<u>**Test 1**</u>

Question 1

By using *"d/dt"* differential block and *"a_1"* scalar block shown in Fig Q1, draw the block diagram representation of the following functions:

$$(a)\quad x_2 = a_1\left(\frac{dx_1}{dt}\right) \qquad (b)\quad x_3 = \frac{d^2x_2}{dt^2} + \frac{dx_1}{dt} - x_1$$

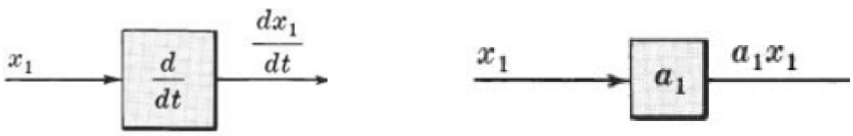


Fig Q1

Question 2

Obtain the Laplace Inverse Transform of the following equation:

$$F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)}$$

Question 3

Simplify the block diagram shown in Figure Q3, and calculate the closed loop transfer function

$E_o(s)/E_i(s)$.

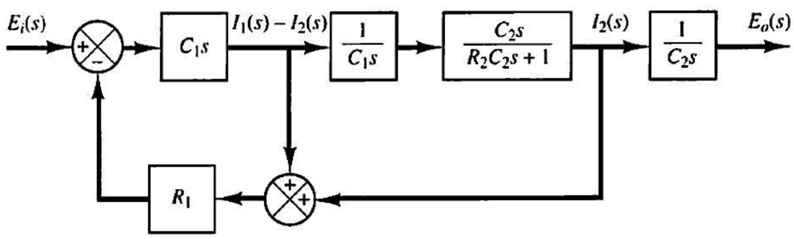


Fig. Q3

Question 4

Use the general gain formula to find the transfer function *H(s)/Q(s)* on Figure Q4

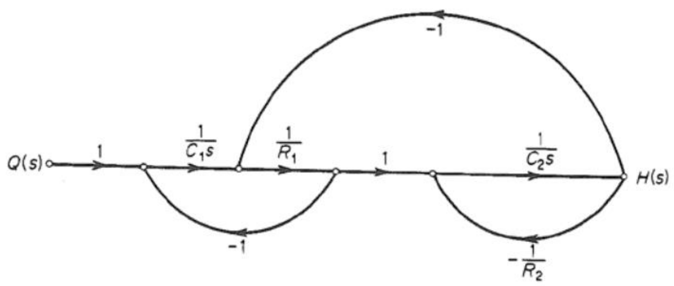
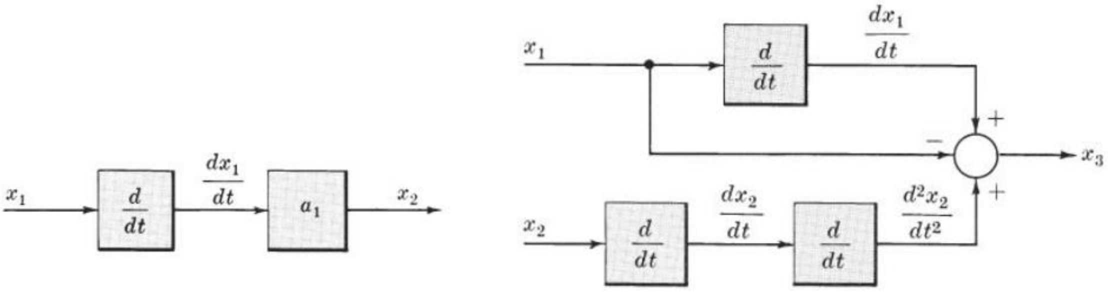


Fig. Q4

Solution

Question 1



(a) (b)

Question 2

$$F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)} = \frac{b_1}{s} + \frac{b_2}{s^2} + \frac{a_1}{s+1} + \frac{a_2}{s+3}$$

$$a_1 = \left.\frac{5(s+2)}{s^2(s+3)}\right|_{s=-1} = \frac{5}{2}$$

$$a_2 = \left.\frac{5(s+2)}{s^2(s+1)}\right|_{s=-3} = \frac{5}{18}$$

$$b_2 = \left.\frac{5(s+2)}{(s+1)(s+3)}\right|_{s=0} = \frac{10}{3}$$

$$b_1 = \frac{d}{ds}\left[\frac{5(s+2)}{(s+1)(s+3)}\right]_{s=0}$$

$$= \left.\frac{5(s+1)(s+3) - 5(s+2)(2s+4)}{(s+1)^2(s+3)^2}\right|_{s=0} = -\frac{25}{9}$$

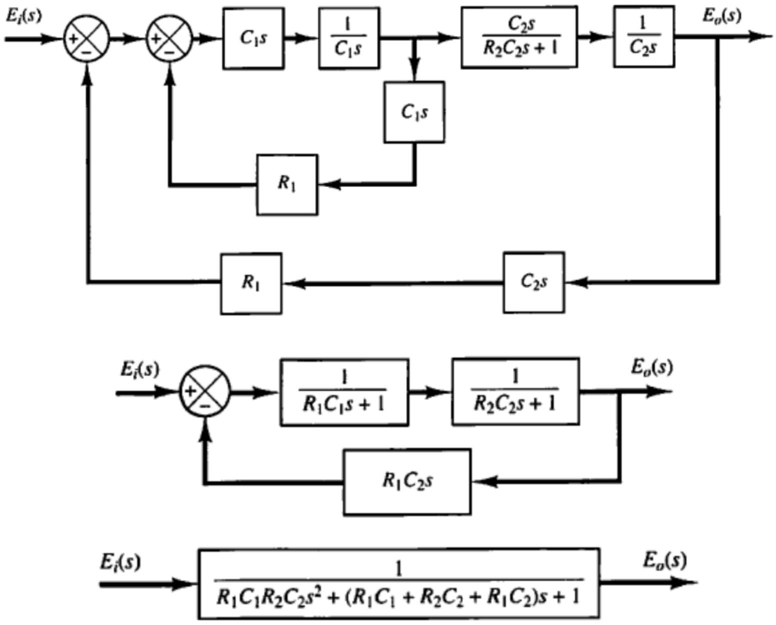
Thus

$$F(s) = -\frac{25}{9}\frac{1}{s} + \frac{10}{3}\frac{1}{s^2} + \frac{5}{2}\frac{1}{s+1} + \frac{5}{18}\frac{1}{s+3}$$

The inverse Laplace transform of $F(s)$ is

$$f(t) = -\frac{25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}, \qquad \text{for } t \geq 0$$

Question 3



Question 4

Loop L_1 does not touch loop L_2. (Loop L_1 touches loop L_3, and loop L_2 touches loop L_3). Hence the determinant Δ is given by

$$\begin{aligned}\Delta &= 1 - (L_1 + L_2 + L_3) + (L_1L_2) \\ &= 1 + \frac{1}{R_1C_1s} + \frac{1}{R_2C_2s} + \frac{1}{R_1C_2s} + \frac{1}{R_1C_1R_2C_2s^2}\end{aligned}$$

Since all three loops touch the forward path P_1, we remove L_1, L_2, L_3, and L_1L_2 from Δ and evaluate the cofactor Δ_1 as follows:

$$\Delta_1 = 1$$

Thus we obtain the closed-loop transfer function as shown:

$$\begin{aligned}\frac{H(s)}{Q(s)} &= \frac{P_1\,\Delta_1}{\Delta} \\ &= \frac{\dfrac{1}{R_1C_1C_2s^2}}{1 + \dfrac{1}{R_1C_1s} + \dfrac{1}{R_2C_2s} + \dfrac{1}{R_1C_2s} + \dfrac{1}{R_1C_1R_2C_2s^2}} \\ &= \frac{R_2}{R_1C_1R_2C_2s^2 + (R_1C_1 + R_2C_2 + R_2C_1)s + 1}\end{aligned}$$