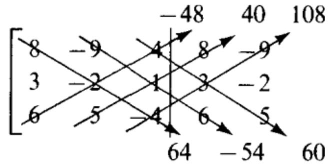
Tutorial Solution 1-02-c

Question 1

One method of evaluation is to repeat the first two columns to the right of the third column and then find the products of the numbers on the diagonals, as indicated:

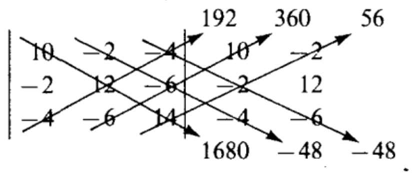


The value of the determinant is the sum of the products for the downward-pointing arrows minus the sum of the products for the upward-pointing arrows:

$$(64 - 54 + 60) - (-48 + 40 + 108) = -30$$

Question 2

All three unknowns have the same denominator determinant of coefficients, which evaluates to



$$1680 - 48 - 48 - (192 + 360 + 56) = 976$$

In the numerator determinants, the right-hand sides of the equations replace the first column for I_1, the second column for I_2, and the third column for I_3:

$$I_1 = \frac{\begin{vmatrix} 10 & -2 & -4 \\ -34 & 12 & -6 \\ 40 & -6 & 14 \end{vmatrix}}{976} = \frac{1952}{976} = 2\ \text{A} \qquad I_2 = \frac{\begin{vmatrix} 10 & 10 & -4 \\ -2 & -34 & -6 \\ -4 & 40 & 14 \end{vmatrix}}{976} = \frac{-976}{976} = -1\ \text{A}$$

$$I_3 = \frac{\begin{vmatrix} 10 & -2 & 10 \\ -2 & 12 & -34 \\ -4 & -6 & 40 \end{vmatrix}}{976} = \frac{2928}{976} = 3\ \text{A}$$

Question 3

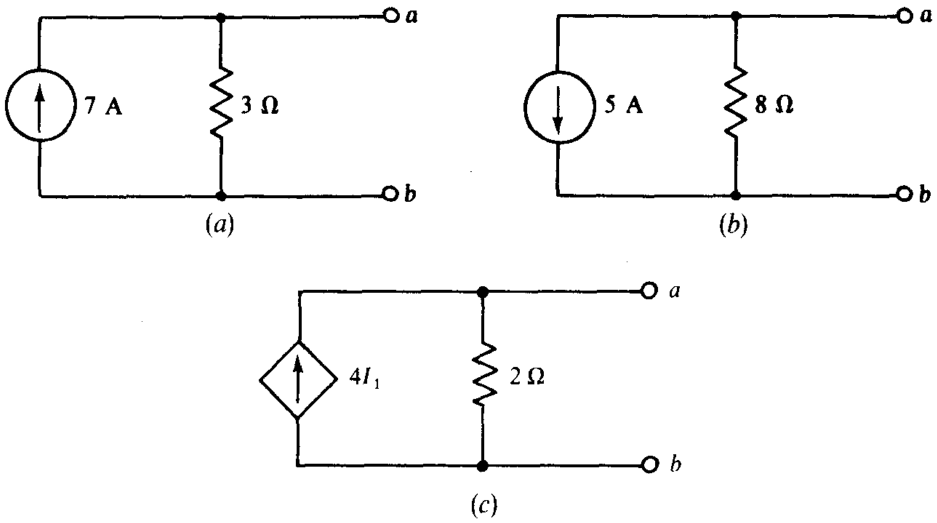


Fig. 4-5

Question 4

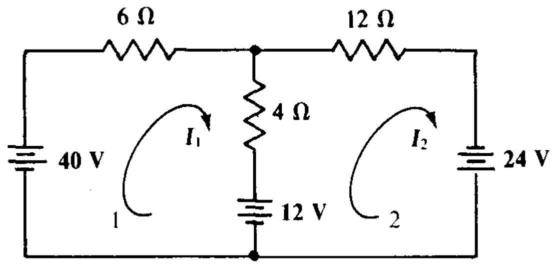


Fig. 4-13

The self-resistance of mesh 1 is $6 + 4 = 10\ \Omega$, the mutual resistance with mesh 2 is 4 Ω, and the sum of the source voltage rises in the direction of I_1 is $40 - 12 = 28$ V. So, the mesh 1 KVL equation is $10I_1 - 4I_2 = 28$.

Similarly, for mesh 2 the self-resistance is $4 + 12 = 16\ \Omega$, the mutual resistance is 4 Ω, and the sum of the voltage rises from voltage sources is $24 + 12 = 36$ V. These give a mesh 2 KVL equation of $-4I_1 + 16I_2 = 36$.

Placing the two mesh equations together shows the symmetry of coefficients (here -4) about the principal diagonal as a result of the common mutual resistance:

$$10I_1 - 4I_2 = 28$$

$$-4I_1 + 16I_2 = 36$$

A good way to solve these two equations is to add four times the first equation to the second equation to eliminate I_2. The result is

$$40I_1 - 4I_1 = 112 + 36 \qquad \text{from which} \qquad I_1 = \frac{148}{36} = 4.11\text{ A}$$

This substituted into the second equation gives

$$-4(4.11) + 16I_2 = 36 \qquad \text{and} \qquad I_2 = \frac{52.44}{16} = 3.28\text{ A}$$

Question 5

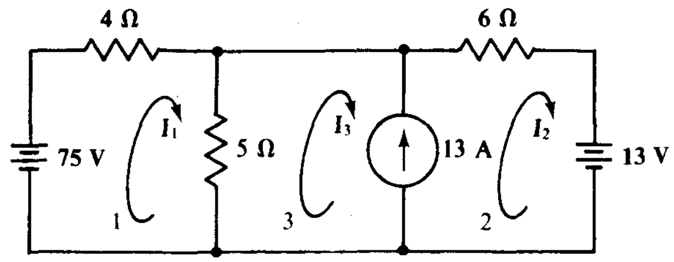


Fig. 4-15

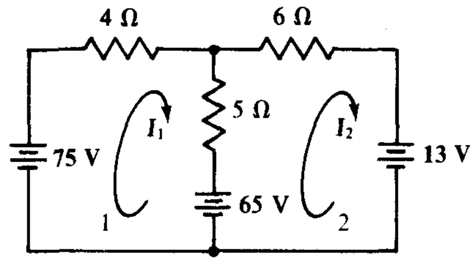


Fig. 4-16

One analysis approach is to transform the 13-A current source and parallel 5-Ω resistor into a voltage source, as shown in the circuit of Fig. 4-16.

The self-resistance of mesh 1 is $4 + 5 = 9\ \Omega$, and that of mesh 2 is $6 + 5 = 11\ \Omega$. The mutual resistance is 5 Ω. The voltage rises from sources are $75 - 65 = 10$ V for mesh 1 and $65 - 13 = 52$ V for mesh 2. The corresponding mesh equations are

$$9I_1 - 5I_2 = 10$$

$$-5I_1 + 11I_2 = 52$$

Multiplying the first equation by 5 and the second by 9 and then adding them eliminates I_1:

$$-25I_2 + 99I_2 = 50 + 468 \qquad \text{from which} \qquad I_2 = \frac{518}{74} = 7\ \text{A}$$

This substituted into the first equation produces

$$9I_1 - 5(7) = 10 \qquad \text{or} \qquad I_1 = \frac{10 + 35}{9} = 5\ \text{A}$$

From the original circuit shown in Fig. 4-15, the current through the current source is $I_2 - I_3 = 13$ A, and so

$$I_3 = I_2 - 13 = 7 - 13 = -6\ \text{A}$$

Question 6

Three loop currents are required because the circuit has three meshes. Only one loop current should flow through the 5-kΩ resistor so that only one current needs to be solved for. The paths for the two other loop currents can be selected as shown, but there are other suitable paths.

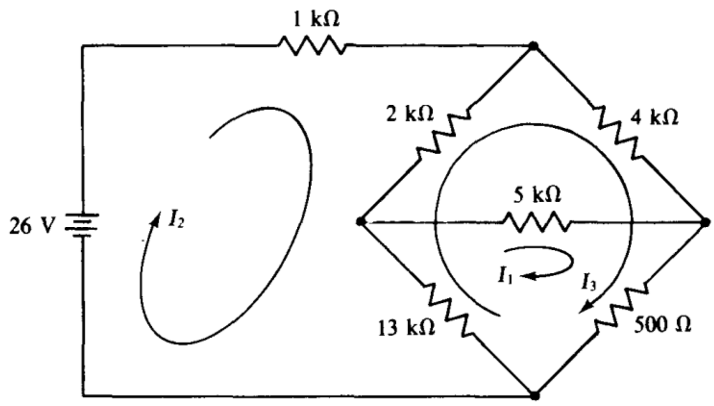


Fig. 4-21

As has been mentioned, since working with kilohms is inconvenient, a common practice is to drop those units—to divide each resistance by 1000. But then the current answers will be in milliamperes. With this approach, and from self-resistances, mutual resistances, and aiding source voltages, the loop equations are

$$\begin{aligned} 18.5I_1 - 13I_2 + 13.5I_3 &= 0 \\ -13I_1 + 16I_2 - 15I_3 &= 26 \\ 13.5I_1 - 15I_2 + 19.5I_3 &= 0 \end{aligned}$$

Notice the symmetry of the I coefficients about the principal diagonal, just as for mesh equations. But there is the difference that some of these coefficients are positive. This is the result of two loop currents flowing through a mutual resistor in the same direction—something that cannot happen in mesh analysis if all mesh currents are selected in the clockwise direction, as is conventional.

From Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} 0 & -13 & 13.5 \\ 26 & 16 & -15 \\ 0 & -15 & 19.5 \end{vmatrix}}{\begin{vmatrix} 18.5 & -13 & 13.5 \\ -13 & 16 & -15 \\ 13.5 & -15 & 19.5 \end{vmatrix}} = \frac{1326}{663} = 2\text{ mA}$$

Question 7

The controlling quantity I in terms of node voltages is $I = V_2/6$. Consequently, the dependent current source provides a current of $0.5I = 0.5(V_2/6) = V_2/12$, and the dependent voltage source provides a voltage of $12I = 12(V_2/6) = 2V_2$.

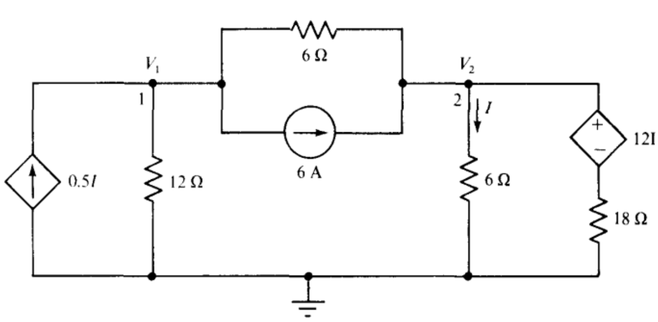


Fig. 4-25

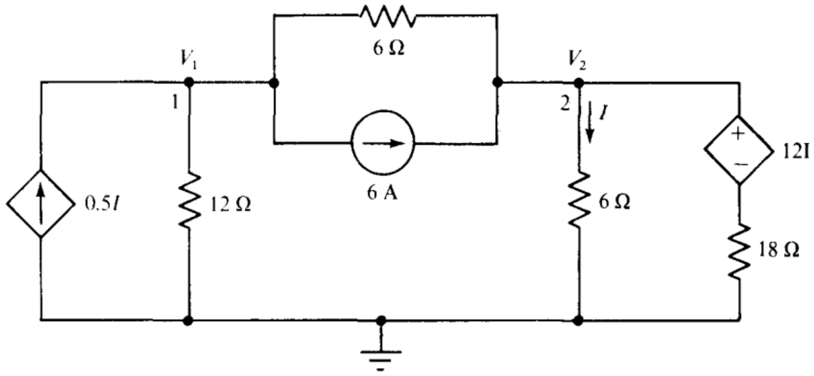


Fig. 4-25

Because of the presence of the dependent sources, it may be best to apply KCL at nodes 1 and 2 on a branch-to-branch basis instead of attempting to use a shortcut method. Doing this gives

$$\frac{-V_2}{12} + \frac{V_1}{12} + \frac{V_1 - V_2}{6} = -6 \qquad \text{and} \qquad \frac{V_2 - V_1}{6} + \frac{V_2}{6} + \frac{V_2 - 2V_2}{18} = 6$$

These simplify to

$$3V_1 - 3V_2 = -72 \qquad \text{and} \qquad -3V_1 + 5V_2 = 108$$

Adding these equations eliminates V_1 and results in $2V_2 = 36$ or $V_2 = 18$ V. Finally,

$$I = \frac{V_2}{6} = \frac{18}{6} = 3\text{ A}$$