Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-f

RC and RL Transient Circuits

Contents

Reference:

Introductory Circuit Analysis 14th edition, Boylesad & Olivari
Basic Circuit Analysis – Schaum's Outline Series

Email: norbertcheung@szu.edu.cn
Web Site: http://norbert.idv.hk
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1. Instantaneous Value od RC Circuits

Finding the Voltage or Current at a Particular Instant of Time

Occasionally, you may need to determine the voltage or current at a particular instant of time that is not an integral multiple of τ, as in the previous sections. For example, if

$$v_C = 20\text{ V}(1 - e^{(-t/2\text{ ms})})$$

the voltage v_C may be required at $t = 5$ ms, which does not correspond to a particular value of τ. Fig. 10.30 reveals that $(1 - e^{t/\tau})$ is approximately 0.93 at $t = 5\text{ ms} = 2.5\tau$, resulting in $v_C - 20(0.93) - 18.6$ V. Additional accuracy can be obtained by substituting $v = 5$ ms into the equation and solving for v_C using a calculator or table to determine $e^{-2.5}$. Thus,

$$v_C = 20\text{ V}(1 - e^{-5\text{ ms}/2\text{ ms}}) = (20\text{ V})(1 - e^{-2.5}) = (20\text{ V})(1 - 0.082)$$
$$= (20\text{ V})(0.918) = \mathbf{18.36\text{ V}}$$

Find the Time to Reach a Particular Level of Voltage or Current

For example, solving for t in the equation

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$

results in

$$t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)} = \tau(\ln)\frac{(V_i - V_f)}{(v_C - V_f)} \qquad \textbf{(10.23)}$$

noting than

$$\log_e = \ln$$

For example, suppose that

$$v_C = 20\text{ V}(1 - e^{-t/2\text{ ms}})$$

and the time t to reach 10 V is desired. Since $V_i = 0$ V, and $V_f = 20$ V, we have

$$t = \tau(\ln)\frac{(V_i - V_f)}{(v_C - V_f)} = (2\text{ ms})(\ln)\frac{(0\text{ V} - 20\text{ V})}{(10\text{ V} - 20\text{ V})}$$
$$= (2\text{ ms})\left[\ln\left(\frac{-20\text{ V}}{-10\text{ V}}\right)\right] = (2\text{ ms})(\ln 2) = (2\text{ ms})(0.693)$$
$$= \mathbf{1.386\text{ ms}}$$

For the discharge equation,

$$\upsilon_C = Ee^{-t/\tau} = V_i\,(e^{-t/\tau})\text{with } V_f = 0\text{ V}$$

Using Eq. (10.23) gives

$$t = \tau(\ln)\frac{(V_i - V_f)}{(\upsilon_C - V_f)} = \tau(\ln)\frac{(V_i - 0\text{ V})}{(\upsilon_C - 0\text{ V})}$$

and

$$t = \tau \ln \frac{V_i}{\upsilon_C} \qquad \textbf{(10.24)}$$

For the current equation,

$$i_C = \frac{E}{R}e^{-t/\tau} \qquad I_i = \frac{E}{R} \qquad I_f = 0\text{ A}$$

and

$$t = \ln \frac{I_i}{i_C} \qquad \textbf{(10.25)}$$

10.9 THÉVENIN EQUIVALENT: $\tau = R_{TH}C$

EXAMPLE 10.12 The capacitor in Fig. 10.64 is initially charged to 40 V. Find the mathematical expression for υ_C after the closing of the switch. Plot the waveform for υ_C.

Solution: The network is redrawn in Fig. 10.65.

E_{Th}:

$$E_{Th} = \frac{R_3E}{R_3 + R_1 + R_4} = \frac{(18\text{ k}\Omega)(120\text{ V})}{18\text{ k}\Omega + 7\text{ k}\Omega + 2\text{ k}\Omega} = 80\text{ V}$$

R_{Th}:

$$R_{Th} = R_2 + R_3 \| (R_1 + R_4)$$

$$R_{Th} = 5\text{ k}\Omega + (18\text{ k}\Omega)\|(7\text{ k}\Omega + 2\text{ k}\Omega)$$

$$= 5\text{ k}\Omega + 6\text{ k}\Omega = 11\text{ k}\Omega$$

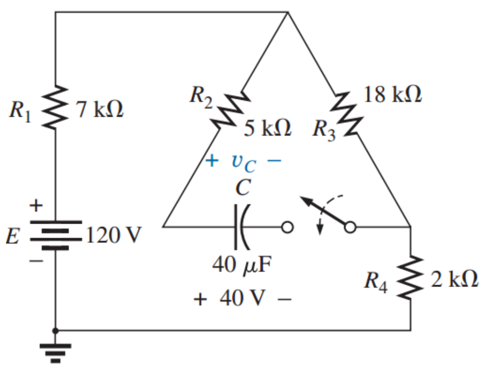

FIG. 10.64

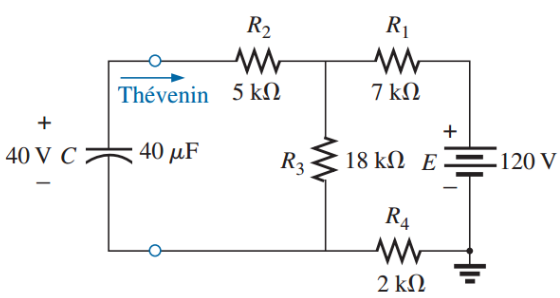

FIG. 10.65
Network in Fig. 10.64 redrawn.

Therefore, $V_i = 40\text{ V}$ and $V_f = 80\text{ V}$

and $\tau = R_{Th}C = (11\text{ k}\Omega)(40\ \mu\text{F}) = 0.44\text{ s}$

Eq. (10.21): $$\upsilon_C = V_f + (V_i - V_f)e^{-t/\tau}$$
$$= 80\text{ V} + (40\text{ V} - 80\text{ V})e^{-t/0.44\text{ s}}$$

and $$\upsilon_C = \mathbf{80\ V - 40\ V}e^{-t/0.44\text{ s}}$$

The waveform appears as in Fig. 10.66.

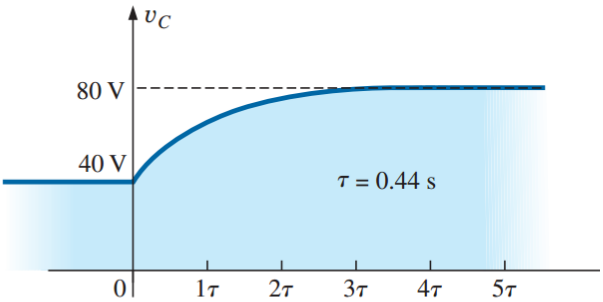

FIG. 10.66
υ_C for the network in Fig. 10.64.

EXAMPLE 10.13 For the network in Fig. 10.67, find the mathematical expression for the voltage υ_C after the closing of the switch (at $t = 0$).

Solution:

$$R_{Th} = R_1 + R_2 = 6\ \Omega + 10\ \Omega = 16\ \Omega$$

$$E_{Th} = V_1 + V_2 = IR_1 + 0$$
$$= (20 \times 10^{-3}\text{ A})(6\ \Omega) = 120 \times 10^{-3}\text{ V} = 0.12\text{ V}$$

and $\tau = R_{Th}C = (16\ \Omega)(500 \times 10^{-6}\text{F}) = 8\text{ ms}$

so that $$\upsilon_C = \mathbf{0.12\ V(1 - e^{-t/8\ ms})}$$

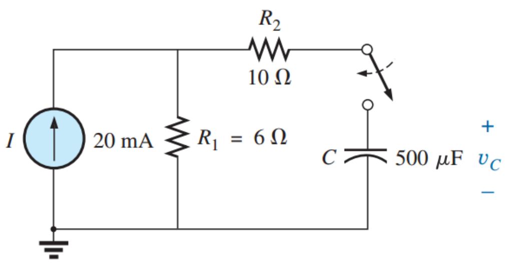

FIG. 10.67

10.10 THE CURRENT i_C

$$i_C = C\frac{dv_C}{dt} \qquad (10.26)$$

The capacitive current is directly related to the rate of change of the voltage across the capacitor, not the levels of voltage involved.

EXAMPLE 10.14 Find the waveform for the average current if the voltage across a 2 μF capacitor is as shown in Fig. 10.68.

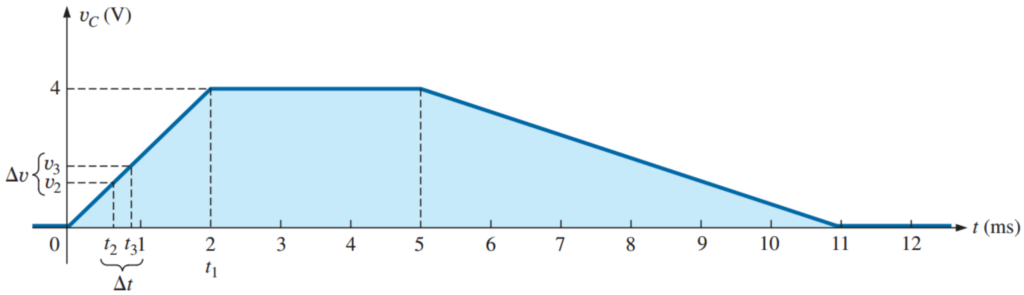

FIG. 10.68
v_C for Example 10.14.

Solutions:

a. From 0 ms to 2 ms, the voltage increases linearly from 0 V to 4 V; the change in voltage $\Delta v = 4\text{ V} - 0 = 4\text{ V}$ (with a positive sign since the voltage increases with time). The change in time $\Delta t = 2\text{ ms} - 0 = 2\text{ ms}$, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = (2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{2 \times 10^{-3}\text{ s}}\right)$$
$$= 4 \times 10^{-3}\text{ A} = \mathbf{4\ mA}$$

b. From 2 ms to 5 ms, the voltage remains constant at 4 V; the change in voltage $\Delta v = 0$. The change in time $\Delta t = 3$ ms, and

c. From 5 ms to 11 ms, the voltage decreases from 4 V to 0 V. The change in voltage Δv is, therefore, $4\text{ V} - 0 = 4\text{ V}$ (with a negative sign since the voltage is decreasing with time). The change in time $\Delta t = 11\text{ ms} - 5\text{ ms} = 6\text{ ms}$, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = -(2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{6 \times 10^{-3}\text{ s}}\right)$$
$$i_{C_{av}} = -1.33 \times 10^{-3}\text{ A} = \mathbf{-1.33\ mA}$$

d. From 11 ms on, the voltage remains constant at 0 and $\Delta v = 0$, so $i_{C_{av}} = 0$ mA. The waveform for the average current for the impressed voltage is as shown in Fig. 10.69.

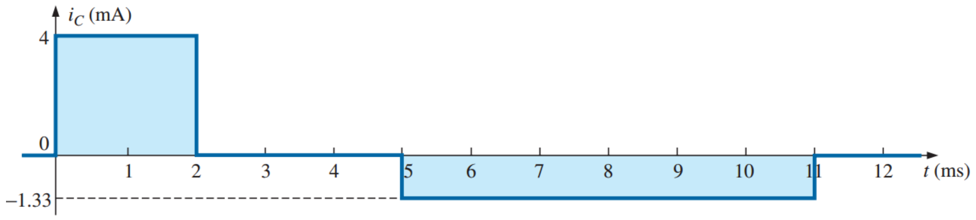

FIG. 10.69
The resulting current i_C for the applied voltage in Fig. 10.68.

3. Instantaneous Value of RL Circuits

11.9 INSTANTANEOUS VALUES

The similarity between the equations

$$v_C = V_f + (V_i + V_f)e^{-t/\tau}$$

and

$$i_L = I_f + (I_i - I_f)e^{-t/\tau}$$

results in a derivation of the following for t that is identical to that used to obtain Eq. (10.23):

$$t = \tau \log_e \frac{(I_i - I_f)}{(i_L - I_f)} \quad \text{(seconds, s)} \qquad \textbf{(11.26)}$$

For the other form, the equation $v_C = Ee^{-t/\tau}$ is a close match with $v_L = Ee^{-t/\tau} = V_i e^{-t/\tau}$, permitting a derivation similar to that employed for Eq. (10.23):

$$t = \tau \log_e \frac{V_i}{v_L} \quad \text{(seconds, s)} \qquad \textbf{(11.27)}$$

For the voltage v_R, $V_i = 0$ V and $V_f = E$V since $v_R = E(1 - e^{-t/\tau})$. Solving for t yields

$$t = \tau \log_e \left(\frac{E}{E - v_R}\right)$$

or

$$t = \tau \log_e \left(\frac{V_f}{V_f - v_R}\right) \quad \text{(seconds, s)} \qquad \textbf{(11.28)}$$

11.10 AVERAGE INDUCED VOLTAGE: $v_{L_{av}}$

average induced voltage is defined by

$$v_{L_{av}} = L\frac{\Delta i_L}{\Delta t} \quad \text{(volts, V)} \tag{11.29}$$

where Δ indicates a finite (measurable) change in current or time. Eq. (11.12) for the instantaneous voltage across a coil can be derived from Eq. (11.29) by letting V_L become vanishingly small. That is,

$$v_{L_{inst}} = \lim_{\Delta t \to 0} L\frac{\Delta i_L}{\Delta t} = L\frac{di_L}{dt}$$

EXAMPLE 11.8 Find the waveform for the average voltage across the coil if the current through a 4 mH coil is as shown in Fig. 11.54.

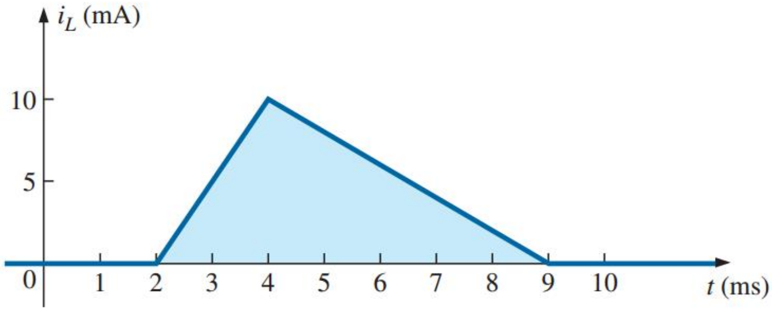

FIG. 11.54
Current i_L to be applied to a 4 mH coil in Example 11.8.

Solutions:

a. *0 to 2 ms:* Since there is no change in current through the coil, there is no voltage induced across the coil. That is,

$$v_L = L\frac{\Delta i_L}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0\ V}$$

b. *2 ms to 4 ms:*

$$v_L = L\frac{\Delta i_L}{\Delta t} = (4 \times 10^{-3}\,\text{H})\left(\frac{10 \times 10^{-3}\,\text{A}}{2 \times 10^{-3}\,\text{s}}\right) = 20 \times 10^{-3}\,\text{V} = \mathbf{20\ mV}$$

c. *4 ms to 9 ms:*

$$v_L = L\frac{\Delta i_L}{\Delta t} = (-4 \times 10^{-3}\,\text{H})\left(\frac{10 \times 10^{-3}\,\text{A}}{5 \times 10^{-3}\,\text{s}}\right) = -8 \times 10^{-3}\,\text{V} = \mathbf{-8\ mV}$$

d. *9 ms to ∞*:

$$\upsilon_L = L\frac{\Delta i_L}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0\ V}$$

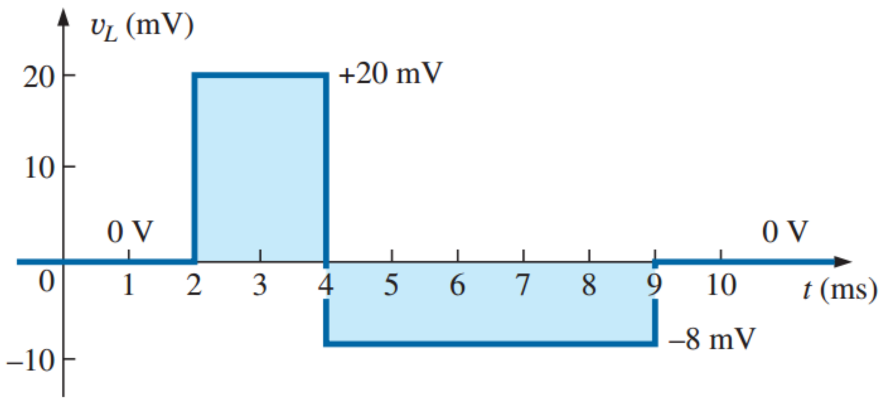

FIG. 11.55
Voltage across a 4 mH coil due to the current in Fig. 11.54.

the voltage across the coil is not determined solely by the magnitude of the change in current through the coil (Δi_L), but by the rate of change of current through the coil ($\Delta i_L/\Delta t$).

11.12 STEADY-STATE CONDITIONS

$$I_1 = \frac{E}{R_1} = \frac{10\ \text{V}}{2\ \Omega} = \mathbf{5\ A}$$

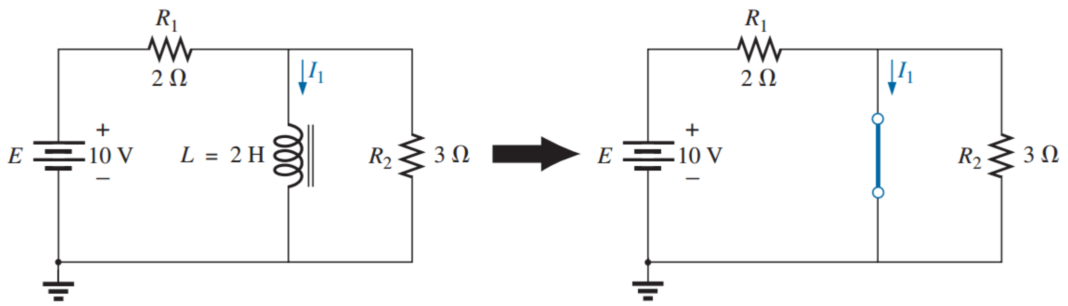

FIG. 11.60
Substituting the short-circuit equivalent for the inductor for t > 5τ.

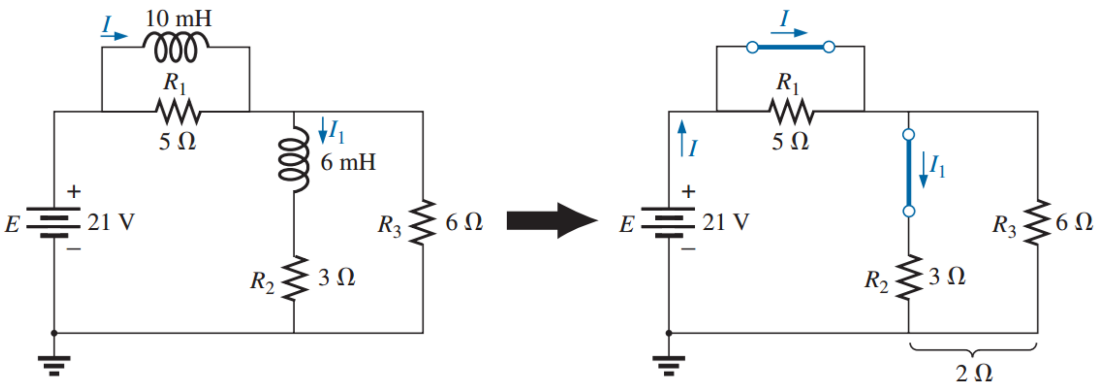

FIG. 11.61
Establishing the equivalent network for $t > 5\tau$.

Applying the current divider rule yields

$$I_1 = \frac{R_3 I}{R_3 + R_2} = \frac{(6\,\Omega)(10.5\text{ A})}{6\,\Omega + 3\,\Omega} = \frac{63}{9}\text{A} = \mathbf{7\text{ A}}$$

EXAMPLE 11.10 Find the current I_L and the voltage V_C for the network in Fig. 11.62.

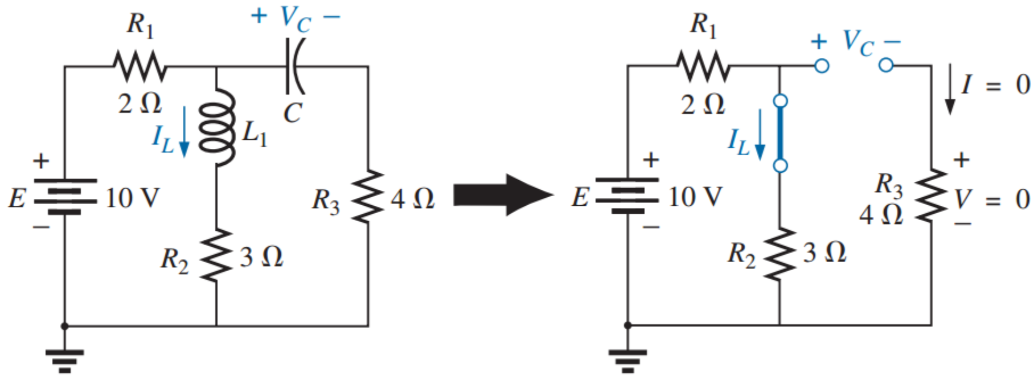

FIG. 11.62
Example 11.10.

Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10\text{ V}}{5\,\Omega} = \mathbf{2\text{ A}}$$

$$V_C = \frac{R_2 E}{R_2 + R_1} = \frac{(3\,\Omega)(10\text{ V})}{3\,\Omega + 2\,\Omega} = \mathbf{6\text{ V}}$$

EXAMPLE 11.11 Find currents I_1 and I_2 and voltages V_1 and V_2 for the network in Fig. 11.63.

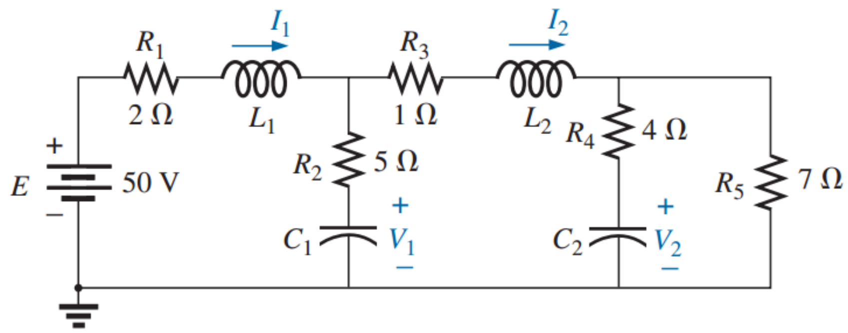

FIG. 11.63

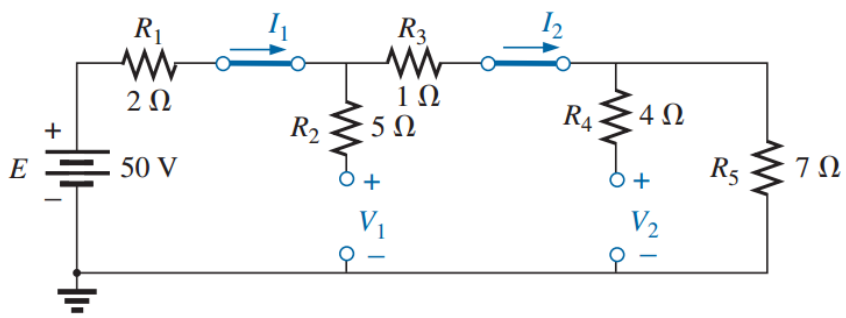

FIG. 11.64
Substituting the short-circuit equivalents for the inductors and the open-circuit equivalents for the capacitor for $t > 5\tau$.

Solution: Note Fig. 11.64.

$$I_1 = I_2$$
$$= \frac{E}{R_1 + R_3 + R_5} = \frac{50\text{ V}}{2\,\Omega + 1\,\Omega + 7\,\Omega} = \frac{50\text{ V}}{10\,\Omega} = \mathbf{5\text{ A}}$$
$$V_2 = I_2 R_5 = (5\text{ A})(7\,\Omega) = \mathbf{35\text{ V}}$$

Applying the voltage divider rule yields

$$V_1 = \frac{(R_3 + R_5)E}{R_1 + R_3 + R_5} = \frac{(1\,\Omega + 7\,\Omega)(50\text{ V})}{2\,\Omega + 1\,\Omega + 7\,\Omega} = \frac{(8\,\Omega)(50\text{ V})}{10\,\Omega} = \mathbf{40\text{ V}}$$

7. Glossary – English/Chinese Translation

English	Chinese
RL circuit	RL 电路
RC circuit	RC 电路
Instantaneous value	瞬时值
transient circuit	瞬态电路
Thevenin equivalent circuit	戴维宁等效电路
induced voltage	感应电压
steady state condition	稳态状态

Your Notes: