Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-e

Capacitors and Inductors

Contents

Reference:

Introductory Circuit Analysis 14th edition, Boylesad & Olivari
Basic Circuit Analysis – Schaum's Outline Series

Email: norbertcheung@szu.edu.cn
Web Site: http://norbert.idv.hk
Last Updated: 2024-03

1. Capacitors and Capacitance

A capacitor consists of two conductors separated by an insulator. The chief feature of a capacitor is its ability to store electric charge, with negative charge on one of its two conductors and positive charge on the other. Accompanying this charge is energy, which a capacitor can release.

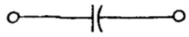


Fig. 8-1

Capacitance, the electrical property of capacitors, is a measure of the ability of a capacitor to store charge on its two conductors. Specifically, if the potential difference between the two conductors is V volts when there is a positive charge of Q coulombs on one conductor and a negative charge of the same amount on the other, the capacitor has a capacitance of:

$$C = \frac{Q}{V}$$

The SI unit of capacitance is the farad, with symbol F. Unfortunately, the farad is much too large a unit for practical applications, and the microfarad (μF) and picofarad (pF) are much more common.

One common type of capacitor is the parallel-plate capacitor of Fig. 8-2a.

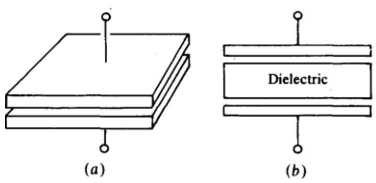


Fig. 8-2

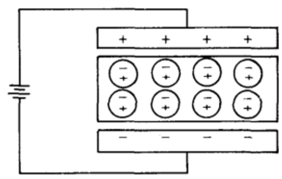


Fig. 8-3

For the parallel-plate capacitor, the capacitance in farads is:

$$C = \varepsilon \frac{A}{d}$$

where A is the area of either plate in square meters, d is the separation in meters, and e is the permittivity in farads per meter (F/m) of the dielectric. The larger the plate area or the smaller the plate separation, or the greater the dielectric permittivity, the greater the capacitance.

The permittivity of vacuum, designated by ε_0, is 8.85 pF/m. Permittivities of other dielectrics are related to that of vacuum by a factor called the *dielectric constant* or *relative permittivity*, designated by ε_r. The relation is $\varepsilon = \varepsilon_r \varepsilon_0$. The dielectric constants of some common dielectrics are 1.0006 for air, 2.5 for paraffined paper, 5 for mica, 7.5 for glass, and 7500 for ceramic.

The total or equivalent capacitance (C_T or C_{eq}) of parallel capacitors, as seen in Fig. 8-4*a*, can be found from the total stored charge and the $Q = CV$ formula. The total stored charge Q_T equals the sum of the individual stored charges: $Q_T = Q_1 + Q_2 + Q_3$. With the substitution of the appropriate $Q = CV$ for each Q, this equation becomes $C_T V = C_1 V + C_2 V + C_3 V$. Upon division by V, it reduces to $C_T = C_1 + C_2 + C_3$. Because the number of capacitors is not significant in this derivation, this result can be generalized to any number of parallel capacitors:

$$C_T = C_1 + C_2 + C_3 + C_4 + \cdots$$

So, the total or equivalent capacitance of parallel capacitors is the sum of the individual capacitances.

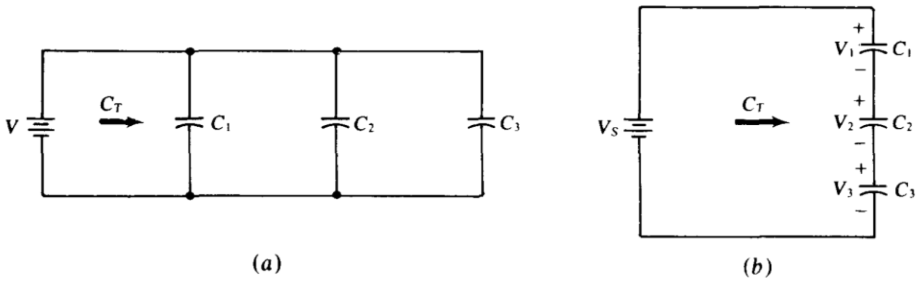


Fig. 8-4

For series capacitors, as shown in Fig. 8-4*b*, the formula for the total capacitance is derived by substituting Q/C for each V in the KVL equation. The Q in each term is the same. This is because the charge gained by a plate of any capacitor must have come from a plate of an adjacent capacitor. The KVL equation for the circuit shown in Fig. 8-4*b* is $V_S = V_1 + V_2 + V_3$. With the substitution of the appropriate Q/C for each V, this equation becomes

$$\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad \text{or} \qquad \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Generalizing,

$$C_T = \frac{1}{1/C_1 + 1/C_2 + 1/C_3 + 1/C_4 + \cdots}$$

Energy Storage

$$W_C = \tfrac{1}{2} C V^2$$

where W_C is in joules, C is in farads, and V is in volts. Notice that this stored energy does not depend on the capacitor current.

Capacitor Current

An equation for capacitor current can be found by substituting $q = Cv$ into $i = dq/dt$:

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv)$$

But C is a constant, and a constant can be factored from a derivative. The result is

$$i = C\frac{dv}{dt}$$

2. Inductor and Inductance

Relationship between Capacitors and Inductors

The following material on inductors and inductance is similar to that on capacitors and capacitance presented in Chap. 8. The reason for this similarity is that, mathematically speaking, the capacitor and inductor formulas are the same. Only the symbols differ. Where one has v, the other has i, and vice versa; where one has the capacitance quantity symbol C, the other has the inductance quantity symbol L; and where one has R, the other has G. It follows then that the basic inductor voltage-current formula is $v = L\,di/dt$ in place of $i = C\,dv/dt$, that the energy stored is $\frac{1}{2}Li^2$ instead of $\frac{1}{2}Cv^2$, that, inductor currents, instead of capacitor voltages, cannot jump, that inductors are short circuits, instead of open circuits, to dc, and that the time constant is $LG = L/R$ instead of CR. Although it is possible to approach the study of inductor action on the basis of this duality, the standard approach is to use magnetic flux.

Magnetic Flux

Magnetic phenomena are explained using *magnetic flux*, or just flux, which relates to magnetic lines of force that, for a magnet, extend in continuous lines from the magnetic north pole to the south pole outside the magnet and from the south pole to the north pole inside the magnet, as is shown in Fig. 9-1*a*. The SI unit of flux is the *weber*, with unit symbol Wb. The quantity symbol is Φ for a constant flux and ϕ for a time-varying flux.

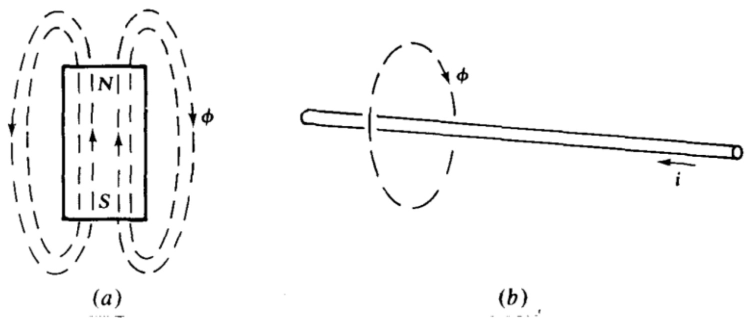


Fig. 9-1

Inductance and Inductors

If a coil of N turns is linked by a ϕ amount of flux, this coil has a flux linkage of $N\phi$. Any change in flux linkages induces a voltage in the coil of

$$v = \lim_{\Delta t \to 0} \frac{\Delta N\phi}{\Delta t} = \frac{d}{dt}(N\phi) = N\frac{d\phi}{dt}$$

This is known as *Faraday's law*. The voltage polarity is such that any current resulting from this voltage produces a flux that opposes the original change in flux.

The inductance of a coil depends on the shape of the coil, the permeability of the surrounding material, the number of turns, the spacing of the turns, and other factors. For the single-layer coil shown in Fig. 9-3, the inductance is approximately $L = N^2\mu A/l$, where N is the number of turns of wire, A is the core cross-sectional area in square meters, l is the coil length in meters, and μ is the core permeability. The greater the length to diameter, the more accurate the formula. For a length of 10 times the diameter, the actual inductance is 4 percent less than the value given by the formula.

Fig. 9-2

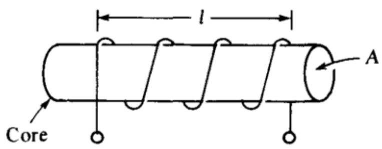


Fig. 9-3

Inductance instead of flux is used in analyzing circuits containing inductors. The equation relating inductor voltage, current, and inductance can be found from substituting $N\phi = Li$ into $v = d(N\phi)/dt$. The result is $v = L\,di/dt$, with associated references assumed. If the voltage and current references are not associated, a negative sign must be included. Notice that the voltage at any instant depends on the rate of change of inductor current at that instant, but not at all on the value of current then.

One important fact from $v = L\,di/dt$ is that if an inductor current is constant, not changing, then the inductor voltage is zero because $di/dt = 0$. With a current flowing through it, but zero voltage across it, an inductor acts as a short circuit: *An inductor is a short circuit to dc.* Remember, though, that it is only after an inductor current becomes constant that an inductor acts as a short circuit.

Total Inductance

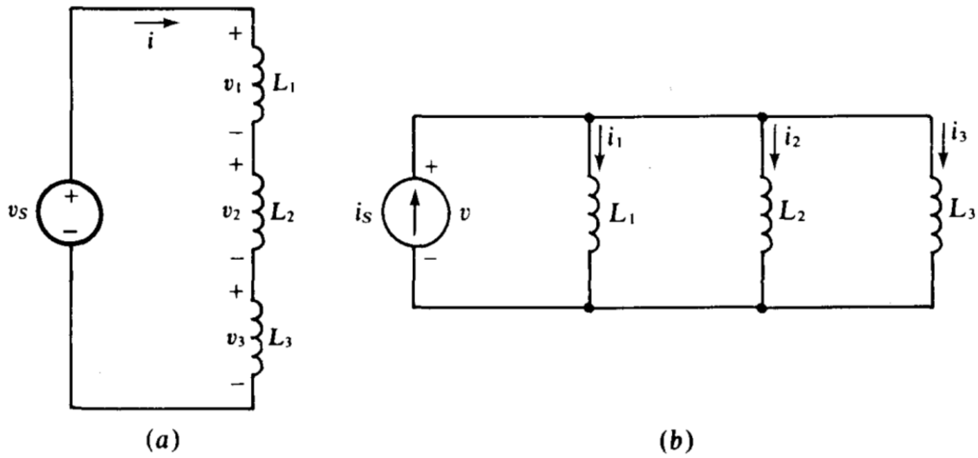


Fig. 9-4

For (a):

$$L_T = L_1 + L_2 + L_3 + L_4 + \cdots$$

For (b)

$$L_T = \frac{1}{1/L_1 + 1/L_2 + 1/L_3 + 1/L_4 + \cdots}$$

Energy Storage:

As can be shown by using calculus, the energy stored in an inductor is

$$w_L = \tfrac{1}{2} L i^2$$

in which w_L is in joules, L is in henries, and i is in amperes. This energy is considered to be stored in the magnetic field surrounding the inductor.

3. Charge and Discharge of a Capacitor

Charging a Capacitor

The transfer of electrons is very rapid at first, slowing down as the potential across the plates approaches the applied voltage.

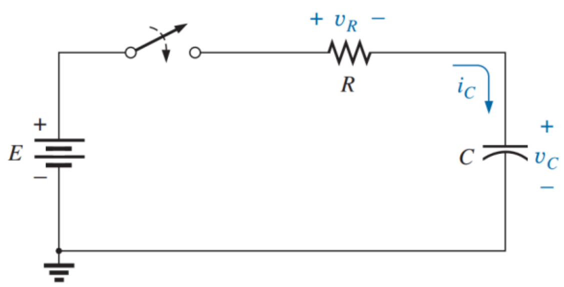


FIG. 10.28
Basic R-C charging network.

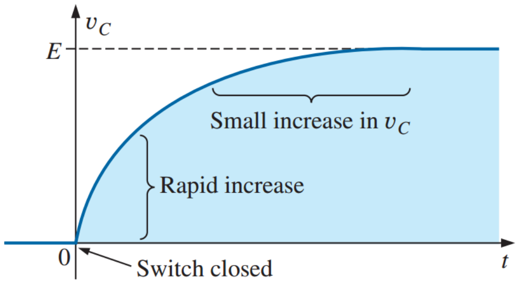


FIG. 10.29
v_C during the charging phase.

$$v_C = E(1 - e^{-t/\tau}) \quad \text{charging} \qquad \text{(volts, V)} \qquad \textbf{(10.13)}$$

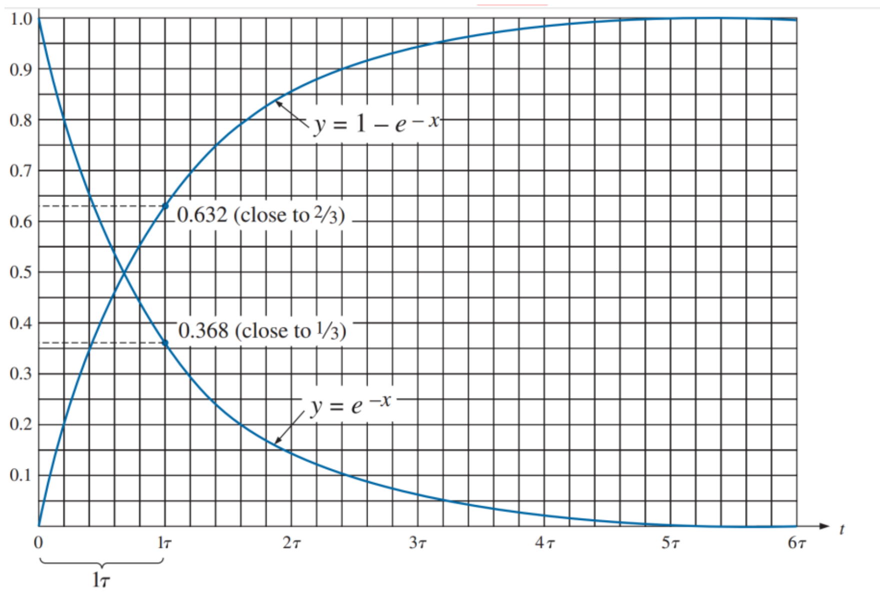


FIG. 10.30
Universal time constant chart.

$$\tau = RC \qquad \text{(time, s)} \qquad \textbf{(10.14)}$$

The factor τ, called the time constant of the network, has the units of time. The transient or charging phase of a capacitor has essentially ended after five time constants.

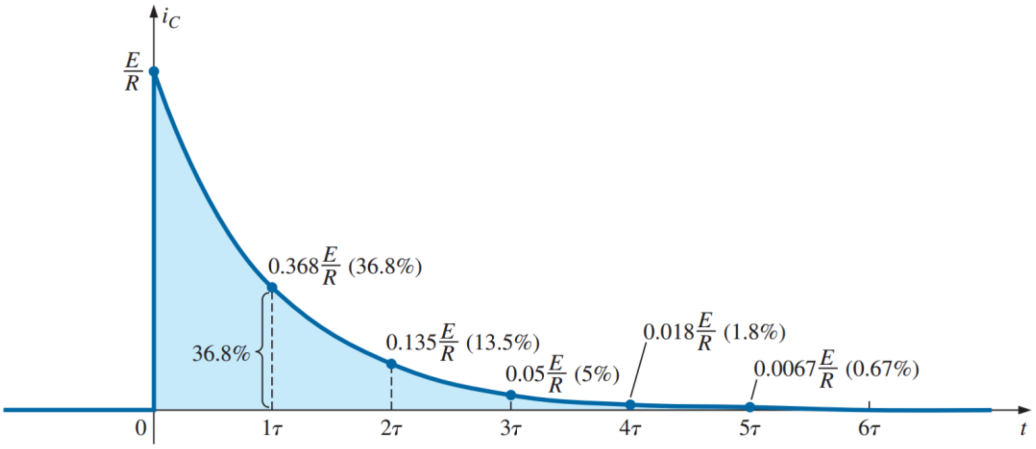
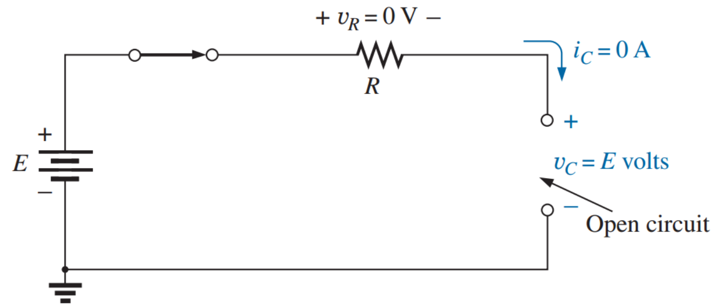


FIG. 10.32

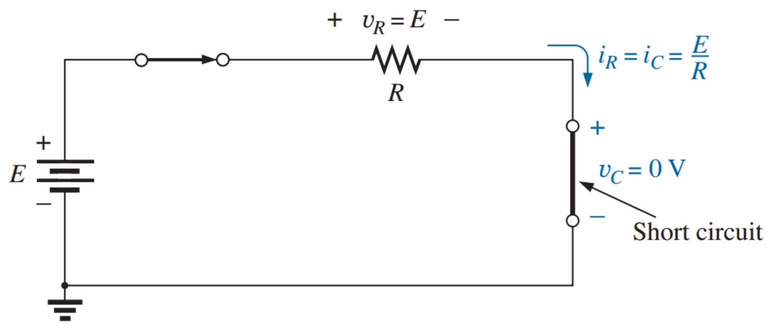
Plotting the equation $i_C = \frac{E}{R}e^{-t/\tau}$ versus time (t).

$$i_C = \frac{E}{R}e^{-t/\tau}_{\text{charging}} \qquad \text{(amperes, A)} \qquad \textbf{(10.15)}$$

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.



A capacitor has the characteristics of a short-circuit equivalent at the instant the switch is closed in an uncharged series R-C circuit.



The voltage across a capacitor cannot change instantaneously.

Discharging a capacitor

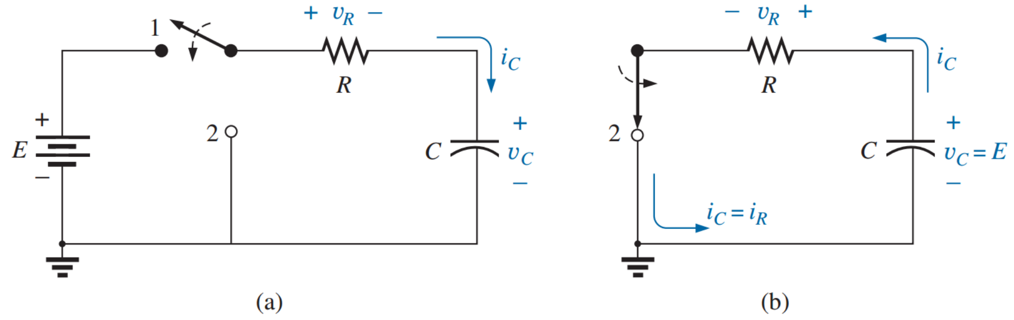


FIG. 10.42
(a) Charging network; (b) discharging configuration.

For the voltage across the capacitor that is decreasing with time, the mathematical expression is

$$v_C = Ee^{-t/\tau} \quad \text{discharging} \tag{10.17}$$

$$\tau = RC \quad \text{discharging} \tag{10.18}$$

Since the current decreases with time, it will have a similar format:

$$i_C = \frac{E}{R} e^{-t/\tau} \quad \text{discharging} \tag{10.19}$$

Since $\upsilon_R = \upsilon_C$ (in parallel), the equation for the voltage υ_R has the same format:

$$\upsilon_R = Ee^{-t/\tau} \quad \text{discharging} \tag{10.20}$$

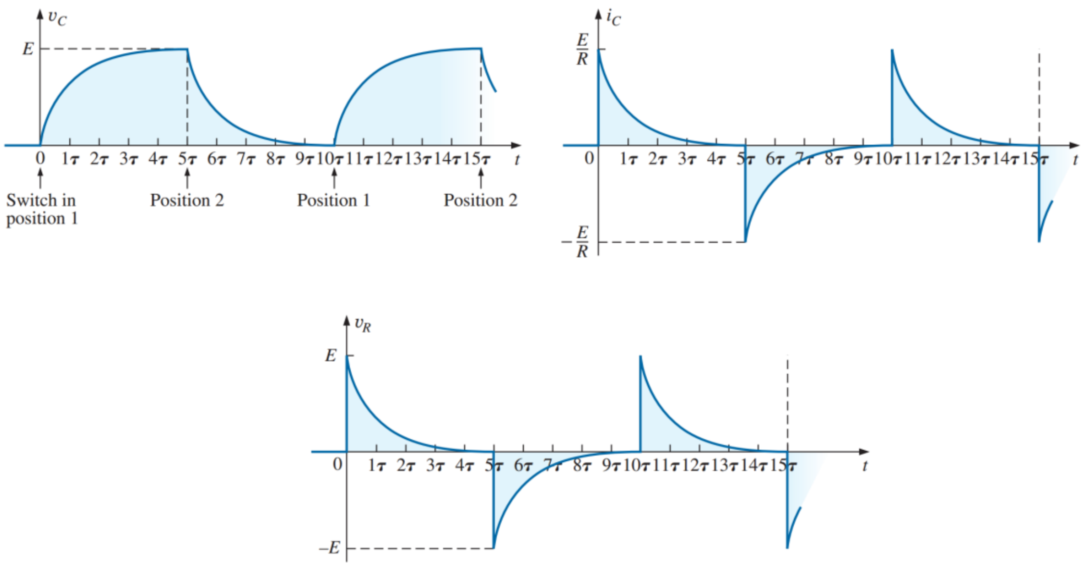


FIG. 10.43
v_C, i_C, and v_R for 5τ switching between contacts in Fig. 10.42(a).

Initial Condition of a Charged Capacitor

We now examine the effect of a charge, and therefore a voltage (V=Q/C) on the plates at the instant the switching action takes place. The voltage across the capacitor at this instant is called the initial value.

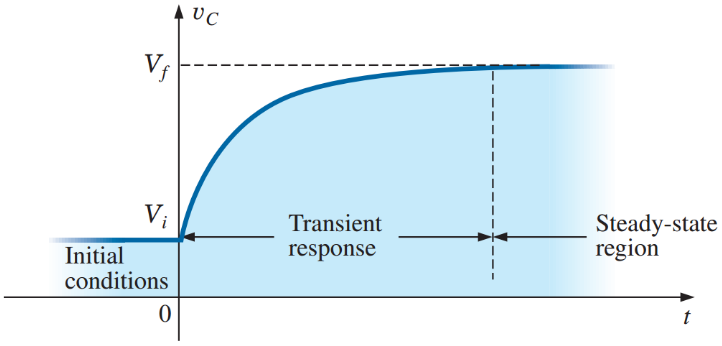


FIG. 10.54

$$v_C = V_f + (V_i - V_f)e^{-t/\tau} \tag{10.21}$$

Eq. (10.21) is a universal equation for the transient response of a capacitor

4. Charge and Discharge of an Inductor

R-L Transient: the storage phase

The circuit in Fig. 11.31 is used to describe the storage phase. Note that it is the same circuit used to describe the charging phase of capacitors, with a simple replacement of the capacitor by an ideal inductor.

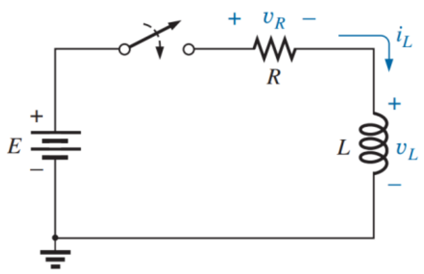


FIG. 11.31
Basic R-L transient network.

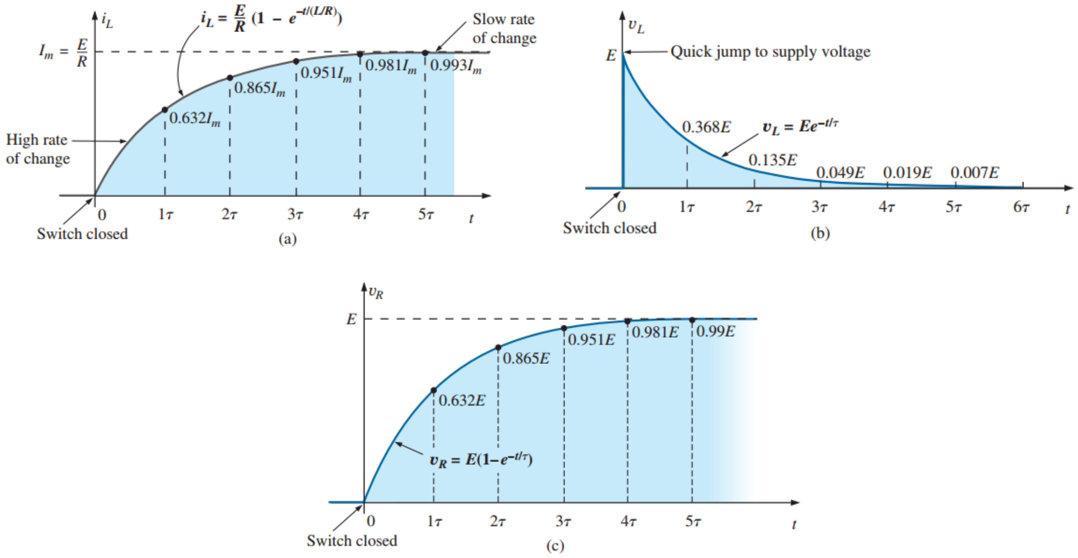


FIG. 11.32
i_L, v_L, and v_R for the circuit in Fig. 11.31 following the closing of the switch.

The equation for the transient response of the current through an inductor is:

$$i_L = \frac{E}{R}(1 - e^{-t/\tau}) \quad \text{(amperes, A)} \qquad \textbf{(11.13)}$$

with the time constant now defined by:

$$\tau = \frac{L}{R} \quad \text{(seconds, s)} \qquad \textbf{(11.14)}$$

The equation for the voltage across the coil is:

$$v_L = Ee^{-t/\tau} \quad \text{(volts, V)} \qquad \textbf{(11.15)}$$

and the equation for the voltage across the resistor is:

$$v_R = E(1 - e^{-t/\tau}) \quad \text{(volts, V)} \qquad \textbf{(11.16)}$$

The storage phase has passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.

The current cannot change instantaneously in an inductive network.

The inductor takes on the characteristics of an open circuit, at the instant the switch is closed.

The inductor takes on the characteristics of a short circuit when steady-state conditions have been established.

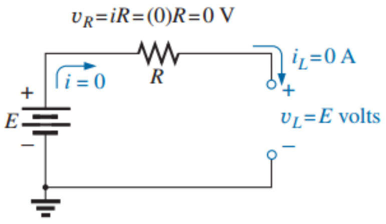


FIG. 11.34
Circuit in Figure 11.31 the instant the switch is closed.

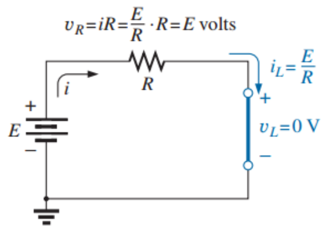


FIG. 11.35
Circuit in Fig. 11.31 under steady-state conditions.

Initial Conditions

Since the current through a coil cannot change instantaneously, the current through a coil begins the transient phase at the initial value established by the network (note Fig. 11.38) before the switch was closed.

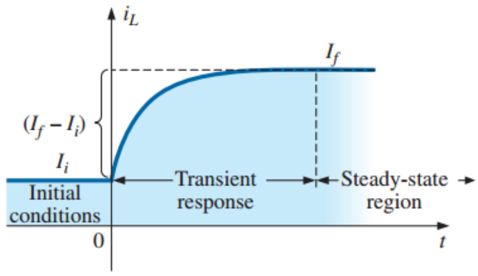


FIG. 11.38
Defining the three phases of a transient waveform.

Using the transient equation developed in the previous section, we can write an equation for the current i_L for the entire time interval in Fig. 11.38.

$$i_L = I_f + (I_i - I_f)e^{-t/\tau} \qquad \textbf{(11.17)}$$

R-L Transients: the release phase

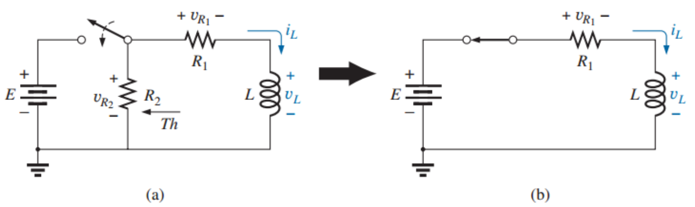


FIG. 11.42
Initiating the storage phase for an inductor by closing the switch.

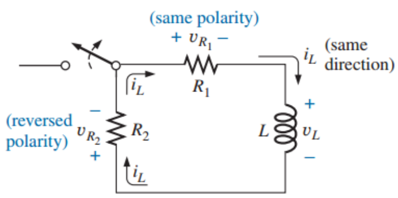


FIG. 11.43
Network in Fig. 11.42 the instant the switch is opened.

$$v_L = -(v_{R_1} + v_{R_2}) \tag{11.18}$$

$$v_L = -(v_{R_1} + v_{R_2}) = -(i_1 R_1 + i_2 R_2)$$
$$= -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2) = -\left(\frac{R_1}{R_1} + \frac{R_2}{R_1}\right)E$$

and

$$v_L = -\left(1 + \frac{R_2}{R_1}\right)E \quad \text{(switch opened)} \tag{11.19}$$

$$v_L = -V_i e^{-t/\tau'} \tag{11.20}$$

$$i_L = \frac{E}{R_1} e^{-t/\tau'} \tag{11.21}$$

$$v_{R_1} = E e^{-t/\tau'} \tag{11.22}$$

$$v_{R_2} = -\frac{R_2}{R_1} E e^{-t/\tau'} \tag{11.23}$$

In the preceding analysis, it was assumed that steady-state conditions were established during the charging. However, if the switch in Fig. 11.42 is opened before i_L reaches its maximum value, the equation for the decaying current of Fig. 11.42 must change to

$$i_L = I_i e^{-t/\tau'} \tag{11.24}$$

where I_i is the starting or initial current. The voltage across the coil is defined by the following:

$$v_L = -V_i e^{-t/\tau'} \tag{11.25}$$

with

$$V_i = I_i(R_1 + R_2)$$

7. Glossary – English/Chinese Translation

English	Chinese
Capacitor and Capacitance	电容器和电容
Inductor and Inductance	电感和电感
Charge and Discharge	充电和放电
Micro Farad	微法拉
Pico Farad	皮科·法拉德
Permittivity	介电常数
Parallel Connection	并联
Series Connection	串联
Magnetic Flux	磁通量
Permeability	渗透性
Time Constant	时间常数
Initial Condition	初始条件

Your Notes: