Tutorial and Solution - 1-02-d

Question 1

5.2 Find the Thevenin equivalent circuit for a dc power supply that has a 30-V terminal voltage when delivering 400 mA and a 27-V terminal voltage when delivering 600 mA.

SOLUTION

For the Thévenin equivalent circuit, the terminal voltage is the Thévenin voltage minus the drop across the Thévenin resistor. Consequently, from the two specified conditions of operation,

$$V_{Th} - (400 \times 10^{-3})R_{Th} = 30$$

$$V_{Th} - (600 \times 10^{-3})R_{Th} = 27$$

Subtracting,

$$-(400 \times 10^{-3})R_{Th} + (600 \times 10^{-3})R_{Th} = 30 - 27$$

from which $$R_{Th} = \frac{3}{200 \times 10^{-3}} = 15\ \Omega$$

This value of R_{Th} substituted into the first equation gives

$$V_{Th} - (400 \times 10^{-3})(15) = 30 \qquad \text{or} \qquad V_{Th} = 36\ \text{V}$$

Question 2

5.4 Find the Norton equivalent circuit for the power supply of Prob. 5.2 if the terminal voltage is 28 V instead of 27 V when the power supply delivers 600 mA.

SOLUTION

For the Norton equivalent circuit, the load current is the Norton current minus the loss of current through the Norton resistor. Consequently, from the two specified conditions of operation,

$$I_N - \frac{30}{R_N} = 400 \times 10^{-3}$$

$$I_N - \frac{28}{R_N} = 600 \times 10^{-3}$$

Subtracting,

$$-\frac{30}{R_N} + \frac{28}{R_N} = 400 \times 10^{-3} - 600 \times 10^{-3}$$

or $$-\frac{2}{R_N} = -200 \times 10^{-3} \qquad \text{from which} \qquad R_N = \frac{2}{200 \times 10^{-3}} = 10\ \Omega$$

Substituting this into the first equation gives

$$I_N - \frac{30}{10} = 400 \times 10^{-3} \qquad \text{and so} \qquad I_N = 3.4\ \text{A}$$

Question 3

5.5 What resistor draws a current of 5 A when connected across terminals a and b of the circuit shown in Fig. 5-10?

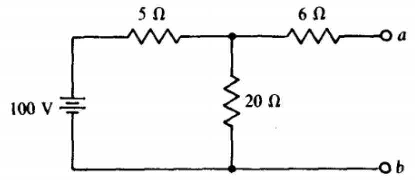

Fig. 5-10

SOLUTION

A good approach is to use Thévenin's theorem to simplify the circuit to the Thévenin equivalent of a V_{Th} voltage source in series with an R_{Th} resistor. Then the load resistor R is in series with these, and Ohm's law can be used to find R:

$$5 = \frac{V_{Th}}{R_{Th} + R} \qquad \text{from which} \qquad R = \frac{V_{Th}}{5} - R_{Th}$$

The open-circuit voltage at terminals a and b is the voltage across the 20-Ω resistor since there is 0 V across the 6-Ω resistor because no current flows through it. By voltage division this voltage is

$$V_{Th} = \frac{20}{20 + 5} \times 100 = 80 \text{ V}$$

R_{Th} is the resistance at terminals a and b with the 100-V source replaced by a short circuit. This short circuit places the 5- and 20-Ω resistors in parallel for a net resistance of $5 \| 20 = 4\ \Omega$. So, ~~$R_{Th} = 6 + 4 = 10\ \Omega$.~~

With V_{Th} and R_{Th} known, the load resistance R for a 5-A current can be found from the previously derived equation:

$$R = \frac{V_{Th}}{5} - R_{Th} = \frac{80}{5} - 10 = 6\ \Omega$$

Question 4

5.7 Find the Thevenin equivalent circuit at terminals a and h of the circuit with transistor model shown in Fig. 5-13.

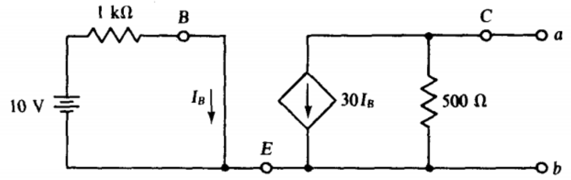

Fig. 5-13

The open-circuit voltage is $500 \times 30I_B = 15\,000 I_B$, positive at terminal b. From the base circuit, $I_B = 10/1000$ A $= 10$ mA. Substituting in for I_B gives

$$V_{Th} = 15\,000(10 \times 10^{-3}) = 150 \text{ V}$$

The best way to find R_{Th} is to deactivate the independent 10-V source and determine the resistance at terminals a and b. With this source deactivated, $I_B = 0$ A, and so $30I_B = 0$ A, which means that the dependent current source acts as an open circuit—it produces zero current regardless of the voltage across it. The result is that the resistance at terminals a and b is just the shown 500 Ω.

The Thévenin equivalent circuit is a 500-Ω resistor in series with a 150-V source that has its positive terminal toward terminal b, as shown in Fig. 5-14.

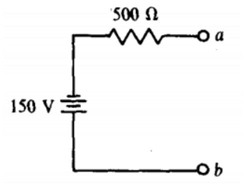

Fig. 5-14

Question 5

5.20 In the circuit of Fig. 5-28, what resistor R_L will absorb maximum power and what is this power?

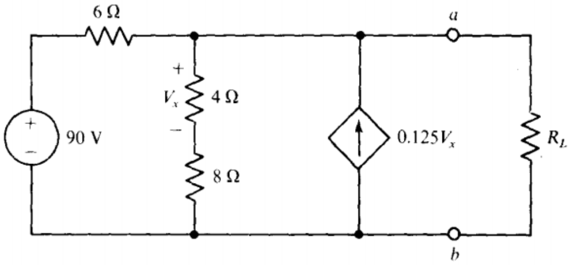

Fig. 5-28

SOLUTION

It is, of course, necessary to obtain the Thévenin equivalent to the left of the a and b terminals. The Thévenin voltage V_{Th} will be obtained first. Observe that the voltage drop across the 4-Ω resistor is V_x, and that this resistor is in series with an 8-Ω resistor. Consequently, by voltage division performed in a reverse manner, the open-circuit voltage is $V_{Th} = V_{ab} = 3V_x$. Next, with R_L removed, applying KCL at the node that includes terminal a gives

$$\frac{3V_x - 90}{6} + \frac{V_x}{4} - 0.125V_x = 0$$

the solution to which is $V_x = 24$ V. So, $V_{Th} = 3V_x = 3(24) = 72$ V.

By inspection of the circuit, it should be fairly apparent that it is easier to use I_{SC} to obtain R_{Th} than it is to determine R_{Th} directly. If a short circuit is placed across terminals a and b, then $V_x = 0$ V, and so no current flows in the 4-Ω resistor and there is no current flow in the dependent current source. Consequently, $I_{SC} = 90/6 = 15$ A. Then,

$$R_{Th} = \frac{V_{Th}}{I_{SC}} = \frac{72}{15} = 4.8\ \Omega$$

which is the resistance that R_L should have for maximum power absorption. Finally,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{72^2}{4(4.8)} = 270\ \text{W}$$

Question 6

5.23 For the circuit shown, use superposition to find *Vn* referenced positive on terminal a.

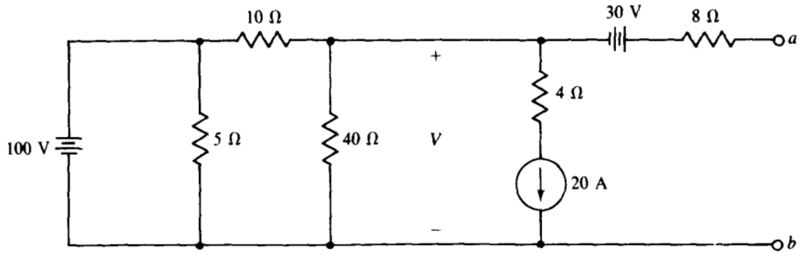

Fig. 5-18

Clearly, the 30-V source contributes 30 V to V_{Th} because this source, being in series with an open circuit, cannot cause any currents to flow. Zero currents mean zero resistor voltage drops, and so the only voltage in the circuit is that of the source.

Figure 5-31*a* shows the circuit with all independent sources deactivated except the 100-V source. Notice that the voltage across the 40-Ω resistor appears across terminals *a* and *b* because there is a zero voltage drop across the 8-Ω resistor. By voltage division this component of V_{Th} is

$$V_{Th_V} = \frac{40}{40 + 10} \times 100 = 80 \text{ V}$$

Figure 5-31*b* shows the circuit with the current source as the only independent source. The voltage across the 40-Ω resistor is the open-circuit voltage since there is a zero voltage drop across the 8-Ω resistor. Note that the short circuit replacing the 100-V source prevents the 5-Ω resistor from having an effect, and also it places the 40- and 10-Ω resistors in parallel for a net resistance of $40 \| 10 = 8$ Ω. So, the component of V_{Th} from the current source is $V_{Th_C} = -20 \times 8 = -160$ V.

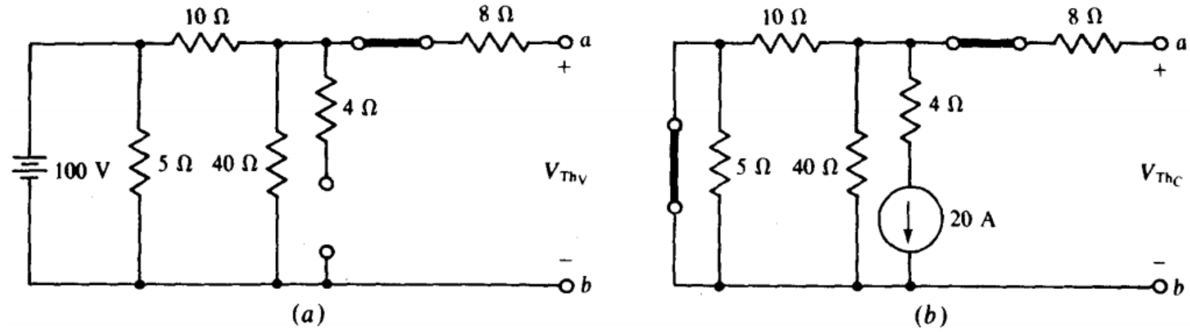

Fig. 5-31

V_{Th} is the algebraic sum of the three components of voltage:

$$V_{Th} = 30 + 80 - 160 = -50 \text{ V}$$

<u>Question 7</u>

5.30 Use a Δ-to-Y transformation in finding the currents I_1, I_2, and I_3 for the circuit shown in Fig. 5-34.

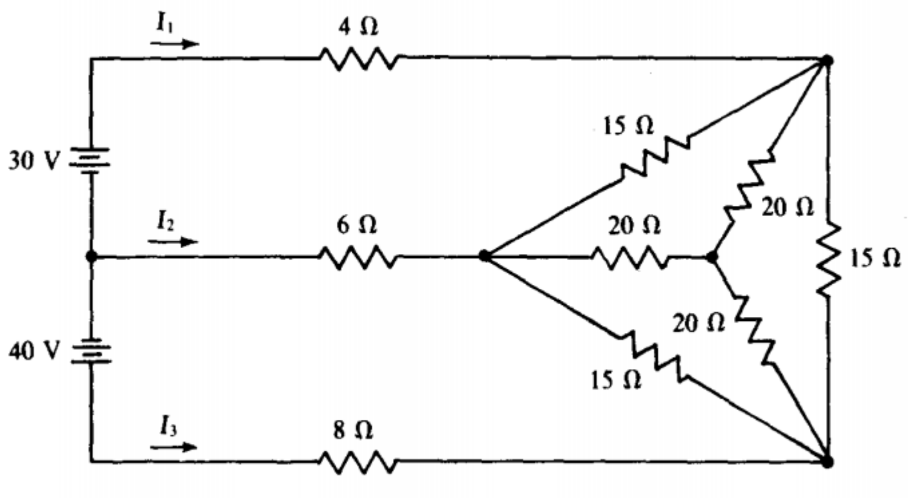

Fig. 5-34

SOLUTION

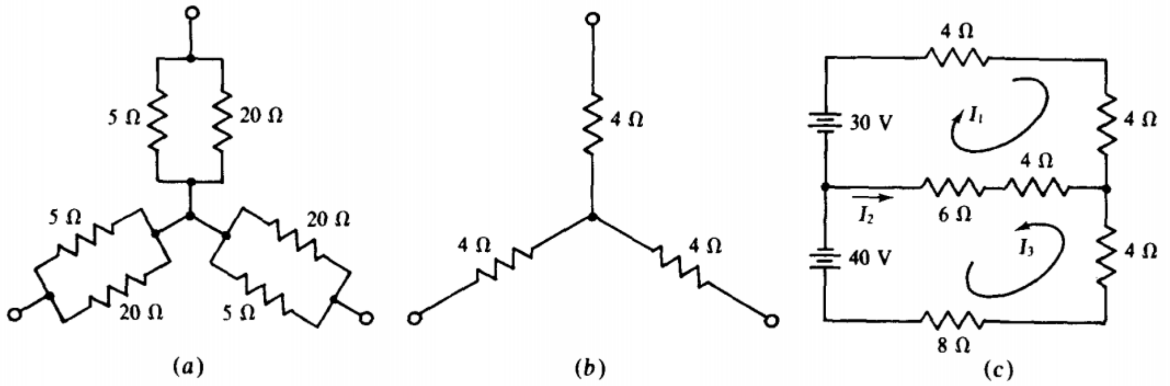

Fig. 5-35

The Δ of 15-Ω resistors transforms to a Y of $15/3 = 5\text{-}\Omega$ resistors that are in parallel with the Y of 20-Ω resistors. It is not obvious that they are in parallel, and in fact they would not be if the resistances for each Y were not all the same value. When, as here, they are the same value, an analysis would show that the middle nodes are at the same potential, just as if a wire were connected between them. So, corresponding resistors of the two Y's are in parallel, as shown in Fig. 5-35*a*. The two Y's can be reduced to the single Y shown in Fig. 5-35*b*, in which each Y resistance is $5\|20 = 4\ \Omega$. With this Y replacing the Δ-Y combination, the circuit is as shown in Fig. 5-35*c*.

With the consideration of I_1 and I_3 as loop currents, the corresponding KVL equations are

$$30 = 18I_1 + 10I_3 \qquad \text{and} \qquad 40 = 10I_1 + 22I_3$$

the solutions to which are $I_1 = 0.88$ A and $I_3 = 1.42$ A. Then, from KCL applied at the right-hand node, $I_2 = -I_1 - I_3 = -2.3$ A.