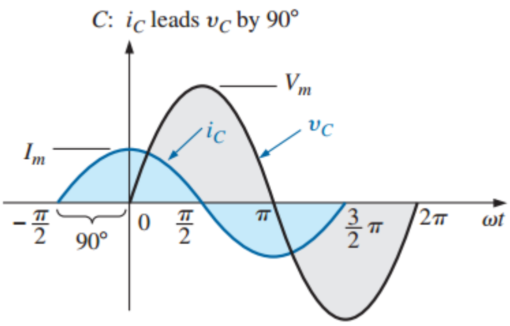

FIG. 14.9
The current of a purely capacitive element leads the voltage across the element by 90°.

For the capacitor of Fig. 14.8, we recall from Chapter 10 that

$$i_C = C\frac{dv_C}{dt}$$

Substituting

$$v_C = V_m \sin \omega t$$

and, applying differentiation, we obtain

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t$$

so that

$$i_C = \omega C V_m \sin(\omega t + 90°)$$

The peak value for the current of a capacitor is directly related to the applied frequency ($\omega = 2\pi f$), the capacitance of the capacitor (C), and the peak value of the applied voltage V_m. The plot of Fig. 14.9 reveals that for a capacitor, i_C leads v_C, or v_C lags i_C by 90°.

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by X_C and is measured in ohms; that is,

$$\boxed{X_C = \frac{1}{\omega C}} \quad (\text{ohms, } \Omega) \qquad \textbf{(14.8)}$$

In an Ohm's law format, its magnitude can be determined from

$$\boxed{X_C = \frac{V_m}{I_m}} \quad (\text{ohms, } \Omega) \qquad \textbf{(14.9)}$$

Once the reactance is known, the peak value of the voltage or current can be found from the other by simply applying Ohm's law as follows:

$$\boxed{I_m = \frac{V_m}{X_C}} \qquad \textbf{(14.10)}$$

and

$$\boxed{V_m = I_m X_C} \qquad \textbf{(14.11)}$$

To summarize…

In the inductive circuit,

$$\boxed{v_L = L\frac{di_L}{dt}} \qquad \textbf{(14.12a)}$$

and through integration:

$$\boxed{i_L = \frac{1}{L}\int v_L \, dt} \qquad \textbf{(14.12b)}$$

In the capacitive circuit,

$$\boxed{i_C = C\frac{dv_C}{dt}} \qquad \textbf{(14.13a)}$$

and through integration:

$$\boxed{v_C = \frac{1}{C}\int i_C \, dt} \qquad \textbf{(14.13b)}$$

If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

EXAMPLE 14.3 The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$
b. $i = 7\sin(377t - 70°)$

Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7\ \Omega$
Eq. (14.7): $V_m = I_m X_L = (10 \text{ A})(37.7\ \Omega) = 377 \text{ V}$

and we know that for a coil v leads i by 90°. Therefore,

$$v = \mathbf{377 \sin(377}\boldsymbol{t} \mathbf{+ 90°)}$$

The curves are sketched in Fig. 14.12.

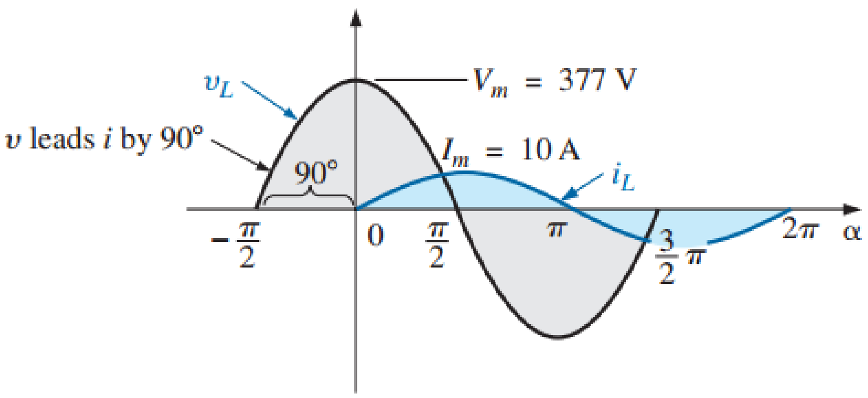

FIG. 14.12
Example 14.3(a).

b. X_L remains at 37.7 Ω.

$$V_m = I_m X_L = (7 \text{ A})(37.7\ \Omega) = 263.9 \text{ V}$$

and we know that for a coil v leads i by 90°. Therefore,

$$v = 263.9 \sin(377t - 70° + 90°)$$

and

$$v = \mathbf{263.9 \sin(377}\boldsymbol{t} \mathbf{+ 20°)}$$

The curves are sketched in Fig. 14.13.

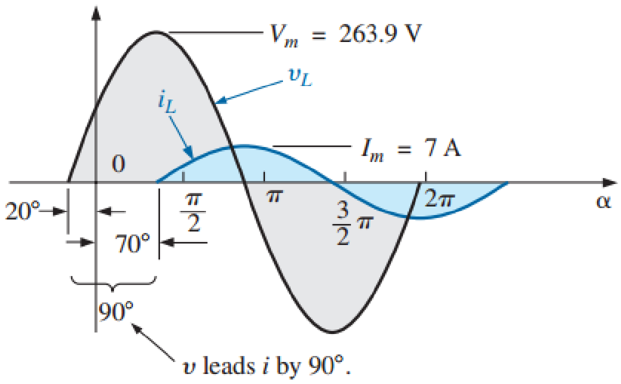

FIG. 14.13
Example 14.3(b).

5. Average Power and Power Factor

To demonstrate this, consider the relatively simple configuration in Fig. 14.26, where an 8 V peak sinusoidal voltage is applied across a 2 Ω resistor. When the voltage is at its positive peak, the power delivered at that instant is 32 W, as shown in the figure. At the midpoint of 4 V, the instantaneous power delivered drops to 8 W; when the voltage crosses the axis, it drops to 0 W. Note, however, that when the applied voltage is at its negative peak, the current may reverse, but at that instant, 32 W is still being delivered to the resistor.

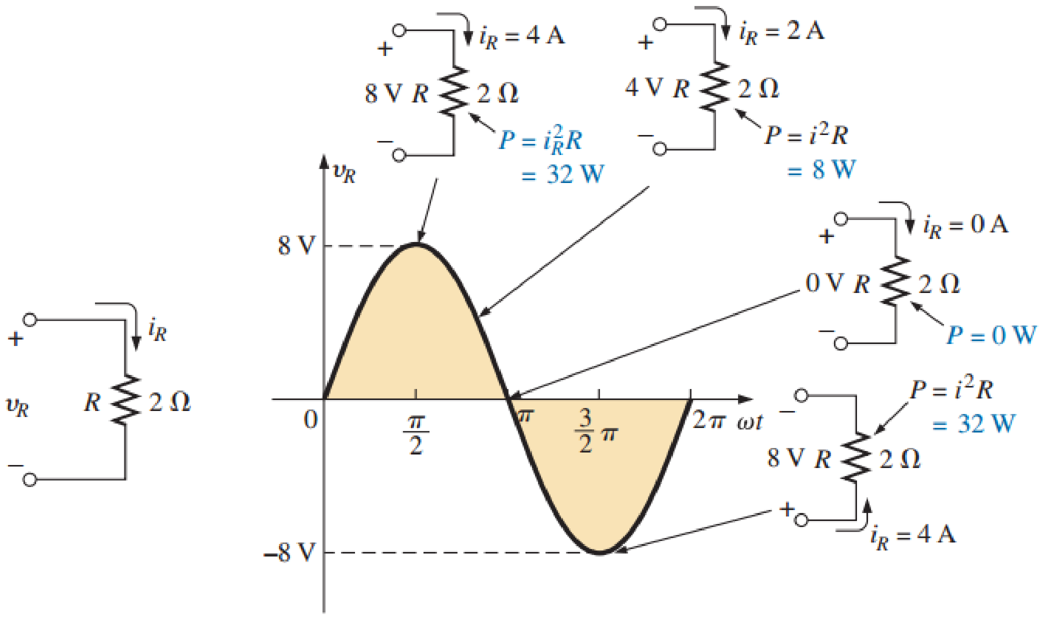

FIG. 14.26
Demonstrating that power is delivered at every instant of a sinusoidal voltage waveform (except v_R = 0 V).

The fact that the power curve is always above the horizontal axis reveals that power is being delivered to the load at each instant of time of the applied sinusoidal voltage.

Any portion of the power curve below the axis reveals that power is being returned to the source. The average value of the power curve occurs at a level equal to $V_m I_m/2$, as shown in Fig. 14.27. This power

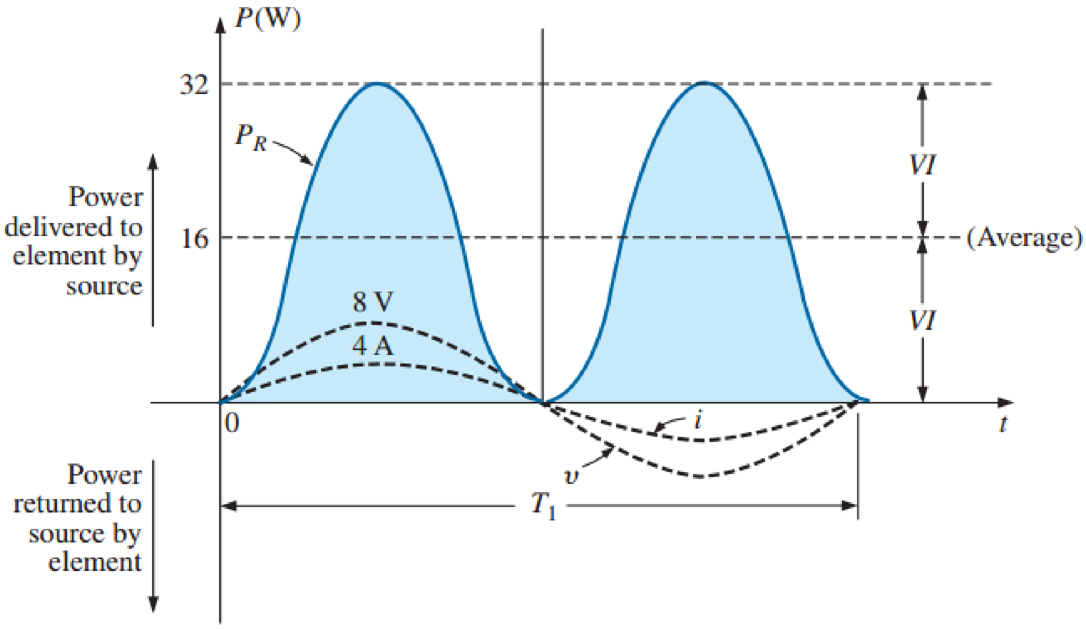

FIG. 14.27
Power versus time for a purely resistive load.

Average or Real Power

If we substitute the equation for the peak value in terms of the rms value as

$$P_{av} = \frac{V_m I_m}{2} = \frac{(\sqrt{2}\, V_{rms})(\sqrt{2}\, I_{rms})}{2} = \frac{2\, V_{rms} I_{rms}}{2}$$

we find that the average or real power delivered to a resistor takes on the following very convenient form:

$$\boxed{P_{av} = V_{rms} I_{rms}} \qquad \textbf{(14.14)}$$

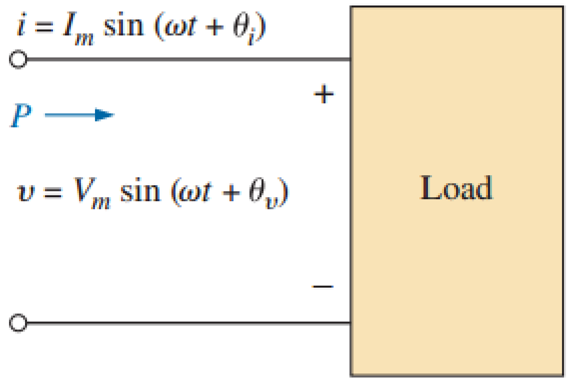

FIG. 14.28

The power delivered at each instant of time is then defined by

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$$

where θ_v is simply the phase angle associated with the applied voltage and θ_i is the phase angle associated with the resulting current.

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

we see that the function $\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$ becomes

$$\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$$
$$= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2}$$
$$= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2}$$

so that

$$p = \overbrace{\left[\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)\right]}^{\text{Time-varying (due to } \omega t \text{ in equation)}}$$

simply the difference in phase angles

A plot of v, i, and ρ on the same set of axes is shown in Fig. 14.29.

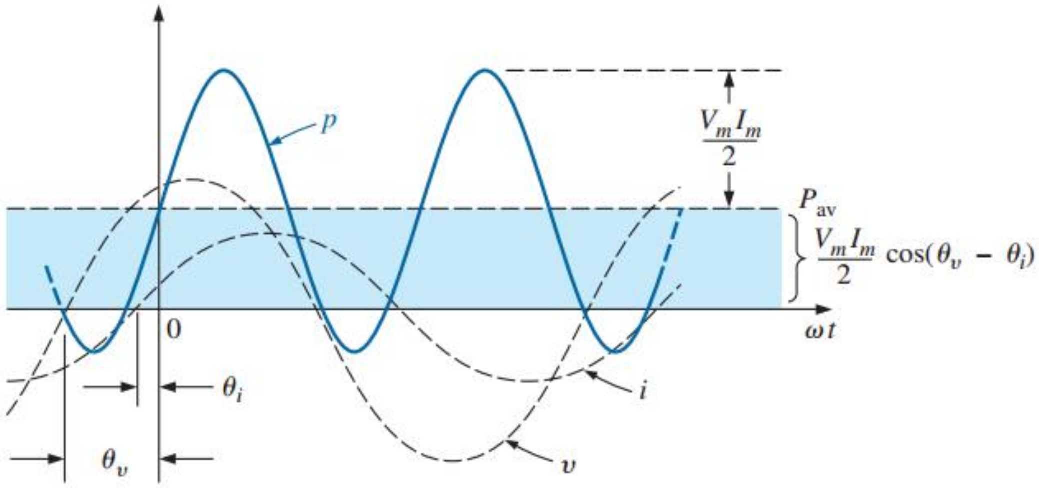

FIG. 14.29
Defining the average power for a sinusoidal ac network.

energy. This term is referred to as the **average power** or **real power** as introduced earlier. The angle $(\theta_v - \theta_i)$ is the phase angle between v and i. Since $\cos(-\alpha) = \cos\alpha$,

the magnitude of average power delivered is independent of whether v leads i or i leads v.

Defining θ as equal to $|\theta_v - \theta_i|$, where $||$ indicates that only the magnitude is important and the sign is immaterial, we have

$$\boxed{P = \frac{V_m I_m}{2}\cos\theta} \quad (\text{watts, W}) \tag{14.15}$$

where P is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

or, since $$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

Eq. (14.15) becomes

$$\boxed{P = V_{\text{rms}} I_{\text{rms}} \cos\theta} \tag{14.16}$$

Let us now apply Eqs. (14.15) and (14.16) to the basic R, L, and C elements.

Resistor

In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0°$, and $\cos\theta = \cos 0° = 1$, so that

$$\boxed{P = \frac{V_m I_m}{2} = V_{\text{rms}} I_{\text{rms}}} \quad (\text{W}) \tag{14.17}$$

or, since

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

then

$$\boxed{P = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R} \quad (\text{W}) \tag{14.18}$$

Inductor

In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2} (0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90°) = \frac{V_m I_m}{2} (0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

EXAMPLE 14.11 Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100\sin(\omega t + 40°)$
$i = 20\sin(\omega t + 70°)$

b. $v = 150\sin(\omega t - 70°)$
$i = 3\sin(\omega t - 50°)$

Solutions:

a. $V_m = 100, \theta_v = 40°$
$I_m = 20 \text{ A}, \theta_i = 70°$
$\theta = |\theta_v - \theta_i| = |40° - 70°| = |-30°| = 30°$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(100 \text{ V})(20 \text{ A})}{2}\cos(30°) = (1000 \text{ W})(0.866)$$
$$= \mathbf{866\ W}$$

b. $V_m = 150 \text{ V}, \quad \theta_v = -70°$
$I_m = 3 \text{ A}, \quad \theta_i = -50°$
$\theta = |\theta_v - \theta_i| = |-70° - (-50°)|$
$= |-70° + 50°| = |-20°| = 20°$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(150 \text{ V})(3 \text{ A})}{2}\cos(20°) = (225 \text{ W})(0.9397)$$
$$= \mathbf{211.43\ W}$$

Power Factor

In the equation $P = (V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name **power factor** and is defined by

$$\boxed{\text{Power factor} = F_p = \cos\theta} \qquad \textbf{(14.19)}$$

In terms of the average power and the terminal voltage and current,

$$\boxed{F_p = \cos\theta = \frac{P}{V_{rms} I_{rms}}} \qquad (14.20)$$

The terms leading and lagging are often written in conjunction with the power factor. They are defined by the current through the load. If the current leads the voltage across a load, the load has a leading power factor. If the current lags the voltage across the load, the load has a lagging power factor. In other words, capacitive networks have leading power factors, and inductive networks have lagging power factors.

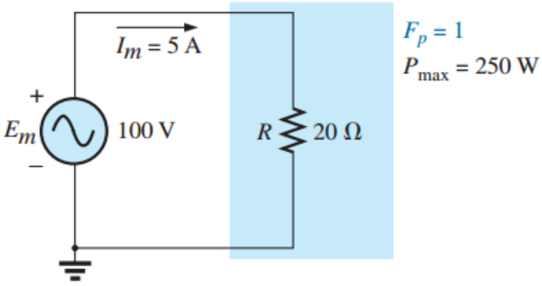

FIG. 14.30
Purely resistive load with $F_p = 1$.

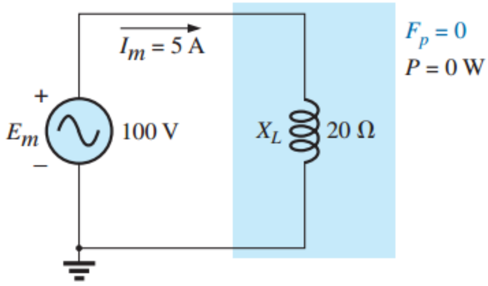

FIG. 14.31
Purely inductive load with $F_p = 0$.

EXAMPLE 14.12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

a. Fig. 14.32
b. Fig. 14.33
c. Fig. 14.34

Solutions:

a. $F_p = \cos\theta = \cos|\theta_v - \theta_i| = \cos|-20° - 40°| = \cos 60° = \mathbf{0.5\ leading}$
b. $F_p = \cos|\theta_v - \theta_i| = \cos|80° - 30°| = \cos 50° = \mathbf{0.64\ lagging}$

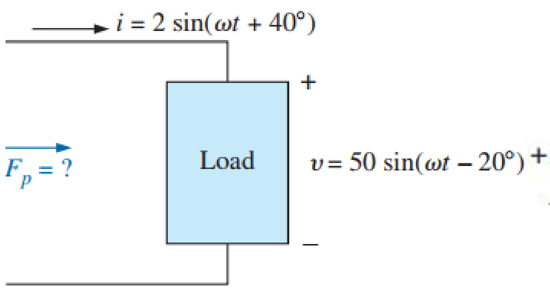

FIG. 14.32

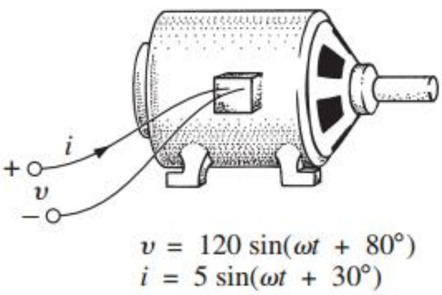

FIG. 14.33

7. Glossary – English/Chinese Translation

English	Chinese
sinusoidal waveform	正弦波形
average and rms values	平均值和 RMS 值
average power and rms power	平均功率和 RMS 功率
degrees and radians	度数和弧度
angular velocity	角速度
frequency and period	频率和周期
phase lead and phase lag	相位超前和相位滞后
peak value	峰值
power factor	功率因数