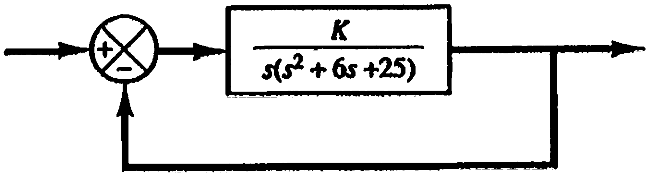
Test 3 自动控制原理

姓名:____________ 学号:____________ 日期:____________

Q1 Root Locus

Find the root locus of the system shown below:



Your Answer

Solution. The open-loop poles are located at $s = 0, s = -3 + j4$, and $s = -3 - j4$. A root locus branch exists on the real axis between the origin and $-\infty$. There are three asymptotes for the root loci. The angles of asymptotes are

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ(2k+1)}{3} = 60^\circ, -60^\circ, 180^\circ$$

Referring to Equation (6–15), the intersection of the asymptotes and the real axis is obtained as

$$\sigma_a = -\frac{0+3+3}{3} = -2$$

Next we check the breakaway and break-in points. For this system we have

$$K = -s(s^2 + 6s + 25)$$

Now we set

$$\frac{dK}{ds} = -(3s^2 + 12s + 25) = 0$$

which yields

$$s = -2 + j2.0817, \qquad s = -2 - j2.0817$$

Notice that at points $s = -2 \pm j2.0817$ the angle condition is not satisfied. Hence, they are neither breakaway nor break-in points. In fact, if we calculate the value of K, we obtain

$$K = -s(s^2 + 6s + 25)\Big|_{s=-2\pm j2.0817} = 34 \pm j18.04$$

(To be an actual breakaway or break-in point, the corresponding value of K must be real and positive.)

The angle of departure from the complex pole in the upper half s plane is

$$\theta = 180^\circ - 126.87^\circ - 90^\circ$$

or

$$\theta = -36.87^\circ$$

The points where root-locus branches cross the imaginary axis may be found by substituting $s = j\omega$ into the characteristic equation and solving the equation for ω and K as follows: Noting that the characteristic equation is

$$s^3 + 6s^2 + 25s + K = 0$$

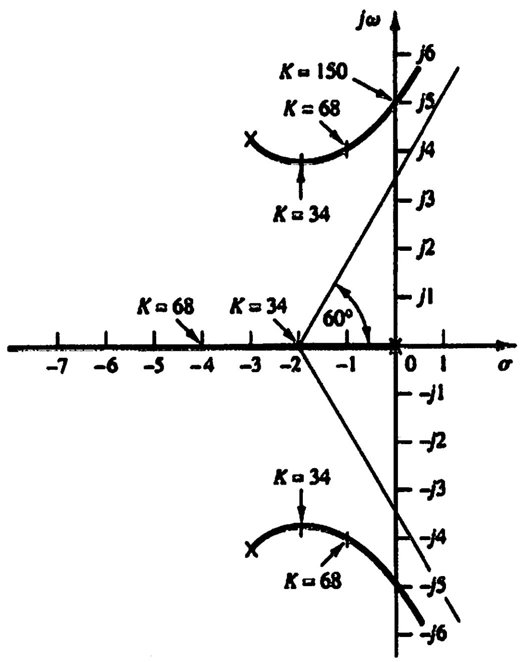
we have

$$(j\omega)^3 + 6(j\omega)^2 + 25(j\omega) + K = (-6\omega^2 + K) + j\omega(25 - \omega^2) = 0$$

which yields

$$\omega = \pm 5, \quad K = 150 \quad \text{or} \quad \omega = 0, \quad K = 0$$

Root-locus branches cross the imaginary axis at $\omega = 5$ and $\omega = -5$. The value of gain K at the crossing points is 150. Also, the root-locus branch on the real axis touches the imaginary axis at $\omega = 0$. Figure 6–44(b) shows a root-locus plot for the system.

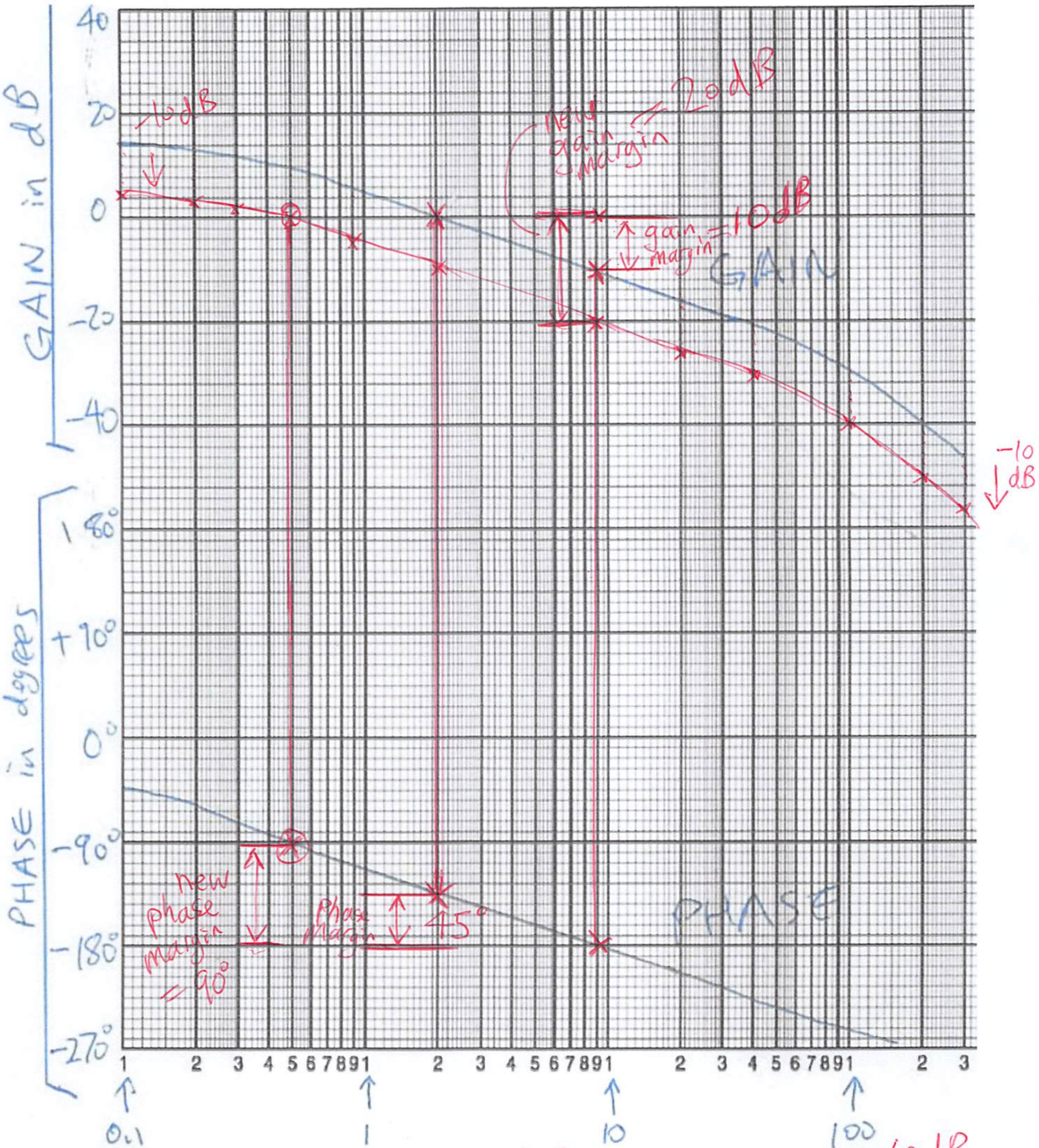


Q2 Bode Plot Design - ANSWER ON THIS GRAPH PAPER

(a) From the Bode plot, find the approximate gain and phase margin.

(b) By changing the overall gain of the system K, increase the gain margin by 10dB more. Sketch the new gain plot. What is the value of the new phase margin?

Your Answer



(a) phase margin = 45°, gain margin = 10dB

(b) new gain margin = 10+10 = 20dB (@ -180°)

new gain plot : red line

new phase margin = 90° (@ 0dB)

Q4 State Space Control

Derive the state space representation of the following function (in controllable or observable form)

$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

Your Answer

B-3-5.

$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

Define

$$x_1 = y$$
$$x_2 = \dot{y}$$
$$x_3 = \ddot{y}$$

Then

$$\dot{x}_3 + 3x_3 + 2x_2 = u$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_1 = x_2$$

or

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q3 PID Control

Briefly explain:

(a) Transient Response Method (Ziegler Nichols Tuning)

(b) Stability Limit Method (Ziegler Nichols Tuning)

Your Answer

Summary: Transient Response Method

1. Measure the open loop step response of the plant
2. Obtain L and R
3. Calculate the P, I, and D values according to the table below.

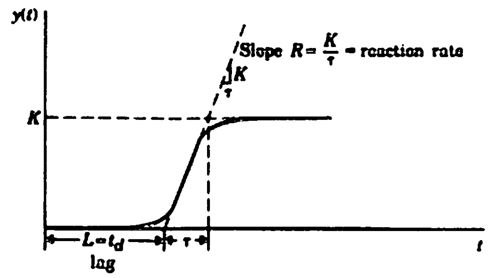


Figure 5.30 Process open-loop step response.

Table 5.2 Ziegler-Nichols tuning parameters using transient response.

	K_p	T_I	T_D
P	$1/RL$		
PI	$0.9/RL$	$3L$	
PID	$1.2/RL$	$2L$	$0.5L$

Summary: Stability Limit Method

1. Switch off I and D, increase P until it oscillates.
2. The P gain (K_u) and the oscillation period (P_u) are recorded
3. Calculate P, I, and D from the table below

Table 5.3 Ziegler-Nichols tuning parameters using stability limit.

	K_p	T_I	T_D
P	$0.5K_u$		
PI	$0.45K_u$	$P_u/1.2$	
PID	$0.6K_u$	$P_u/2$	$P_u/8$