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Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-d

DC Circuit Theorems

Contents

Reference:
Basic Circuit Analysis – Schaum's Outline Series

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1. Thevenin's Theorem and Norton's Theorem

Thevenin's and Norton's theorems are probably the most important network theorems. For the application of either of them, a network is divided into two parts, A and B, as shown in Fig. 5-la, with two joining wires. One part must be linear, but the other part can be anything.

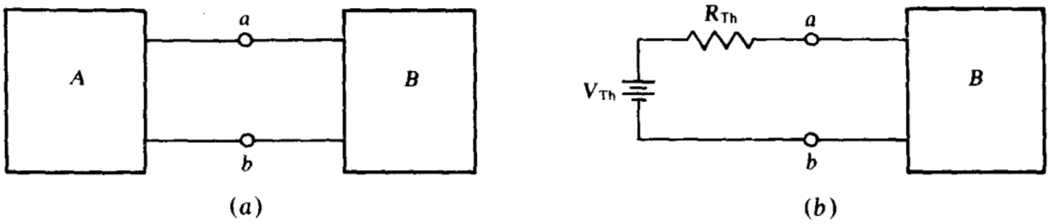


Fig. 5-1

Thevenin's theorem specifies that part A, can be replaced by a

Thevenin equivalent circuit consisting of a voltage source and a resistor in series, without any changes in voltages or currents in part B. The voltage source is called the Thevenin voltage, and the resistor is called the Thevenin resistance.

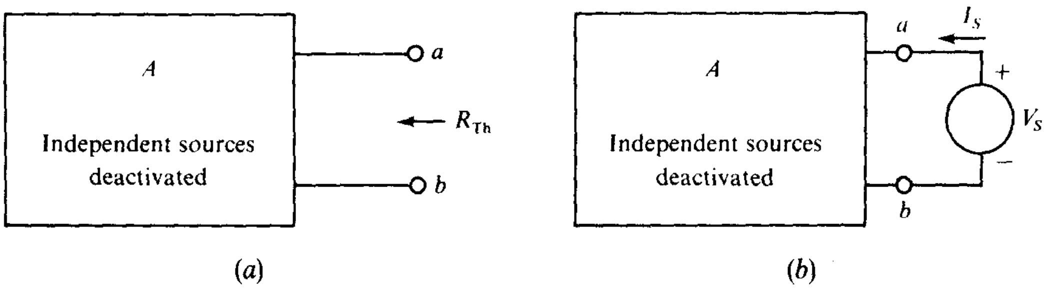


Fig. 5-2

With all internal source deactivated, and with an independent source is applied, the resistance "seen" by this source, using this equation:

$$R_{Th} = \frac{V_s}{I_s}$$

It can also be found from the current I_{SC} that flows in a short circuit placed across terminals a and b, as shown in Fig. 5-3a.

$$R_{Th} = \frac{V_{Th}}{I_{SC}}$$

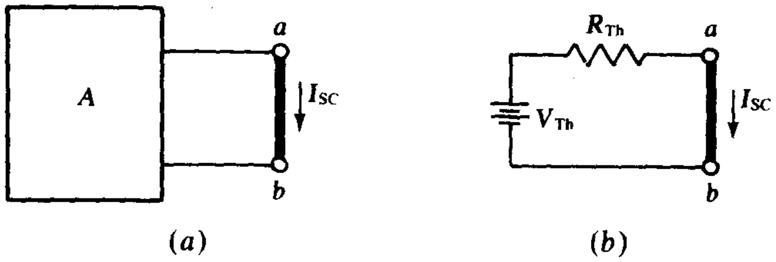


Fig. 5-3

The Norton equivalent circuit can be derived by applying a source transformation to the Thevenin equivalent circuit, as illustrated in Fig. 5-4a.

The Norton equivalent circuit is sometimes illustrated as in Fig. 5-4b, in which $I_N = V_{Th}/R_{Th}$ and $R_N = R_{Th}$.

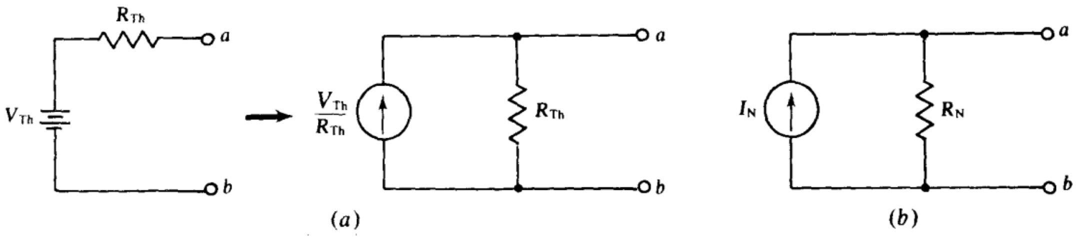
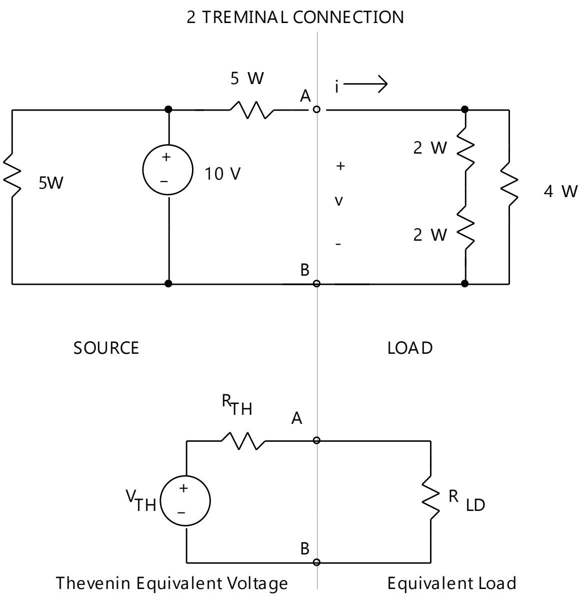


Fig. 5-4

Using Thevenin to solve circuit problems

Any 2-port linear circuit can be broken down into 2 components:

(i) Thevenin voltage source and (ii) equivalent load.



The source can be simplified into a Thevenin equivalent circuit. The circuit contains a voltage source v_{TH} and a resistor R_{TH}.

v_{Th} : Thevenin Equivalent Voltage.
(i.e. open circuit voltage across A-B, load removed)

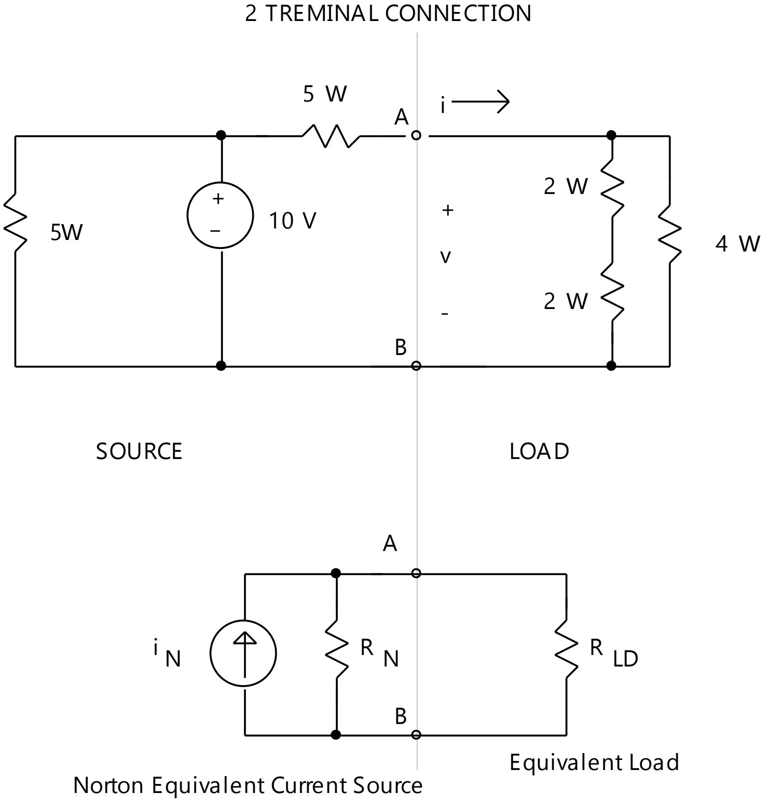
R_{Th}: Thevenin Equivalent Resistance.
(i.e. resistance across A-B, by setting V source as open circuit and i-source as close circuit, load removed)

Methodology

1. Remove the load
2. Find the open circuit voltage across a-b, (i.e. v_{Th})
3. Zero all v and i sources
4. Calculate the resistance across a-b, (i.e. R_{Th})
5. Replace the circuit with v_{Th} and R_{Th}.
6. Put the load back, and calculate the required v and i.

Using Norton to solve circuit problems

Any 2-port linear circuit can be broken down into 2 components: (i) Norton current source and (ii) equivalent load.



The source can be simplified into a Norton equivalent circuit. The circuit contains a current source i_N and a resistor R_N.

i_N: Norton Equivalent Current.
(i.e. short circuit current across A-B, load removed)

R_N: Norton Equivalent Resistance.
(i.e. resistance across A-B, by setting V source as open circuit and i-source as close circuit, load removed)

Methodology

7. Remove the load
8. Find the short circuit current across a-b, (i.e. i_N)
9. Zero all v and i sources
10. Calculate the resistance across a-b, (i.e. R_N)
11. Replace the circuit with i_N and R_N.
12. Put the load back, and calculate the required v and i.

Examples

EXAMPLE 3.21 Norton Equivalent Circuit

LO4

Problem

Determine the Norton current and the Norton equivalent for the circuit of Figure 3.55.

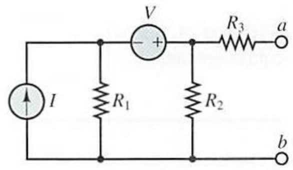


Figure 3.55

Solution

Known Quantities: Source voltage and current; resistor values.

Find: Equivalent resistance R_T; Norton current $i_N = i_{SC}$.

Schematics, Diagrams, Circuits, and Given Data: $V = 6$ V; $I = 2$ A; $R_1 = 6\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 2\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: We first compute the Thévenin equivalent resistance. We zero the two sources by shorting the voltage source and opening the current source. The resulting circuit is shown in Figure 3.56. We can clearly see that $R_T = R_1 \| R_2 + R_3 = 6\|3 + 2 = 4\ \Omega$.

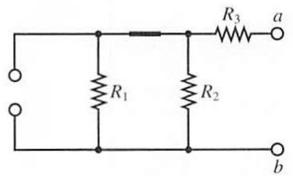


Figure 3.56

Next we compute the Norton current. Following the Norton current Focus on Methodology box, first we replace the load with a short circuit and label the short-circuit current i_{SC}. The circuit is shown in Figure 3.57 ready for node voltage analysis. Note that we have identified two node voltages v_1 and v_2, and that the voltage source requires that $v_2 - v_1 = V$. The unknown current flowing through the voltage source is labeled i.

Now we are ready to apply the nodal analysis method.

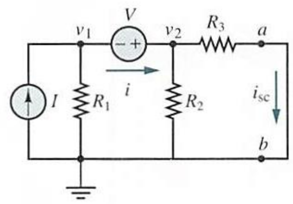


Figure 3.57 Circuit of Example 3.21 ready for nodal analysis

1. The reference node is the ground node in Figure 3.57.
2. The two nodes v_1 and v_2 are also identified in the figure; note that the voltage source imposes the constraint $v_2 = v_1 + V$. Thus only one of the two nodes leads to an independent equation. The unknown current i provides the second independent variable, as you will see in the next step.
3. Applying KCL at nodes 1 and 2, we obtain the following set of equations:

$$I - \frac{v_1}{R_1} - i = 0 \qquad \text{node 1}$$

$$i - \frac{v_2}{R_2} - \frac{v_2}{R_3} = 0 \qquad \text{node 2}$$

Next, we eliminate v_1 by substituting $v_1 = v_2 - V$ in the first equation:

$$I - \frac{v_2 - V}{R_1} - i = 0 \qquad \text{node 1}$$

and we rewrite the equations in matrix form, recognizing that the unknowns are i and v_2. Note that the short-circuit current is $i_{SC} = v_2/R_3$; thus we will seek to solve for v_2.

$$\begin{bmatrix} 1 & \dfrac{1}{R_1} \\ -1 & \dfrac{1}{R_2} + \dfrac{1}{R_3} \end{bmatrix} \begin{bmatrix} i \\ v_2 \end{bmatrix} = \begin{bmatrix} I + \dfrac{V}{R_1} \\ 0 \end{bmatrix}$$

Substituting numerical values, we obtain

$$\begin{bmatrix} 1 & 0.1667 \\ -1 & 0.8333 \end{bmatrix} \begin{bmatrix} i \\ v_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

and we can numerically solve for the two unknowns to find that $i = 2.5$ A and $v_2 = 3$ V. Finally, the Norton or short-circuit current is $i_N = i_{SC} = v_2/R_3 = 1.5$ A.

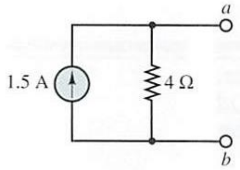


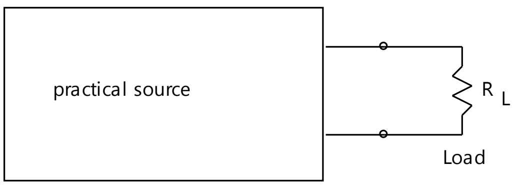
Figure 3.58 Norton equivalent circuit

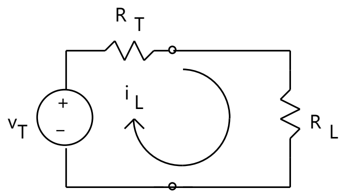
Comments: In this example it was not obvious whether nodal analysis, mesh analysis, or superposition might be the quickest method to arrive at the answer. It would be a very good exercise to try the other two methods and compare the complexity of the three solutions. The complete Norton equivalent circuit is shown in Figure 3.58.

2. Maximum Power Transfer Theorem

The maximum power transfer theorem specifies that a resistive load receives maximum power from a linear, bilateral dc circuit if the load resistance equals the Thevenin resistance of the circuit

Given the values of v_T and R_T, what value of R_L will allow for maximum power transfer?





Power of load: $P_L = i_L^2 R_L$

Current through load: $i_L = \dfrac{v_T}{R_L + R_T}$

Load power: $P_L = \dfrac{v_T^2}{(R_L + R_T)^2} R_L$

Maximize power: $\dfrac{dP_L}{dR_L} = 0 = \dfrac{v_T^2 (R_L + R_T)^2 - 2v_T^2 R_L (R_L + R_T)}{(R_L + R_T)^4}$

Then simplifies… $(R_L + R_T)^2 - 2R_L(R_L + R_T) = 0$

Therefore, the conclusion: $R_L = R_T$

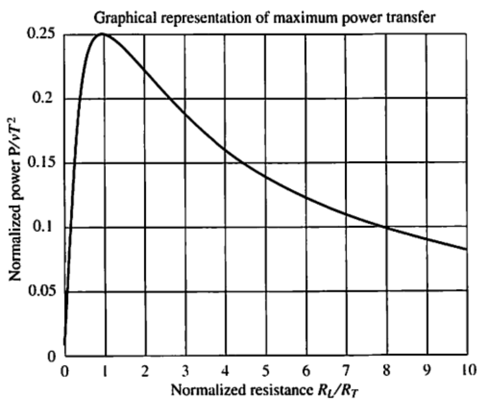
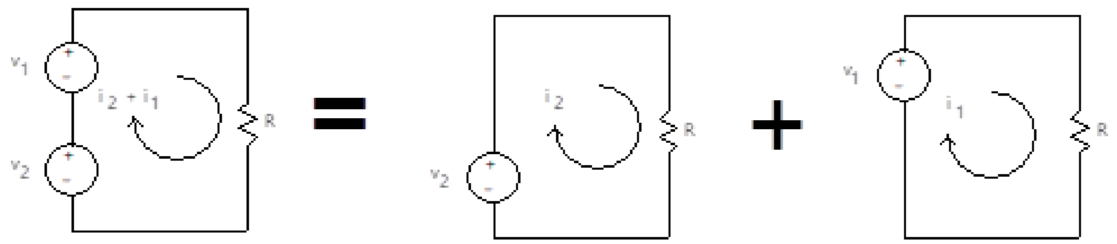


Figure 3.70 Graphical representation of maximum power transfer

For maximum power output to the load, the load resistance must be equal to the source resistance.

3. Superposition Theorem

The superposition theorem specifies that, in a linear circuit containing several independent sources, the current or voltage of a circuit element equals the algebraic sum of the component voltages or currents produced by the independent sources acting alone. This theorem applies only to independent sources—not to dependent ones.



Example

Find the current i_2 below:

Solution

Known Quantities: Source voltage and current values; resistor values.

Find: Unknown current i_2.

Given Data: $V_S = 10$ V; $I_S = 2$ A; $R_1 = 5\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 4\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: *Part 1:* Zero the current source. Once the current source has been set to zero (replaced by an open circuit), the resulting circuit is a simple series circuit shown in Figure 3.29(b); the current flowing in this circuit i_{2-V} is the current we seek. Since the total series resistance is $5 + 2 + 4 = 11\ \Omega$, we find that $i_{2-V} = 10/11 = 0.909$ A.

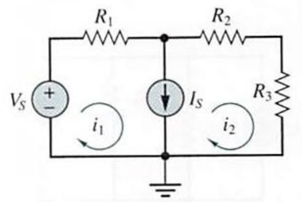
Part 2: Zero the voltage source. After we zero the voltage source by replacing it with a short circuit, the resulting circuit consists of three parallel branches shown in Figure 3.29(c): On the left we have a single 5-Ω resistor; in the center we have a −2-A current source (negative because the source current is shown to flow into the ground node); on the right we have a total resistance of $2 + 4 = 6\ \Omega$. Using the current divider rule, we find that the current flowing in the right branch i_{2-I} is given by

$$i_{2-I} = \frac{\frac{1}{6}}{\frac{1}{5} + \frac{1}{6}}(-2) = -0.909 \text{ A}$$

And, finally, the unknown current i_2 is found to be

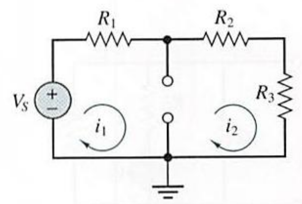
$$i_2 = i_{2\text{-}V} + i_{2-I} = 0 \text{ A}$$

Comments: Superposition is not always a very efficient tool. Beginners may find it preferable to rely on more systematic methods, such as nodal analysis, to solve circuits. Eventually, experience will suggest the preferred method for any given circuit.



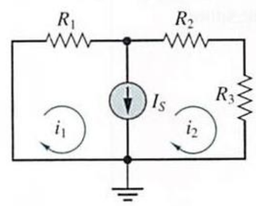
(a)

Figure 3.29 (a) Circuit for the illustration of the principle of superposition



(b)

Figure 3.29 (b) Circuit with current source set to zero



4. Millman's Theorem

Millman's theorem is a method for reducing a circuit by combining parallel voltage sources into a single voltage source. It is just a special case of the application of Thevenin's theorem.

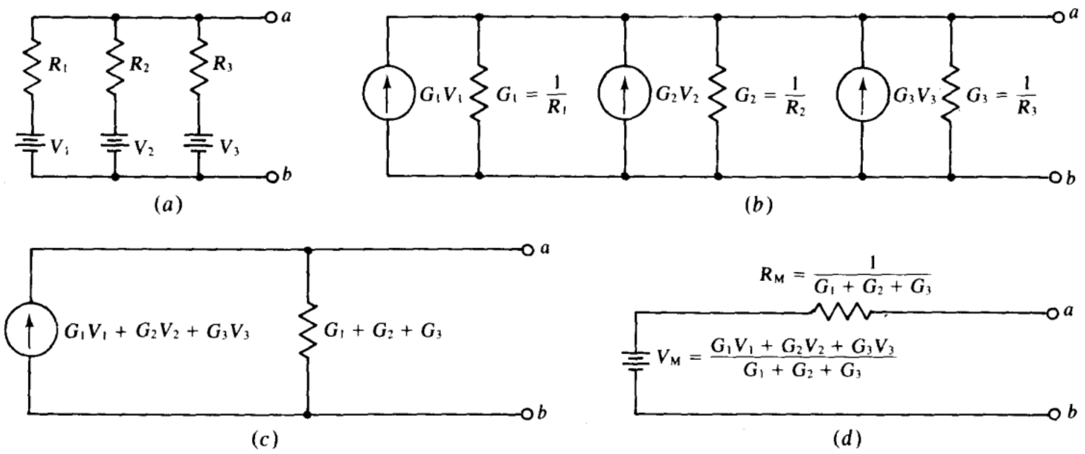


Fig. 5-5

In general, for N parallel voltage sources the Millman voltage source has a voltage of:

$$V_M = \frac{G_1 V_1 + G_2 V_2 + \cdots + G_N V_N}{G_1 + G_2 + \cdots + G_N}$$

and the Millman series resistor has a resistance of:

$$R_M = \frac{1}{G_1 + G_2 + \cdots + G_N}$$

5. Δ-Y Transformation

Figure 5-6a shows a Y (wye) resistor circuit and Fig. 5-6b a Δ (delta) resistor circuit.

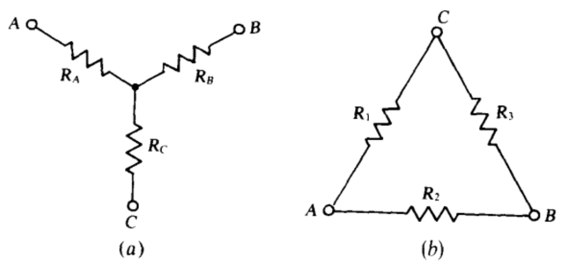


Fig. 5-6

Some algebraic manipulation of the results produces the following A-to-Y transformation formulas:

$$R_A = \frac{R_1R_2}{R_1 + R_2 + R_3} \qquad R_B = \frac{R_2R_3}{R_1 + R_2 + R_3} \qquad R_C = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

Also produced are the following Y-to-Δ transformation formulas:

$$R_1 = \frac{R_AR_B + R_AR_C + R_BR_C}{R_B} \qquad R_2 = \frac{R_AR_B + R_AR_C + R_BR_C}{R_C} \qquad R_3 = \frac{R_AR_B + R_AR_C + R_BR_C}{R_A}$$

Suitable problem to apply the Y-Δ Transformation:

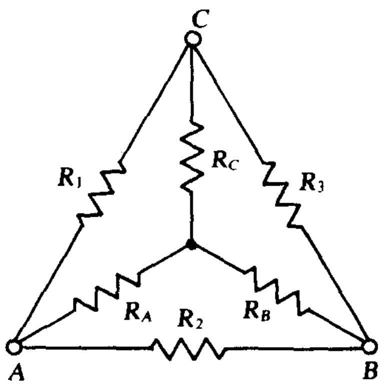


Fig. 5-7

6. Bridge Circuits

A bridge circuit can be used for precision resistance measurements. A Wheatstone bridge has a center branch that is a sensitive current indicator such as a galvanometer, as shown in Fig. 5-9. Three of the

other branches are precision resistors, one of which is variable as indicated. The fourth branch is the resistor with the unknown resistance Rx that is to be measured.

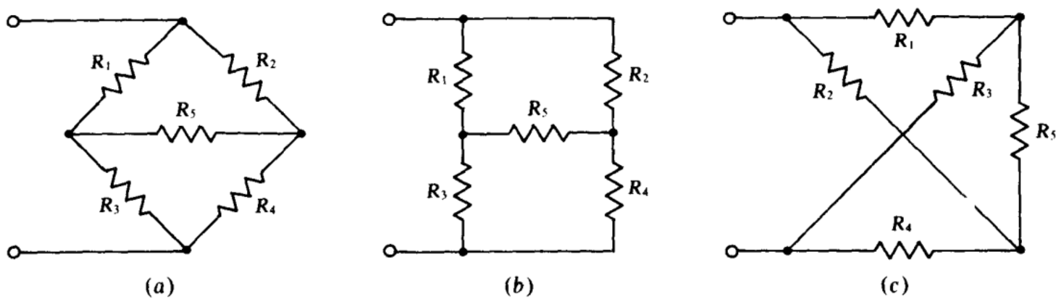


Fig. 5-8

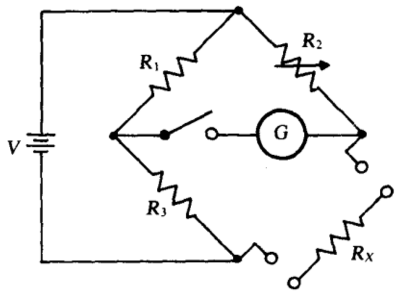


Fig. 5-9

For a resistance measurement, the resistance R_2 of the variable resistor is adjusted until the galvanometer needle does not deflect when the switch in the center branch is closed. This means that, even with the switch open,

the voltage across R_t equals that across R_2, and the voltage across R_3 equals that across R_x. In this condition the bridge is said to be balanced. By voltage division,

$$\frac{R_1 V}{R_1 + R_3} = \frac{R_2 V}{R_2 + R_X} \qquad \text{and} \qquad \frac{R_3 V}{R_1 + R_3} = \frac{R_X V}{R_2 + R_X}$$

Taking the ratio of the two equations produces the bridge balance equation:

$$R_X = \frac{R_2 R_3}{R_1}$$

7. Glossary – English/Chinese Translation

English	Chinese
Thevenin's Theorem	戴维南定理
Norton's Theorem	诺顿定理
Maximum Power Transfer	最大功率传输
Superposition Theorem	叠加定理
Millman's Theorem	米尔曼定理
Delta-Wye Transformation	Delta-Wye 变换
Bridge Circuits	桥接电路
Thevenin Voltage Source	戴维南电压源
Norton Current Source	诺顿电流源
Thevenin Resistance	戴维宁抗性
Equivalent Circuit	等效电路
Galvanometer	检流计
Wheatstone Bridge Circuit	惠斯通桥电路