Dr. Norbert Cheung's Lecture Series

Level 1 Topic no: 02-b

Resistance and Parallel/Series DC Circuits

Contents

Reference:
Basic Circuit Analysis – Schaum's Outline Series

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1. Resistance

Ohm's Law

If the applied voltage *V* and resulting current *I* have associated references, the relation between *V* and *I* is:

$$I(\text{amperes}) = \frac{\text{V (volts)}}{R(\text{ohms})}$$

The SI unit of resistance is the *ohm* with symbol Ω. The inverse of resistance is called *conductance* and its quantity symbol is *G,* and the unit is ℧.

$$G = 1/R$$

$$I(amperes) = G(siemens) \ x \ V(volts)$$

Resistivity

At a fixed temperature the resistance of a conductor is:

$$R = \rho \frac{l}{A}$$

where *l* is the conductor length in meters and *A* is the cross-sectional area in square meters. The constant of proportionality *p,* the Greek lowercase *ρ* is the quantity symbol for *resistivity,* the factor that depends on the type of material.

Resistor Power Absorption

Substitution from $V = IR$ into $P = VI$ gives the power absorbed by a linear resistor in terms of resistance:

$$P = \frac{V^2}{R} = I^2 R$$

Nominal Values and Tolerances

The popular carbon-composition resistors have tolerances of 20, 10, and 5 percent, which means that the actual resistances can vary from the nominal values by as much as +20, ± 10, and ±5 percent of the nominal values.

Color Code

The most popular resistance color code has nominal resistance values and tolerances indicated by the colors of either three or four bands around the resistor casing. For tolerance: Gold is 5%, Silver is 10%.

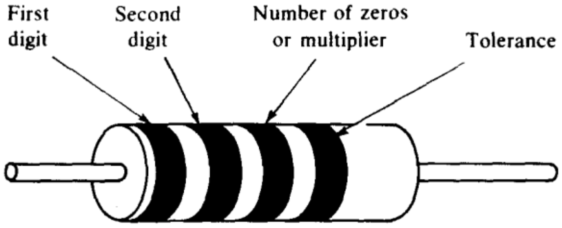

Fig. 2-3

Table 2-4

Color	Number	Color	Number
Black	0	Blue	6
Brown	1	Violet	7
Red	2	Gray	8
Orange	3	White	9
Yellow	4	Gold	0.1
Green	5	Silver	0.01

Open and Short Circuits

An *open circuit* has an infinite resistance, which means that it has zero current flow through it for any finite voltage across it. On a circuit diagram it is indicated by two terminals not connected to Anything.

A *short circuit* is the opposite of an open circuit. It has zero voltage across it for any finite current flow through it. On a circuit diagram a short circuit is designated by an ideal conducting wire—a wire with zero resistance.

Internal Resistance

Every practical voltage or current source has an *internal resistance*.

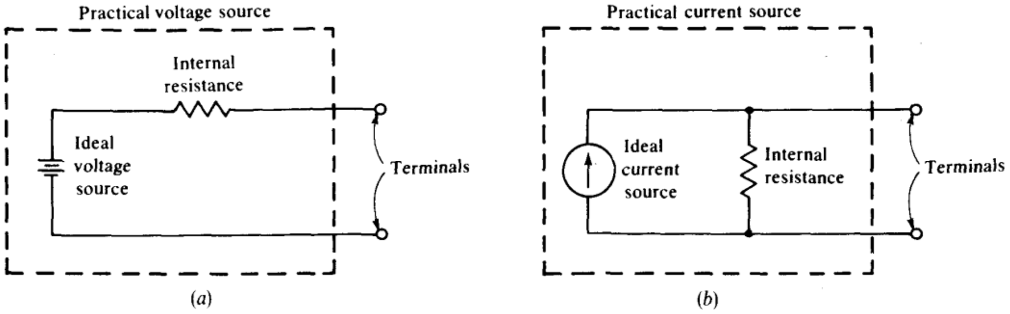

Fig. 2-4

2. Kirchhoff's Voltage Law and Series DC Circuits

Circuit Terms

A *branch* of a circuit is a single component such as a resistor or a source.

A *node* is a connection point between two or more branches. On

A *loop* is any simple closed path in a circuit.

A *mesh* is a loop that does not have a closed path in its interior. No components are inside a mesh.

Components are connected in *series* if they carry the same current.

Components are connected in *parallel* if the same voltage is across them.

Kirchhoff's Voltage Law and Series DC Circuits

Kirchhoff's voltage law, abbreviated KVL, has three equivalent versions: At any instant around a loop, in either a clockwise or counter-clockwise direction:

1. The algebraic sum of the voltage drops is zero.
2. The algebraic sum of the voltage rises is zero.
3. The algebraic sum of the voltage drops equals the algebraic sum of the voltage rises.

In the application of KVL, a loop current is usually referenced clockwise, as shown in the series circuit of Fig. 3-1, and KVL is applied in the direction of the current. (This is a series circuit because the same current I flows through all components.) The sum of the voltage drops across the resistors, $V_1 + V_2 + V_3$, is set equal to the voltage rise V_S across the voltage source: $V_1 + V_2 + V_3 = V_S$. Then the IR Ohm's law relations are substituted for the resistor voltages:

$$V_S = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) = IR_T$$

from which $I = V_S/R_T$ and $R_T = R_1 + R_2 + R_3$. This R_T is the *total resistance* of the series-connected resistors. Another term used is *equivalent resistance*, with symbol R_{eq}.

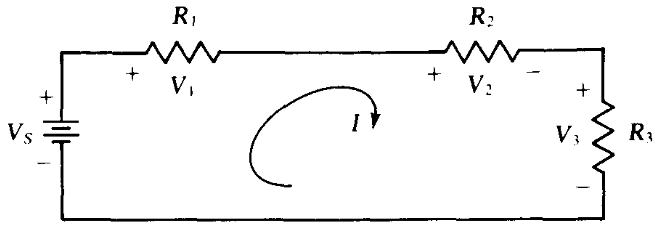

Fig. 3-1

In general, the total resistance of series-connected resistors (series resistors) equals the sum of the individual resistances:

$$R_T = R_1 + R_2 + R_3 + \cdots$$

Voltage Division

The voltage division or voltage divider rule applies to resistors in series.

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_S$$

In general, for any number of series resistors with a total resistance of R_T and with a voltage of V_s across the series combination, the voltage V_x across one of the resistors R_x is:

$$V_X = \frac{R_X}{R_T} V_S$$

3. Kirchhoff's Current Law and Parallel Circuits

Kirchhoff's current law, abbreviated KCL, has three equivalent versions:

At any instant in a circuit,

1. The algebraic sum of the currents leaving a closed surface is zero.
2. The algebraic sum of the currents entering a closed surface is zero.
3. The algebraic sum of the currents entering a closed surface equals the algebraic sum of those leaving.

In the application of KCL, one node is selected as the *ground* or *reference* or *datum node*, which is often indicated by the ground symbol (⏚). Usually, the node at the bottom of the circuit is the ground node, as shown in the parallel circuit of Fig. 3-2. (This is a parallel circuit because the same voltage V is across all circuit components.) The voltages on other nodes are almost always referenced positive with respect to the ground node. At the nongrounded node in the circuit shown in Fig. 3-2, the sum of the currents leaving through resistors, $I_1 + I_2 + I_3$, equals the current I_S entering this node from the current source: $I_1 + I_2 + I_3 = I_S$. The substitution of the $I = GV$ Ohm's law relations for the resistor currents results in

$$I_S = I_1 + I_2 + I_3 = G_1V + G_2V + G_3V = (G_1 + G_2 + G_3)V = G_TV$$

from which $V = I_S/G_T$ and $G_T = G_1 + G_2 + G_3 = 1/R_1 + 1/R_2 + 1/R_3$. This G_T is the *total conductance* of the circuit. Another term used is *equivalent conductance*, with symbol G_{eq}.

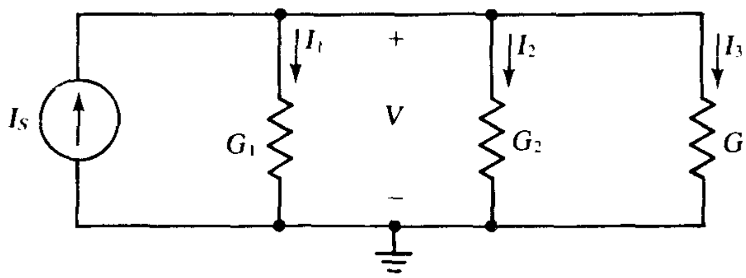

Fig. 3-2

From this result it should be evident that, in general, the total conductance of parallel-connected resistors (parallel resistors) equals the sum of the individual conductances:

$$G_T = G_1 + G_2 + G_3 + \cdots$$

If the conductances are the same (G), and if there are N of them, then $G_T = NG$ and $R_T = 1/G_T = 1/NG = R/N$. Finding the voltage in a parallel circuit is easier using total conductance than applying KCL directly.

Sometimes working with resistances is preferable to conductances. Then from $R_T = 1/G_T = 1/(G_1 + G_2 + G_3 + \cdots)$,

$$R_T = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \cdots}$$

An important check on calculations with this formula is that R_T must always be less than the least resistance of the parallel resistors.

For the special case of just two parallel resistors,

$$R_T = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1R_2}{R_1 + R_2}$$

So, the total or equivalent resistance of two parallel resistors is the product of the resistances divided by the sum.

The symbol $\|$ as in $R_1 \| R_2$ indicates the resistance of two parallel resistors: $R_1 \| R_2 = R_1R_2/(R_1 + R_2)$. It is also sometimes used to indicate that two resistors are in parallel.

If a parallel circuit has more than one current source,

$$(G_1 + G_2 + G_3 + \cdots)V = I_{S_1} + I_{S_2} + I_{S_3} + \cdots$$

in which each I_S term is positive for a source current entering the nongrounded node and is negative for a source current leaving this node.

Current Division

The *current division* or *current divider rule* applies to resistors in parallel. It gives the current through any resistor in terms of the conductances and the current into the parallel combination—the step of finding the resistor voltage is eliminated. The current division formula is easy to derive from the circuit shown in Fig. 3-2. Consider finding the current I_2. By Ohm's law, $I_2 = G_2 V$. Also, $V = I_S/(G_1 + G_2 + G_3)$. Eliminating V results in

$$I_2 = \frac{G_2}{G_1 + G_2 + G_3} I_S$$

In general, for any number of parallel resistors with a total conductance G_T and with a current I_S entering the parallel combination, the current I_X through one of the resistors with conductance G_X is

$$I_X = \frac{G_X}{G_T} I_S$$

This is the formula for the current division or divider rule. For this formula, I_S and I_X must be referenced in the same direction, with I_X referenced away from the node of the parallel resistors that I_S is referenced into. If both currents enter this node, then the formula requires a negative sign. The current I_S need not be that of a source. It is just the total current entering the parallel resistors.

For the special case of two parallel resistors, the current division formula is usually expressed in resistances instead of conductances. If the two resistances are R_1 and R_2, the current I_1 in the resistor with resistance R_1 is

$$I_1 = \frac{G_1}{G_1 + G_2} I_S = \frac{1/R_1}{1/R_1 + 1/R_2} I_S = \frac{R_2}{R_1 + R_2} I_S$$

Example 1

3.2 In fig. 3-3, which components in series and which components are in parallel?

3.3 Identify all loops and meshes in fig. 3-4. Also find out which components are in series and which components are in parallel?

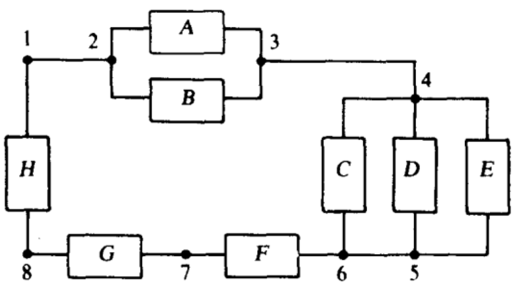

Fig. 3-3

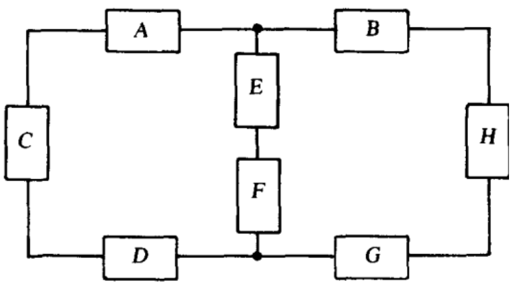

Fig. 3-4

Components F, G, and H are in series because they carry the same current. Components A and B, being connected together at both ends, have the same voltage and so are in parallel. The same is true for components C, D, and E—they are in parallel. Further, the parallel group of A and B is in series with the parallel group of C, D, and E, and both groups are in series with components F, G, and H.

There are three loops: one of components A, E, F, D, and C; a second of components B, H, G, F, and E; and a third of A, B, H, G, D, and C. The first two loops are also meshes, but the third is not because components E and F are inside it. Components A, C, and D are in series because they carry the same current. For the same reason, components E and F are in series, as also are components B, H, and G. No components are in parallel.

Example 2

What is the value of V in the circuit below?

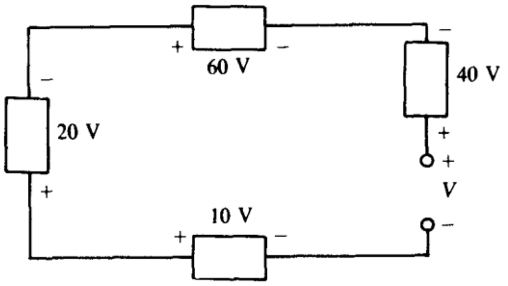

Fig. 3-6

Answer

$$60 - 40 + V - 10 + 20 = 0 \qquad \text{from which} \qquad V = -30\ \text{V}$$

In the summation, the 40 and 10 V are negative because they are voltage rises in a clockwise direction. The negative sign in the answer indicates that the actual open-circuit voltage has a polarity opposite the shown reference polarity.

Example 3

Find the voltage V_{ab} in the circuit below.

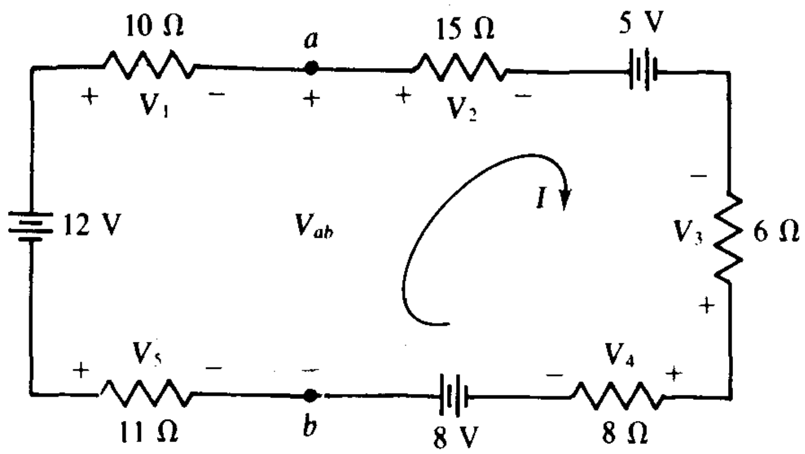

Fig. 3-8

Answer:

V_{ab} is the voltage drop from node *a* to node *b*, which is the sum of the voltage drops across the components connected between nodes *a* and *b* either to the right or to the left of node *a*. It is convenient to choose the path to the right because this is the direction of the $I = 0.3\text{-A}$ current found in the solution of Prob. 3.18. Thus,

$$V_{ab} = (0.3 \times 15) + 5 + (0.3 \times 6) + (0.3 \times 8) - 8 = 5.7\ \text{V}$$

Note that *an IR drop is always positive in the direction of I.* A voltage reference, and that of V_3 in particular here, has no effect on this.

4. Glossary – English/Chinese Translation

English	Chinese
Ohm's Law	欧姆定律
Conductance	电导
Resistance and Resistivity	电阻和电阻率
Tolerance	宽容
Color Code of Resistors	电阻器的颜色代码
Internal Resistance	内阻
Branch	分支
Node Analysis	节点分析
Mesh Analysis	网格分析
Loop Analysis	环路分析
Series and Parallel Connection	串联和并联
Kirchhoff's Voltage Law	基尔霍夫电压定律
Kirchhoff's Current law	基尔霍夫现行定律
Voltage Divider Rule	分压器规则
Current Divider Rule	电流分频器规则

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Your Notes: