SZU - Test 2 - Automatic Control Theory – 1hr 15 min – (25% each)

Question 1

A mechanical system is shown in Fig Q1 where z(t) is input displacement and ϴ(t) is output angular displacement. Assume that the masses involved are negligibly small and all motions are restricted to be small. Find the transfer function between output and input displacements, with zero initial conditions.

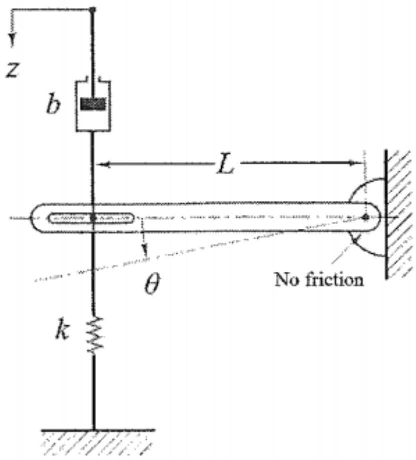


Fig. Q1

Question 2

Use Routh's stability criterion to find the range of value of K for the function below, if it remains to be stable under unity feedback:

$$\frac{C(s)}{R(s)} = \frac{K}{s\left(s^2+s+1\right)\left(s+2\right)+K}$$

Question 3

Sketch the Root Locus of the following transfer function, when K >0.

$$GH = \frac{K}{s(s+1)(s+3)(s+4)}$$

Question 4

Plot the Bode plots (magnitude and phase) of the following functional block, by using the semi-logarithmic graph paper provided.

$$GH(j\omega) = \frac{1+\dfrac{j\omega}{2}}{j\omega\left(1+\dfrac{j\omega}{0.5}\right)\left(1+\dfrac{j\omega}{4}\right)}$$

SOLUTION

Q1

(a) The equation of motion for the system shown in Fig. Q2 is

$$b(\dot{z} - L\dot{\theta}) = kL\theta$$

which can also be written as

$$L\dot{\theta} + \frac{k}{b}L\theta = \dot{z}$$

By taking the Laplace transforms of these two equations with zero initial conditions, we have

$$(Ls + \frac{k}{b}L)\Theta(s) = sZ(s)$$

Hence, the transfer function *is*

$$\frac{\Theta(s)}{Z(s)} = \frac{1}{L}\frac{s}{s+(k/b)}$$

Q2

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2 + s + 1)(s + 2) + K}$$

The characteristic equation is

$$s^4 + 3s^3 + 3s^2 + 2s + K = 0$$

The array of coefficients becomes

$$\begin{array}{llll} s^4 & 1 & 3 & K \\ s^3 & 3 & 2 & 0 \\ s^2 & \frac{7}{3} & K & \\ s^1 & 2 - \frac{9}{7}K & & \\ s^0 & K & & \end{array}$$

For stability, K must be positive, and all coefficients in the first column must be positive. Therefore,

$$\frac{14}{9} > K > 0$$

When $K = \frac{14}{9}$, the system becomes oscillatory and, mathematically, the oscillation is sustained at constant amplitude.

Q3

For this transfer function the center of asymptotes is simply $\sigma_c = -2$; and $n - m = 4$. Therefore for $K > 0$ the asymptotes have angles of 45°, 135°, 225°, and 315°. The real axis sections between 0 and -1 and between -3 and -4 lie on the root-locus for $K > 0$ and it was determined in Problem 13.20 that a breakaway point is located at $\sigma_b = -0.424$. From the symmetry of the pole locations, another breakaway point is located at -3.576. This can be verified by substituting this value into the relation for the breakaway point, Equation (*13.8*). The completed root-locus for $K > 0$ is shown in Fig. 13-31.

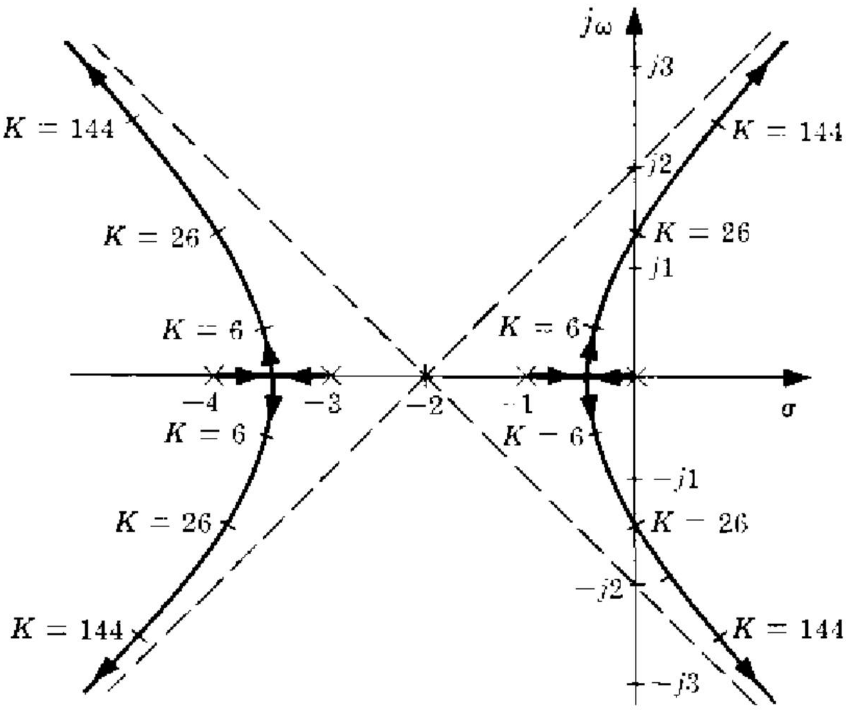
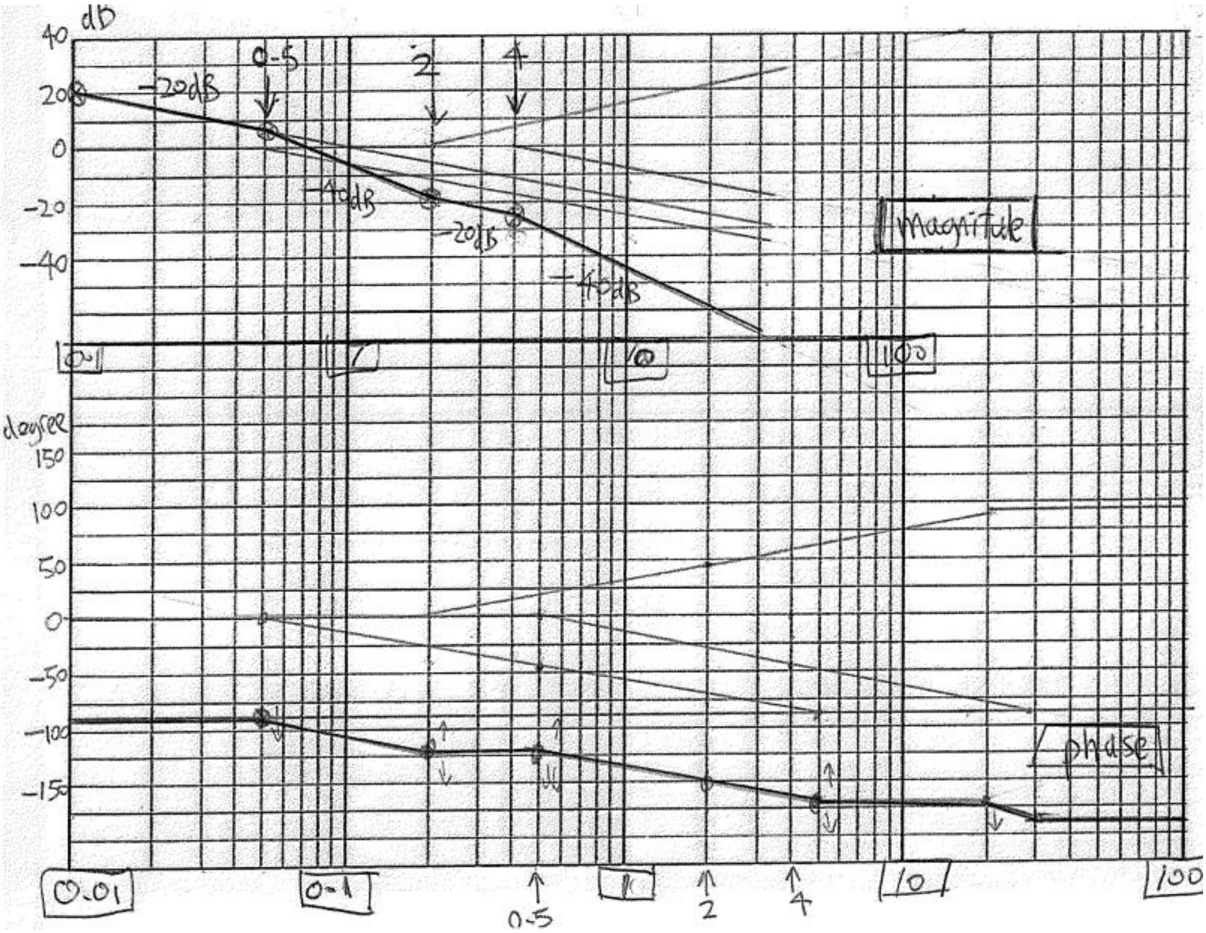


Fig. 13-31

Q4



dB
40
20
0
-20
-40
0.5
2
4
-20dB
-40dB
-20dB
-40dB
Magnitude
0.1
1
10
100
degree
150
100
50
0
-50
-100
-150
phase
0.01
0.1
0.5
1
2
4
10
100