Tutorial - 1-02-e

Question 1

EXAMPLE 10.6 For the circuit in Fig. 10.38:

a. Find the mathematical expression for the transient behavior of v_C, i_C, and v_R if the switch is closed at $t = 0$ s.
b. Plot the waveform of v_C versus the time constant of the network.
c. Plot the waveform of v_C versus time.
d. Plot the waveforms of i_C and v_R versus the time constant of the network.
e. What is the value of v_C at $t = 20$ ms?
f. On a practical basis, how much time must pass before we can assume that the charging phase has passed?
g. When the charging phase has passed, how much charge is sitting on the plates?

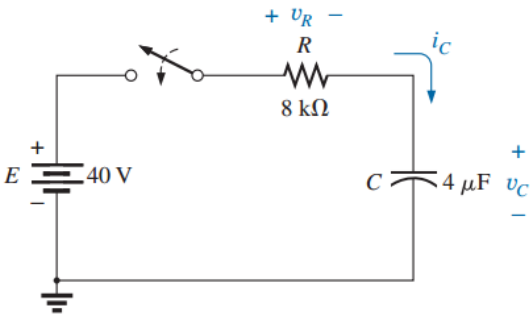


FIG. 10.38
Transient network for Example 10.6.

Question 2

EXAMPLE 10.9 For the network in Fig. 10.49:

a. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is thrown into position 1 at $t = 0$ s.
b. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is moved to position 2 at $t = 1\tau$.

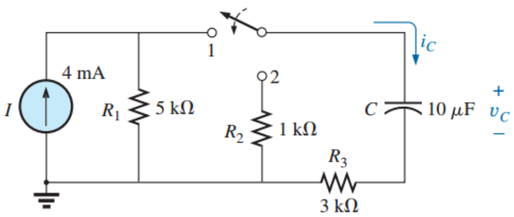


FIG. 10.49
Network to be analyzed in Example 10.9.

c. Plot the resulting waveform for the voltage v_C as determined by parts (a) and (b).
d. Repeat parts (a)–(c) for the current i_C.

Question 3

EXAMPLE 10.11 For the network in Fig. 10.60:

a. Find the mathematical expression for the transient behavior of the voltage v_C and the current i_C following the closing of the switch (position 1 at $t = 0$ s).
b. Find the mathematical expression for the voltage v_C and the current i_C as a function of time if the switch is thrown into position 2 at $t = 9$ ms.
c. Draw the resultant waveforms of parts (a) and (b) on the same time axis.

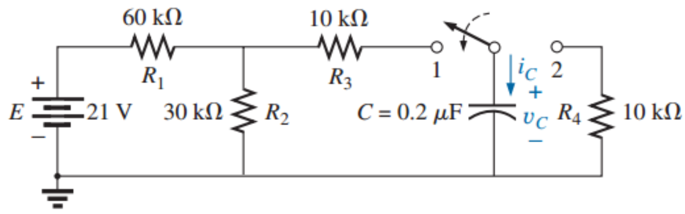


FIG. 10.60
Example 10.11.

Question 4

EXAMPLE 11.7 Switch S_1 in Fig. 11.51 has been closed for a long time. At $t = 0$ s, S_1 is opened at the same instant that S_2 is closed to avoid an interruption in current through the coil.

a. Find the initial current through the coil. Pay particular attention to its direction.

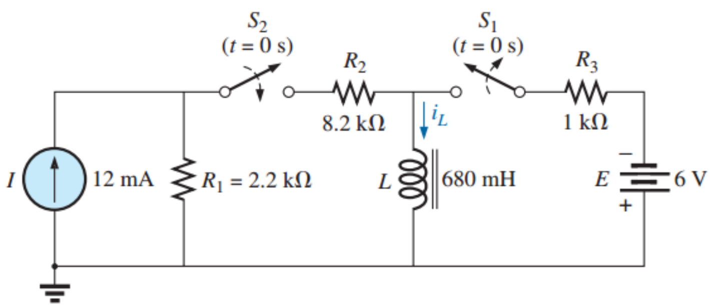


FIG. 11.51
Example 11.7.

b. Find the mathematical expression for the current i_L following the closing of switch S_2.
c. Sketch the waveform for i_L.

Solution

Question 1

a. The time constant of the network is

$$\tau = RC = (8\ \text{k}\Omega)(4\ \mu\text{F}) = 32\ \text{ms}$$

resulting in the following mathematical equations:

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{40\ V(1 - e^{-t/32\ ms})}$$

$$i_C = \frac{E}{R} e^{-t/\tau} = \frac{40\ \text{V}}{8\ \text{k}\Omega} e^{-t/32\ \text{ms}} = \mathbf{5\ mA e^{-t/32\ ms}}$$

$$v_R = E e^{-t/\tau} = \mathbf{40\ V e^{-t/32\ ms}}$$

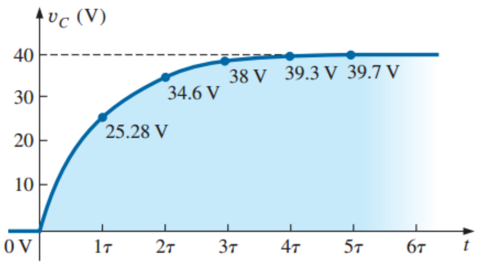


FIG. 10.39
v_C versus time for the charging network in Fig. 10.38.

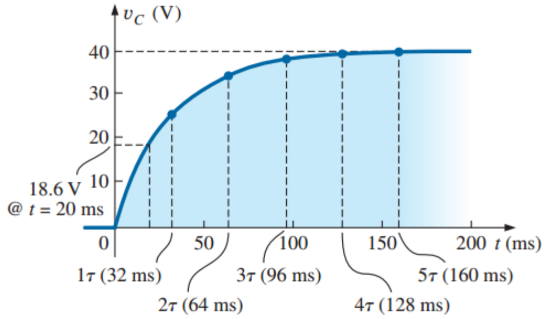


FIG. 10.40
Plotting the waveform in Fig. 10.39 versus time (t).

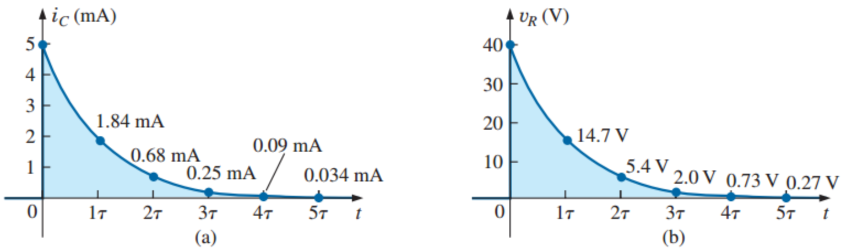


FIG. 10.41
i_C and v_R for the charging network in Fig. 10.39.

b. The resulting plot appears in Fig. 10.39.

c. The horizontal scale will now be against time rather than time constants, as shown in Fig. 10.40. The plot points in Fig. 10.40 were taken from Fig. 10.39.

d. Both plots appear in Fig. 10.41.

e. Substituting the time $t = 20$ ms results in the following for the exponential part of the equation:

$$e^{t/\tau} = e^{-20\ \text{ms}/32\ \text{ms}} = e^{-0.625} = 0.535\ (\text{using a calculator})$$

so that $v_C = 40\ \text{V}(1 - e^{\tau/32\ \text{ms}}) = 40\ \text{V}(1 - 0.535)$

$$= (40\ \text{V})(0.465) = \mathbf{18.6\ V}\ (\text{as verified by Fig. 10.40})$$

f. Assuming a full charge in five time constants results in

$$5\tau = 5(32\ \text{ms}) = \mathbf{160\ ms} = \mathbf{0.16\ s}$$

g. Using Eq. (10.6) gives

$$Q = CV = (4\ \mu\text{F})(40\ \text{V}) = \mathbf{160\ \mu C}$$

Question 2

Solutions:

a. Converting the current source to a voltage source results in the configuration in Fig. 10.50 for the charging phase.

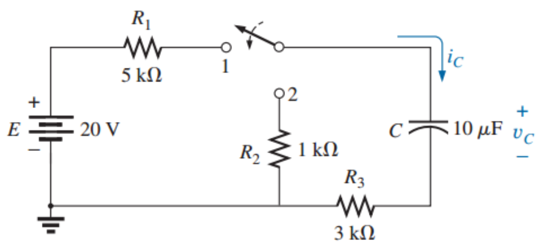


FIG. 10.50
The charging phase for the network in Fig. 10.49.

For the source conversion

$$E = IR = (4\text{ mA})(5\text{ k}\Omega) = 20\text{ V}$$

and $$R_s = R_p = 5\text{ k}\Omega$$

$$\tau = RC = (R_1 + R_3)C = (5\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 80\text{ ms}$$

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{20\ V\ (1 - e^{-t/80\ ms})}$$

For the source conversion

$$E = IR = (4\text{ mA})(5\text{ k}\Omega) = 20\text{ V}$$

and $$R_s = R_p = 5\text{ k}\Omega$$

$$\tau = RC = (R_1 + R_3)C = (5\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 80\text{ ms}$$

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{20\ V\ (1 - e^{-t/80\ ms})}$$

b. With the switch in position 2, the network appears as shown in Fig. 10.51. The voltage at 1 τ can be found by using the fact that the voltage is 63.2% of its final value of 20 V, so that 0.632(20 V) = 12.64 V. Alternatively, you can substitute into the derived equation as follows:

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} = 0.368$$

and $$v_C = 20\text{ V}(1 - e^{-t/80\text{ ms}}) = 20\text{ V}(1 - 0.368)$$
$$= (20\text{ V})(0.632) = 12.64\text{ V}$$

Using this voltage as the starting point and substituting into the discharge equation results in

$$\tau' = RC = (R_2 + R_3)C = (1\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 40\text{ ms}$$

$$v_C = Ee^{-t/\tau'} = \mathbf{12.64\ Ve^{-t/40\ ms}}$$

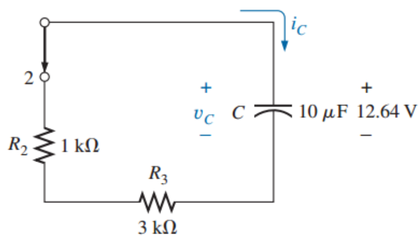


FIG. 10.51
Network in Fig. 10.50 when the switch is moved to position 2 at $t = 1\tau_1$.

c. See Fig. 10.52.

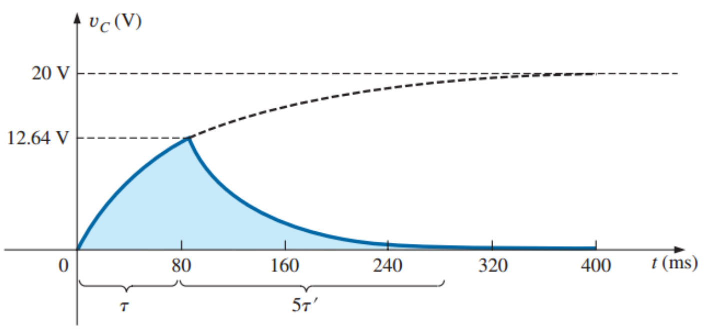


FIG. 10.52

d. The charging equation for the current is

$$i_C = \frac{E}{R}\,e^{-t/\tau} = \frac{E}{R_1 + R_3}\,e^{-t/\tau} = \frac{20\text{ V}}{8\text{ k}\Omega}\,e^{-t/80\text{ ms}} = \mathbf{2.5\ mA}\boldsymbol{e}^{-t/80\ \mathbf{ms}}$$

which, at $t = 80$ ms, results in

$$i_C = 2.5\text{ mA}e^{-80\text{ ms}/80\text{ ms}} = 2.5\text{ mA}e^{-1} = (2.5\text{ mA})(0.368) = 0.92\text{ mA}$$

When the switch is moved to position 2, the 12.64 V across the capacitor appears across the resistor to establish a current of 12.64 V/4 kΩ = 3.16 mA. Substituting into the discharge equation with $V_i = 12.64$ V and $\tau' = 40$ ms yields

$$i_C = -\frac{V_i}{R_2 + R_3}\,e^{-t/\tau'} = -\frac{12.64\text{ V}}{1\text{ k}\Omega + 3\text{ k}\Omega}\,e^{-t/40\text{ ms}}$$
$$= -\frac{12.64\text{ V}}{4\text{ k}\Omega}e^{-t/40\text{ ms}} = \mathbf{-3.16\ mA}\boldsymbol{e}^{-t/40\ \mathbf{ms}}$$

The equation has a minus sign because the direction of the discharge current is opposite to that defined for the current in Fig. 10.51. The resulting plot appears in Fig. 10.53.

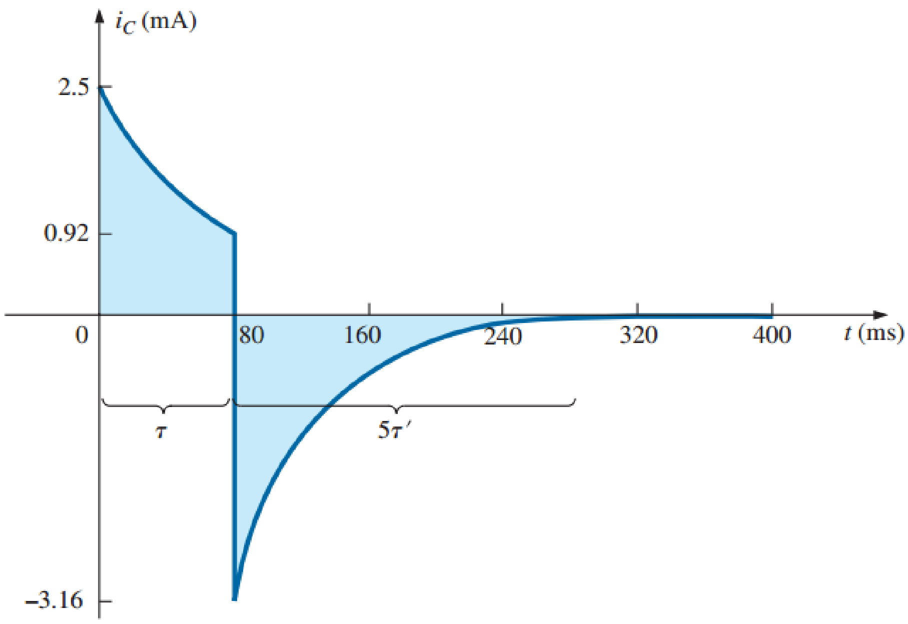


FIG. 10.53

Question 3

Solutions:

a. Applying Thévenin's theorem to the 0.2 μF capacitor, we obtain Fig. 10.61. We have

$$R_{Th} = R_1 \| R_2 + R_3 = \frac{(60\ \text{k}\Omega)(30\ \text{k}\Omega)}{90\ \text{k}\Omega} + 10\ \text{k}\Omega$$
$$= 20\ \text{k}\Omega + 10\ \text{k}\Omega = 30\ \text{k}\Omega$$
$$E_{Th} = \frac{R_2 E}{R_2 + R_1} = \frac{(30\ \text{k}\Omega)(21\ \text{V})}{30\ \text{k}\Omega + 60\ \text{k}\Omega} = \frac{1}{3}(21\ \text{V}) = 7\ \text{V}$$

The resultant Thévenin equivalent circuit with the capacitor replaced is shown in Fig. 10.62.

Using Eq. (10.21) with $V_f = E_{\text{Th}}$ and $V_i = 0$ V, we find that

$$\upsilon_C = V_f + (V_i - V_f)e^{-t/\tau}$$

becomes $$\upsilon_C = E_{Th} + (0\ V - E_{Th})e^{-t/\tau}$$

or $$\upsilon_C = E_{Th}(1 - e^{-t/\tau})$$

with $$\tau = RC = (30\ \text{k}\Omega)(0.2\ \mu\text{F}) = 6\ \text{ms}$$

Therefore, $$\upsilon_C = \mathbf{7\ V(1 - e^{-t/6\ ms})}$$

For the current i_C:

$$i_C = \frac{E_{Th}}{R}e^{-t/RC} = \frac{7\ \text{V}}{30\ \text{k}\Omega}e^{-t/6\ \text{ms}}$$
$$= \mathbf{0.23\ mAe^{-t/6\ ms}}$$

b. At $t = 9$ ms,

$$\upsilon_C = E_{Th}(1 - e^{-t/\tau}) = 7\ \text{V}(1 - e^{-(9\ \text{ms}/6\ \text{ms})})$$
$$= (7\ \text{V})(1 - e^{-1.5}) = (7\ \text{V})(1 - 0.223)$$
$$= (7\ \text{V})(0.777) = 5.44\ \text{V}$$

and $$i_C = \frac{E_{Th}}{R}e^{-t/\tau} = 0.23\ \text{mA}e^{-1.5}$$
$$= (0.23 \times 10^{-3})(0.233) = 0.052 \times 10^{-3} = 0.05\ \text{mA}$$

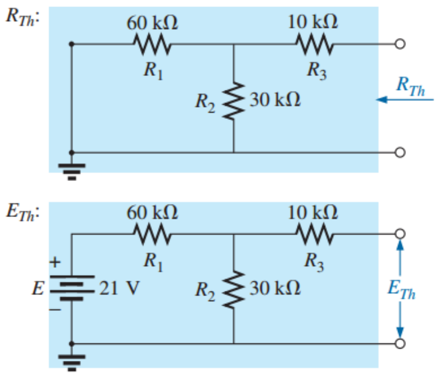


FIG. 10.61
Applying Thévenin's theorem to the networ in Fig. 10.60.

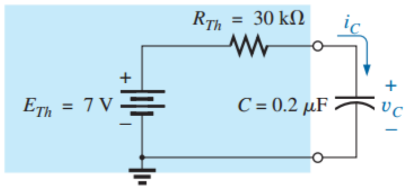


FIG. 10.62
Substituting the Thévenin equivalent for th network in Fig. 10.60.

Using Eq. (10.21) with $V_f = 0$ V and $V_i = 5.44$ V, we find that

$$\upsilon_C = V_f + (V_i - V_f)e^{-t/\tau'}$$

becomes $$\upsilon_C = 0\ \text{V} + (5.44\ \text{V} - 0\ \text{V})e^{-t/\tau'}$$
$$= 5.44\ \text{V}e^{-t/\tau'}$$

with $$\tau' = R_4 C = (10\ \text{k}\Omega)(0.2\ \mu\text{F}) = 2\ \text{ms}$$

and $$\upsilon_C = \mathbf{5.44\ Ve^{-t/2\ ms}}$$

By Eq. (10.19),

$$I_i = \frac{5.44\ \text{V}}{10\ \text{k}\Omega} = 0.54\ \text{mA}$$

and $$i_C = I_i e^{-t/\tau} = \mathbf{0.54\ mAe^{-t/2\ ms}}$$

c. See Fig. 10.63.

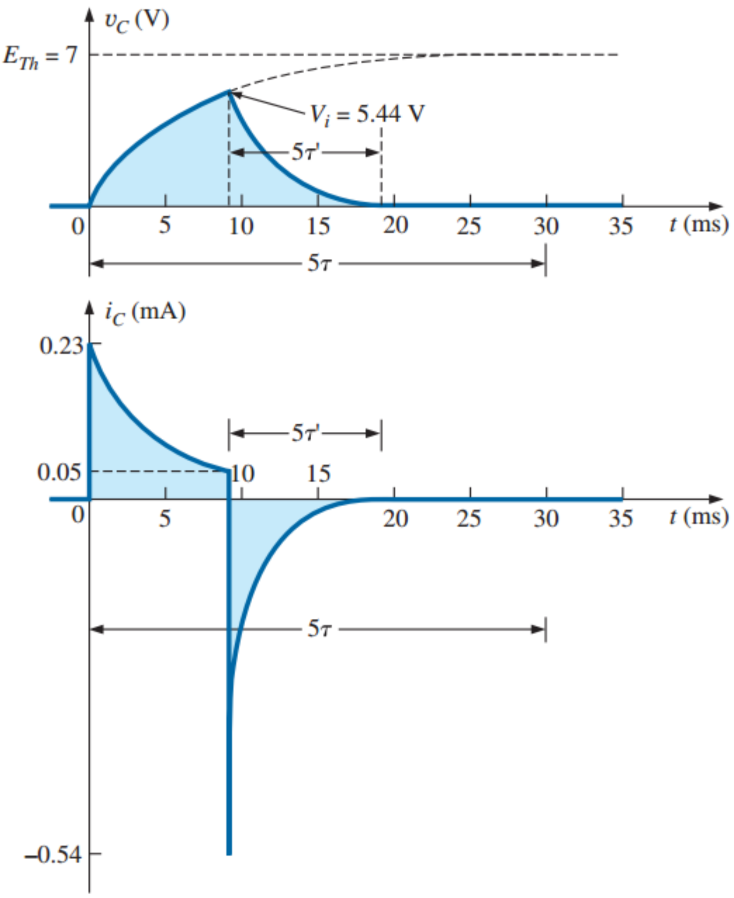
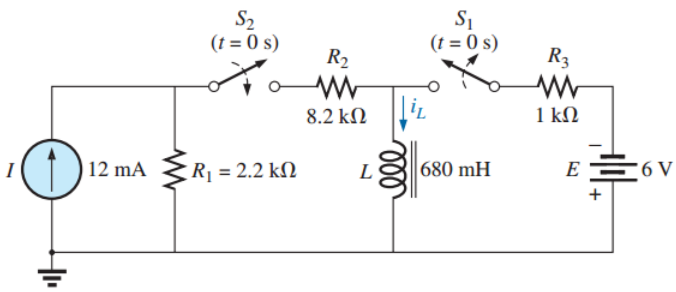


FIG. 10.63
The resulting waveforms for the network in Fig. 10.60.

Question 4



a. Using Ohm's law, we find the initial current through the coil:

$$I_i = -\frac{E}{R_3} = -\frac{6\text{ V}}{1\text{ k}\Omega} = -6\text{ mA}$$

b. Applying Thévenin's theorem gives

$$R_{Th} = R_1 + R_2 = 2.2\text{ k}\Omega + 8.2\text{ k}\Omega = 10.4\text{ k}\Omega$$
$$E_{Th} = IR_1 = (12\text{ mA})(2.2\text{ k}\Omega) = 26.4\text{ V}$$

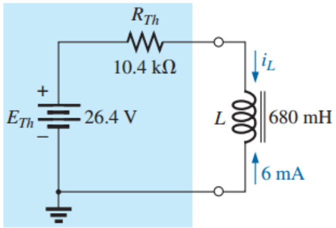


FIG. 11.52
Thévenin equivalent circuit for the network in Fig. 11.51 for $t \geq 0$ s.

The Thévenin equivalent network appears in Fig. 11.52.

The steady-state current can then be determined by substituting the short-circuit equivalent for the inductor:

$$I_f = \frac{E}{R_{Th}} = \frac{26.4\text{ V}}{10.4\text{ k}\Omega} = 2.54\text{ mA}$$

The time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{680\text{ mH}}{10.4\text{ k}\Omega} = 65.39\ \mu\text{s}$$

Applying Eq. (11.17) gives

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} \\ &= 2.54\text{ mA} + (-6\text{ mA} - 2.54\text{ mA})e^{-t/65.39\ \mu\text{s}} \\ &= \mathbf{2.54\ mA - 8.54\ mA}\boldsymbol{e^{-t/65.39\ \mu s}} \end{aligned}$$

c. Note Fig. 11.53.

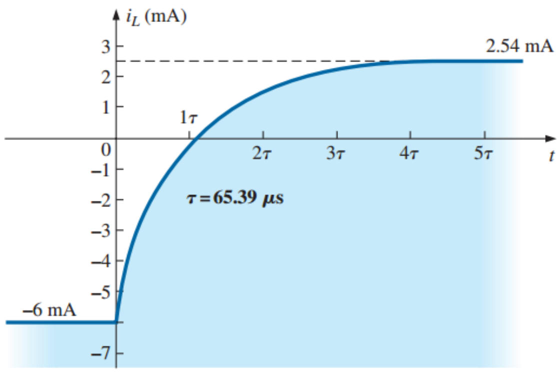


FIG. 11.53
The current i_L for the network in Fig. 11.51.